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STABLE OPTICAL FREQUENCY COMB GENERATION AND APPLICATIONS IN ARBITRARY WAVEFORM GENERATION, SIGNAL PROCESSING AND OPTICAL DATA MINING

by

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ABSTRACT

This thesis focuses on the generation and applications of stable optical frequency combs. Optical frequency combs are defined as equally spaced optical frequencies with a fixed phase relation among themselves. The conventional source of optical frequency combs is the optical spectrum of the modelocked lasers. In this work, we investigated alternative methods for optical comb generation, such as dual sine wave phase modulation, which is more practical and cost effective compared to modelocked lasers stabilized to a reference.

Incorporating these comblines, we have generated tunable RF tones using the serrodyne technique. The tuning range was ±1 MHz, limited by the electronic waveform generator, and the RF carrier frequency is limited by the bandwidth of the photodetector. Similarly, using parabolic phase modulation together with time division multiplexing, RF chirp extension has been realized.

Another application of the optical frequency combs studied in this thesis is real time data mining in a bit stream. A novel optoelectronic logic gate has been developed for this application and used to detect an 8 bit long target pattern. Also another approach based on orthogonal Hadamard codes have been proposed and explained in detail.

Also novel intracavity modulation schemes have been investigated and applied for various applications such as a) improving rational harmonic modelocking for repetition rate multiplication and pulse to pulse amplitude equalization, b) frequency skewed pulse generation for ranging and c) intracavity active phase modulation in amplitude modulated modelocked lasers for supermode noise spur suppression and integrated jitter reduction.
The thesis concludes with comments on the future work and next steps to improve some of the results presented in this work.
for Pınar and my family
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CHAPTER 1: INTRODUCTION

1.1. The Need for Stable Combs

The spectrum of a modelocked laser consists of equally spaced phase-locked longitudinal modes [1]. These coherent frequency components can be accessed independently via filtering [2]. Once they are filtered each individual longitudinal mode will behave as a continuous wave (CW) laser, and will constitute a coherent Fourier basis. Since they can be independently phase and intensity modulated and recombined, this basis can be used for arbitrary waveform generation as shown in figure 1.1.

![Diagram of arbitrary waveform generation](image)

Figure 1-1: Proposed scheme for arbitrary waveform generation, HF WDM Demux:

Hyperfine Filter, PM: Phase Modulator, IM: Intensity Modulator

Even without any modulation scheme just by combining two different frequency components a microwave signal at a frequency equal to the difference frequency of the longitudinal modes can be synthesized. This method has been used to create microwave tones at 37.2 GHz, 24.8 GHz, and 12.4 GHz [3]. This concept of creating RF tones by
beating frequency components is becoming more evolved and practical with the realization of low noise modelocked lasers. Ultra stable equally spaced frequency components with a timing jitter around 63 fs (integrated 10 Hz-10 MHz) that have been observed from a semiconductor laser [4] are very suitable for generating high frequency RF tones (~100 GHz), where electronic methods are limited.

Another application of optical frequency combs is calibrating astrophysical spectrographs [5]. Some applications can make use of the fact that the optical frequency comb lines have a fixed phase relationship among themselves. As a result one can encode data through phase modulation on individual comb lines, where each comb line will correspond to a single channel [6],[7], or one can assign a certain code to phases of all comb lines resulting in a noise-like pulse in the time domain, which can be sent to the shared network and only the user with the right “key” can put the phases of the individual comb lines in order to retrieve the pulse and decode the data [8].

Another aspect to consider is the time domain picture of optical frequency combs. As the optical frequency combs are becoming more stable in terms of their optical frequency, phase and amplitude, the optical pulses they generate in the time domain become more quiet (i.e. the phase and amplitude noise of these pulses will reduce.). Optical pulses with low phase and amplitude noise are very useful for analog to digital conversion, where the optical pulses are used to sample and therefore digitize an input analog signal [9].

Due to the wide spectrum of applications of optical frequency combs, it is important to realize low noise, stable optical frequency combs with high reliability and low cost. In the following subsection, we will look at different approaches for modulating the phase
and amplitude of optical signals, which is an important part of optical comb based technologies.

1.2. Phase and Amplitude Modulation

In order to use optical frequency combs effectively for a specific application such as arbitrary waveform generation, coherent communication, etc., the amplitude and/or phase of its components should be modulated. In this subsection we will look at different ways for modulating the phase and amplitude of a given optical frequency comb line.

1.2.1. Phase Modulation

Unlike amplitude of an optical signal, the phase of an optical signal is a relative parameter and it is defined with respect to another optical signal and/or a reference point of time where the phase has a specific value. Another major difference between the phase and amplitude of a signal is that the phase of a signal constantly evolves as the signal propagates, therefore phase modulation is basically based on modulating the optical path length the optical signal travels.

The most commonly used electro-optic (phase and amplitude) modulators are LiNbO$_3$ (lithium niobate) based modulators. Lithium niobate crystal has many interesting properties such as ferroelectricity, piezoelectricity, and pyroelectricity and also it has high electro-optic coefficients [10]. Pockel’s electro-optic effect simply states that crystals which do not have inversion symmetry (such as lithium niobate) will produce birefringence under an electric field [11]. In other words, the refractive index of the lithium niobate crystal along an axis will change when an electric field is applied; this
will result in a change in optical path length and therefore a change in the phase of optical signal traveling through the lithium niobate crystal. This is the basis of phase modulation with lithium niobate crystals.

Another family of modulators is liquid crystal based modulators. Liquid crystals are basically semi-crystals, where the molecules are arranged in a crystal like structure but still the overall material is in liquid form. By applying an electric field one can change the orientation of the liquid crystal molecules and therefore the refractive index in a given axis, since the crystalline molecules are birefringent, they change both the polarization and the phase of the input light depending on their physical orientation. Liquid crystal based phase modulators are usually used in $nxm$ arrays as spatial light modulators, which are very useful in pulse shaping and spectral phase encoding [12].

The mathematical theory of sinusoidal phase modulation is explained in detail in Chapter 2.

1.2.2. Amplitude Modulation

Amplitude modulation is theoretically simpler, because it can be achieved simply by introducing loss to the optical signal. Two main approaches for introducing a controllable, repeatable and stable loss (i.e. amplitude modulation) are interferometric based and absorption based.

Interferometric based amplitude modulators are simple Mach-Zehnder interferometers where one or both arms are phase modulated. As a result one can control the relative phase between two arms of the interferometer and consecutively control the output power of the interferometer (i.e. amplitude modulator). This type of modulators use also lithium niobate crystals.
Absorption based modulators on the other hand either use direct absorption as in the electro-absorption modulator case, or indirect absorption as amplitude modulation via polarization rotation and polarization filtering. Electro-absorption modulators are based on Franz-Keldysh effect [13], which simply states that the optical absorption of a bulk semiconductor material increases when an electrical field is applied to it, or on quantum confined Stark effect [14], which increases the absorption in a quantum well due to the existence of excitons. Amplitude modulation via polarization rotation and polarization filtering can be realized with a lithium niobate crystal. Since the applied electric field creates a birefringence in the lithium niobate crystal. If the input optical field has a linear polarization at a 45 degree angle with respect to principal axis of the lithium niobate crystal, then due to the electrically induced birefringence, its polarization can be rotated depending on the strength of the applied electric field. If there is a polarization filter at the output of the lithium niobate waveguide, this polarization rotation will be translated into a change in the amplitude of the optical signal. The same principle can be applied to liquid crystal based amplitude modulators, which are very practical for spatial light modulators.

1.3. Overview of the Thesis

In this work, we have investigated different methods to generate stable optical frequency combs and novel external modulation schemes of these combs for various applications, as well as intra-cavity modulation schemes. Comb generation by CW light modulation has been analyzed experimentally and theoretically as an alternative to mode-locked lasers for generating stable optical combs in Chapter 2. The external cavity
modulation schemes are used for arbitrary waveform generation. We have achieved tunable single frequency generation by realizing endless phase modulation and also the limitations of the electronic arbitrary waveform generators have been studied by simulations and experiments as well (see Chapter 3). RF chirp extension by incorporating phase modulation schemes with Time Division Multiplexing approach has been analyzed in Chapter 4. The intracavity modulation schemes are used for high repetition rate pulse generation via rational harmonic modelocking, which is explained in Chapter 5. We have realized a repetition rate multiplication factor up to 15 without suffering from pulse-amplitude fluctuations compared to standard approaches. Another novel intracavity modulation scheme is used for generating a frequency skewed pulse train, which is ideal for laser ranging applications. This approach is discussed in Chapter 6. Chapter 7 discusses the use of optical frequency combs for data mining applications. We propose two distinct approaches. The first approach is based on an opto-electronic logic gate which can act on multiple optical input signals simultaneously. The second approach uses orthogonal Hadamard codes for phase encoding of the optical frequency components which are used for pattern recognition and data mining. In Chapter 8, we have analyzed pulse to pulse timing fluctuation reduction by the use of an intracavity phase modulator based on the theory of Haus and Rana. In the final chapter the results and future directions are discussed. The Appendix A includes the Mathematica code used to determine the phase modulation parameters for dual sine phase modulation in order to generate a flat optical spectrum. The Appendix B includes the Mathematica code for serrodyne simulations with the experimental sawtooth waveform from the electronic arbitrary waveform generator used in our experiments.
CHAPTER 2: STABLE OPTICAL FREQUENCY COMBS

2.1. Introduction

Equally spaced, stable optical frequency combs have many applications such as arbitrary waveform generation [15], and spectral-phase-encoded optical code division multiple access [16]. An optical frequency comb is a set of coherent optical waves which have a well defined phase relation among themselves and are equally spaced in frequency domain. The optical frequency of a comb component with the index \( n \) (not the \( n^{th} \) component), can be defined as:

\[
\nu_n = n \cdot \nu_0 + \nu_{ceo},
\]

(2.1)

where, \( \nu_0 \) is the spacing between the neighboring frequency components, and \( \nu_{ceo} \) is the carrier envelope offset frequency defined in the range of \( 0 \leq \nu_{ceo} < \nu_0 \). Based on equation (1), we can mathematically define an optical frequency comb as follows:

\[
E(\omega) = \sum a_n \cdot e^{i\phi_n} \cdot \delta(\omega - n \cdot \omega_0 - \omega_{ceo})
\]

(2.2)

\( E \) is the scalar electric field, \( a_n \) is the amplitude, \( \phi_n \) is the phase of the corresponding comb component. \( \omega_0 \) and \( \omega_{ceo} \) are the optical frequencies \( \nu_0 \) and \( \nu_{ceo} \) in units of radiance respectively.

Since an optical frequency comb is a set of sine waves at different frequencies (with equal separation), they constitute a Fourier basis. With the advances in the filtering technology, one can access each frequency component and modulate them independently. This leads to a wide variety of applications such as in optical communications (especially
in spectrally phase encoded optical code division multiple access networks), frequency
domain based arbitrary waveform generators and microwave photonics.

There are two main ways to generate an optical frequency comb. These are mode-
locked lasers and side band generation based CW modulation. Mode-locked lasers are
well studied and are very efficient in terms of the frequency comb bandwidth. However,
some extra work should be done in order to improve the stability of the frequency comb
both in terms of amplitude stability and frequency stability. On the other hand, CW
modulation based comb sources are more robust, economically inexpensive, and
relatively easier to stabilize than mode-locked lasers. Both of these systems are analyzed
in detail in the following sections.

2.2. Mode-Locked Lasers

An optical ring cavity given at length \( l \), supports many optical modes separated at a
frequency of \( f_c = \frac{c}{n l} \), where \( c \) is the speed of light, \( n \) is the refractive index which is a
function of wavelength and \( f_c \) is called the cavity fundamental frequency. The total
number of allowed modes and their absolute frequency also depend on other cavity
parameters such as gain bandwidth, intracavity dispersion, and spectral filtering. In a
mode-locked laser, an active mechanism (active modelocking), passive mechanism
(passive modelocking), or a combination of both (hybrid modelocking) distributes energy
of each mode to the neighboring modes hence locking the modes. As a result all lasing
modes (i.e. comb components) are coherent and have a well-defined phase relation.
For active modelocking, there are three distinct families based on the relation among the pulse repetition rate \( (f_p) \), the cavity fundamental frequency \( (f_c) \), and the modulation frequency \( (f_{mod}) \). These three families are fundamental modelocking, harmonic modelocking, and rational harmonic modelocking. Fundamental modelocking is where all three of the abovementioned parameters are equal to each other \( (f_p = f_c = f_{mod}) \). If the pulse repetition rate is equal to modulation frequency and an integer multiple of the cavity fundamental \( (f_p = n \cdot f_c = f_{mod}) \), the resulting modelocking is called harmonic modelocking. Rational harmonic modelocking is called when pulse repetition rate is an integer multiple of modulation frequency and modulation frequency is equal to an integer multiple of cavity fundamental plus an offset term \( (f_p = (m \cdot n \pm 1) \cdot f_c = m \cdot f_{mod}) \). Rational harmonic modelocking is investigated in great detail in chapter 5.

Passive modelocking is usually fundamental modelocking, since the intracavity nonlinear effects (Kerr nonlinearity, saturable absorption, nonlinear polarization rotation, etc.) prefer a pulse repetition rate at the cavity fundamental frequency. However, there have been demonstrations of passively harmonic modelocking as well [17].

Mode-locked lasers referenced to an external or internal optical reference can generate optical frequency combs with large bandwidth and high stability. However, these lasers are very demanding and expensive due to the requirements of good thermal and acoustic isolation, a stable optical reference [4], or a carrier envelope offset stabilization scheme [1] according to the type of laser. An alternative method of generating optical frequency combs is by external modulation of continuous wave (CW) light.
2.3. Comb Generation by CW Modulation

CW modulation method is based on sideband generation and is relatively simple and cost effective [18]. Ho and Kahn proposed to integrate an optical loop with CW modulation in order to broaden the resulting spectrum [19]. Bennett et al. experimentally realized this optical comb generator and generated very broad spectra up to 1.8 THz, however, the amplitude variation among the comb lines of the spectra was ~40 dB which limits the use of these comb lines [20]. Other approaches involve using more than one modulator in series or in parallel. Gheorma et al. used an integrated dual-parallel modulator in order to realize a flat optical spectrum [21]. Since this special modulator requires controlling three DC bias inputs in addition to two RF inputs, it needs extra care to avoid bias drift. Also, as the number of modulators involved in the system increases, so does the loss and the complexity of the system. Another important thing to note is that any kind of amplitude modulation will reduce the efficiency of the system, since it is based on modulating the loss rather than frequency of the input light. Sakamoto et al. uses a dual-drive intensity modulator to generate a flat spectrum, and the resulting optical power efficiency is only 1% including insertion loss of the modulator [22].
2.4. Dual Sine Wave Phase Modulation

2.4.1. Introduction

In this section we demonstrate, both theoretically and experimentally, a novel method to generate an ultra-flat optical comb with the use of a single phase modulator driven with two sine waves at different amplitudes and frequencies. Using our method, we have realized an optical frequency comb with 11 comb lines spaced by 3 GHz and better than 2 dB amplitude variation, and also an optical comb consisting of 9 comb lines spaced by 3 GHz and with better than 1 dB flatness. Since the setup has only one optical element (phase modulator) other than the seed laser, our method is extremely simple and inexpensive. The optical power efficiency is also very high due to the nature of phase modulation. The power efficiency of both cases are calculated theoretically and experimentally.

2.4.2. Theory of Dual Sine Wave Phase Modulation

The phase modulated electric field can be rewritten as a sum of its harmonics where the amplitudes are given by the Bessel functions of the first kind of the corresponding order evaluated at the depth of modulation as shown in the equation below [23]:

$$E_{in} e^{i \cdot A \sin(\omega_{mod}t)} = E_{in} \sum_{n=-\infty}^{\infty} J_n(A) \cdot e^{i \cdot n \cdot \omega_{mod}t}$$

(2.3)

In equation (2.3), $A \cdot \sin(\omega_{mod}t)$ is the sinusoidal modulation waveform at frequency $\omega_{mod}$ with a depth of modulation $A$. $E_{in}$ is the scalar input electric field, $J_n$ is the Bessel function of the first kind and on the order of $n$. Equation (2.3) tells us that the amplitude
of the $n$th harmonic is given by $J_n(A)$. The absolute values of amplitudes of the first 3 harmonics as a function of $A$ are plotted in Figure 2.1. in logarithmic scale.

There are several features to note in Fig. 2.1. First, when there is no phase modulation (i.e. $A=0$), all harmonics other than the $0$th harmonic (i.e. the input signal) are zero as expected. Secondly, the amplitude of $n$th harmonic is equal to the amplitude of $-n$th harmonic since the property $|J_n(A)| = |J_{-n}(A)|$ holds. As a result, the spectrum is always symmetric. However the key point to note is that there is no $A$ value where the neighboring harmonics will satisfy a flatness better than 3 dB. Therefore a sinusoidal phase modulation will not result in a flat spectrum independent of parameter $A$.

![Figure 2-1: Amplitudes of the first 3 harmonics as a function of depth of modulation $A$.](image)

It should be noted that using an RF pulsed waveform in amplitude modulation results in a flat optical spectrum. However, the same approach will not work for phase modulation, because the pulsed waveform is flat and its time derivative is zero between the RF pulses. Therefore the signal is unmodulated between the RF pulses and modulated only while the RF pulse is applied to the modulator. As a result, the optical
spectrum will always have a large center tone, making it impossible to obtain a flat spectrum.

However, if one modulates the phase with more than one sine wave, then the resulting optical spectrum can be optimized for flatness by controlling the amplitudes, the frequencies and the relative phases of these sine wave components independently. This optimization can be done with a minimum of two sine waves at different frequencies and different amplitudes. For the general case, we assume that the phase modulator is driven by the sum of two sine waves; \( A \cdot \sin(\omega_{\text{mod}} t + \Delta \phi) + B \cdot \sin(m \omega_{\text{mod}} t) \). \( A \) and \( B \) are the corresponding depths of modulation, \( \Delta \phi \) is the phase difference between the two sine waves and the modulation frequencies are \( \omega_{\text{mod}} \) and \( m \omega_{\text{mod}} \) respectively, where \( m \) is an integer number. In that case, the equation governing the amplitude of harmonics changes as follows:

\[
E_{in} e^{i (A \cdot \sin(\omega_{\text{mod}} t + \Delta \phi) + B \cdot \sin(m \omega_{\text{mod}} t))} = E_{in} \left( \sum_{k=-\infty}^{\infty} J_k(A) \cdot e^{i k (\omega_{\text{mod}} t + \Delta \phi)} \right) \left( \sum_{l=-\infty}^{\infty} J_l(B) \cdot e^{i l m \omega_{\text{mod}} t} \right)
\]

\[
= E_{in} \sum_{n=-\infty}^{\infty} \left( \sum_{h=-\infty}^{\infty} J_h(B) \cdot J_{n-hm}(A) \cdot e^{i (n-hm) \Delta \phi} \right) \cdot e^{i n m \omega_{\text{mod}} t}
\]

As seen in equation (2.4), if one uses two sine waves for phase modulation, the amplitude of the \( n^{\text{th}} \) harmonic becomes an infinite sum which depends on the depths of modulation of both sine waves (\( A \) and \( B \)), relative phase (\( \Delta \phi \)), and the ratio of their frequencies (\( m \)). It should be noted that for \( m=1 \), the equation (2.4) reduces to the case for two cascaded phase modulators. One interesting feature to note is that when \( m \) is an even number, then the amplitude of the \( n^{\text{th}} \) harmonic and \(-n^{\text{th}}\) harmonic are different and
the resulting optical spectrum is asymmetric. This fact may be exploited for some applications.

We have written a mathematical code, which calculates the flatness \( f \) of the target spectrum as a function of parameters \( A \) and \( B \), for given values of \( \Delta \phi \) and \( m \). For each value of \( A \) and \( B \), the code finds the maximum and minimum amplitudes of the comb lines of the resulting spectrum and calculates the difference in amplitude \( \Delta a \) in dB scale. The flatness parameter \( f \) is defined as \( 1/\Delta a \). The code is presented in Appendix A.

![Figure 2-2: Contour plot showing the flatness as a function of parameters \( A \) and \( B \). The peaks correspond to conditions to obtain flat spectrum, when \( m=3 \) and \( \Delta \phi=0 \) (a), and when \( m=3 \) and \( \Delta \phi=\pi/2 \) (b).](image)

Therefore every peak on the contour plots shown in Figure 2.2. correspond to a flat spectrum where the amount of flatness is given by the peak value. Figure 2.2a. shows the contour plot of flatness when \( m=3 \) and \( \Delta \phi = 0 \) for a target spectrum with 11 comb lines. The peak located at \( A=5.13 \) & \( B=2.49 \), corresponds to an amplitude variation of 1.3 dB. Figure 2.2b. shows the flatness, when \( m=3 \) and \( \Delta \phi = \pi/2 \) for a target spectrum
with 9 comb lines. The peak at A=1.43 & B=1.43, corresponds to an amplitude variation of 0.3 dB among the 9 comb lines. The first case has a theoretical power efficiency of 66%, and the second case has a theoretical power efficiency of 87%. The power efficiency is defined by the ratio of the multiplication of the number of usable comb lines with the power of the weakest usable comb line, to the sum of all the comb lines. The reason is to exclude the insertion loss of the phase modulator which is a device specific parameter (which was 3 dB in our experiments) and to allow further flattening by attenuation. Therefore the power efficiency is a device independent parameter.

2.4.3. Experimental Results

In order to experimentally verify our theoretical expectations, we have chosen the modulation frequency $\omega_{\text{mod}}$ to be 3 GHz. We have realized both optimum cases shown on Figs. 2.2a. and 3.2b. The parameters $A$, $B$, $\Delta \phi$, and $m$ are set as 5.13, 2.49, 0, and 3 respectively in order to realize the first case shown in Fig. 2.2a, and for the second case (Fig. 2.2b.) the parameters are set as $A=1.43$, $B=1.43$, $\Delta \phi = \pi/2$, and $m=3$. The experimental setup is shown in Figure 2.3.

$V_\pi$ of the phase modulator used in the experiment is around 3 Volts at DC. In order to determine the actual required RF powers for each case, we have calculated the theoretically expected spectra for each sine wave individually, and optimized the RF power applied to the phase modulator, such that the experimental spectrum is as close as possible to the theoretical spectrum.
Figure 2-3: Experimental Setup: P.M. is the phase modulator, P.S. is the phase shifter to ensure that both sine waves are in phase. P.C. is the polarization controller, CWL is the CW Laser.

Figure 2.4a. shows the experimental and theoretical optical spectra when the phase modulation waveform is 5.12sin(3GHz). Similarly, Figure 2.4b. shows the optical spectrum when the phase modulation waveform is 2.49sin(9GHz). When both of the sine waves are added together and applied to the phase modulator, the resulting optical spectrum is shown in Figure 2.4c. The final spectrum has 11 comb lines, with 1.9 dB flatness and an arbitrary spacing of 3 GHz. This spectrum corresponds to the peak on Fig. 2.2a.
Figure 2-4: a&b: Experimental and theoretical (dotted) optical spectra when the modulation waveform is $5.12 \sin(3 \text{GHz})$ (4a), and when the modulation waveform is $2.49 \sin(9 \text{GHz})$ (4b). c: The final experimental and theoretical optical spectrum generated by dual sine wave as: $5.12 \sin(3 \text{GHz}) + 2.49 \sin(9 \text{GHz})$. There exist 11 comb lines with 1.9 dB flatness and 3 GHz spacing.

The other optimum modulation scheme corresponds to the peak on Fig. 2.4b. The experimental and theoretical optical spectra, when the phase modulation waveform is $1.43 \sin(3 \text{GHz})$, and $1.43 \sin(9 \text{GHz})$ are shown in Fig. 2.5a., and 2.5b. respectively. When both of these sine waves applied together with a phase difference of $\pi/2$, a flat spectrum with 0.8 dB flatness is generated as shown in Fig. 2.5c.
Figure 2-5: Experimental and theoretical (dotted) optical spectra when the modulation waveform is $1.43\sin(3\text{GHz})$ (5a), and when the modulation waveform is $1.43\sin(9\text{GHz})$ (5b). The final experimental and theoretical optical spectrum generated by dual sine wave as: $1.43\sin(3\text{GHz}+\pi/2)+1.43\sin(9\text{GHz})$. There exist 9 comb lines with 0.8 dB flatness and 3 GHz spacing.

The relative phase in both cases is adjusted by optimizing the resulting spectrum. The mismatch between the theoretically expected optical spectrum and the experimental spectrum can be attributed to the bandwidth limitations of the phase modulator used in the experiment, which has a 10 GHz bandwidth, and also to the nonlinearities coming from the high power RF amplifier, which results in an imperfect sine wave for phase modulation.

The experimental power efficiency is defined the same way as the theoretical power efficiency and is found to be 46% for the first case and 79% for the second case.
2.5. Conclusion

The dual sine wave phase modulation approach is extremely simple and robust for generating an ultra-flat, stable, narrow bandwidth optical spectrum with high power efficiency which will be useful for applications in frequency domain arbitrary waveform generation, optical filter testing, microwave photonics, or line by line shaping based systems where a large optical bandwidth is not necessary. The setup does not require a special modulator and can be incorporated with RF frequency multipliers and RF filters, so that it may require only one RF synthesizer instead of two. The stability of the output combs is limited by the stability of the CW laser used in the setup. Using our dual sine wave phase modulation approach we have realized an optical spectrum of 11 comb lines with 1.9 dB flatness and 3 GHz spacing, and also a spectrum of 9 comb lines with 0.8 dB flatness [24].
CHAPTER 3: TUNABLE SINGLE FREQUENCY GENERATION VIA SERRODYNING

3.1. Introduction

By beating the longitudinal frequency components of a mode-locked laser spectrum without introducing any modulation, one can only produce RF tones at discrete frequencies, because the spectrum of the modelocked laser is discrete, i.e. only the frequencies which are integer multiples of the modelocked frequency can be synthesized [3]. To cover a continuous RF spectrum, one should introduce some tunability to the frequency spacing of the longitudinal modes.

A well known method for arbitrarily shifting the frequency of light is the serrodyne technique. Serrodyning is modulating the phase of the light with a sawtooth waveform. This technique is based on the fact that the frequency of any wave is given by the time derivative of its phase. As a result, if one modulates the phase of the light wave such that it linearly increases (or decreases, based on whether an upshift or downshift is desired) then the frequency shifts up (or down) by a fixed amount given by the slope of the linear phase modulation. This linear phase modulation is in fact a sawtooth modulation, since it is not possible to implement an electric waveform which will modulate the phase linearly from 0 phase to infinite phase (i.e. from 0 Volts to infinite Volts). The limitation of serrodyne technique emerges at this point. Because of the finite reset time of the sawtooth modulation, there occurs a discontinuity in the frequency shift. This finite reset time shows itself as a discontinuity in the time domain, and as spurs in the frequency
domain which limit the spur free dynamic range of the signal. Johnson and Cox showed that the spur suppression is inversely proportional to phase reset time [25], i.e. the shorter the reset time the higher the suppression. A more detailed analysis has been done by Laskoskie. He pointed out other key parameters playing a role in the emergence of the serrodyne spurs [26]. However, all these works have been done at serrodyne frequencies on the order of kHz, because of electronic sawtooth waveform generation limitations. To our knowledge, the methods used to improve the serrodyne spur suppression have been through the improvement of the electronics (better sawtooth waveforms with faster reset time) and the optics (controlling the polarization, coherence, etc.) of the experimental setup. Other work has been performed to increase the frequency of the serrodyning by making use of optics. One such method is the Delay Attenuation Coupling Method [27], which can produce a sawtooth waveform at 20 MHz with spur suppression of 24.5 dB. Another method uses spectral modulation of dispersed pulses [28]. This method gives a shift of 1.28 GHz, however the spur free dynamic range is not reported.

None of these aforementioned works address the main problem of serrodyne modulation, which is the phase reset time. To overcome the limitation due to finite phase reset time, we have introduced a time division multiplexing approach to serrodyning. Our method introduces a new modulation scheme which can be incorporated with any of serrodyne applications and improve the spur suppression.
3.2. Serrodyne Theory

A simple serrodyne setup is given in figure 3.1. Laser light is split into two arms, one arm goes through the acousto-optic modulator (AOM), and shifts the light frequency in order to move the microwave carrier frequency from DC to 99 MHz in our case. The other arm is phase modulated (PM) with a sawtooth waveform. If we denote this waveform as $\Phi(t)$, then the detected signal’s phase at the output will be given by the difference between the phases of the beating waves. Its frequency will be equal to the time derivative of this phase and will be given as:

$$f = \frac{w_0 - w_p}{2\pi} + \frac{A}{2\pi} \frac{d\Phi}{dt}$$

(1)

Figure 3-1: Simple Serrodyne Setup

In equation 1, the parameter A can be seen as the peak to peak phase change due to sawtooth modulation. If it is tuned to give a $2\pi$ phase shift (i.e. $A=2\pi$), then the amount of frequency shift due to serrodyning is directly given by the slope of the modulation. Also, $w_p$ and $w_0$ are the frequencies of the input light to the phase modulator and the shifted output light from the AOM.
Equation 1 points out the mathematical reasoning for the existence of the spurs. The main source of spurs in serrodyne modulation is the discontinuity of the time derivative of the phase, i.e. sawtooth waveform. This discontinuity in the time domain and the resulting frequency spurs can be seen in figure 3.2.

Figure 3-2: Time and Frequency Picture of Simple Serrodyne. (a) Phase Discontinuity in time domain, the dotted line represents the sawtooth phase modulation note phase reset corresponds to the phase discontinuity and (b) the resulting RF spectrum, ordinary serrodyne gives 36 dB of spur free dynamic range

3.3. Time Division Multiplexing Approach to Serrodyning

3.3.1. Phase Delay Method

Our approach is based on tailoring the waveform in the time domain via Time Division Multiplexing (TDM). Our idea is to simply use two “parallel” phase modulators, and modulate them with the same sawtooth waveform but with a phase delay
of $\pi$. As a result, while one phase modulator resets its phase, the other modulator still sees linear modulation. Since these two phase modulators are “parallel”, one can turn off the light going to the first phase modulator when the first modulator resets its phase and use the second phase modulator until the second one needs to be reset, then again light will be directed to the first phase modulator. As a result the input light will never interact with a phase modulator while it is resetting its phase. This process of turning on and off the light is termed as toggling. For this purpose we have used an interferometric switch. The interferometric switch we have incorporated into the setup guides the light to only one phase modulator at a time, and turns off the light for the other phase modulator owing to its square wave modulation. As a result the effective system never interacts with phase resetting. This fact is the key to the endless phase modulation. The setup is given in figure 3.3.

![Experimental Setup](image)

Figure 3-3: Experimental Setup

As seen in the above figure the two outputs of the switch are modulated and detected separately, later they are combined in the RF domain.

The AOM is used just to determine the unshifted carrier tone (i.e. the resulting RF tone when phase modulators are turned off). It is modulated at 99 MHz. The ultimate goal in our work is to synthesize high frequency tones with tunability, and for this reason
the later work uses the frequency comb of the aforementioned low noise modelocked laser instead of the AOM. The AOM has been chosen for its simplicity. Also, equation 1 shows that the carrier frequency \( w_0 - w_p \) does not interact with the serrodyning scheme and can be independently chosen.

In figure 3.3., the upper line after the interferometric switch represents Arm A, and the line below Arm A represents Arm B. These arms are phase modulated in the same way, and are optimized to be as similar as they can be, except they are \( \pi \) phase shifted versions of each other. The waveform from arm A after the switch and phase modulation is shown in figure 3.4a. Note that this waveform is the output of one arm only. The other arm will give the time shifted version of what is shown in figure 3.4a since they are \( \pi \) out of phase, and their addition in the RF domain will give us the desired waveform.

Figure 3.4b is given in order to show the difference between the two cases, when the phase reset is absent and when it is present as in the figure 3.4b. The waveform in figure 3.4b is not the actual waveform which is used for signal generation. If one compares figures 3.4a and 3.4b, one will notice that the existence of the phase glitch on figure 3.4b results in 6 dB increase in the serrodyne spurs’ power. Figure 3.4a where phase glitch is completely eliminated gives a suppression of 39 dB while figure 3.4b gives only 33 dB suppression. These two graphs clearly show the immediate effect of avoiding phase reset time on the spur suppression.
There is just one limitation on the square wave - its frequency should be an integer multiple of the serrodyne modulation frequency. The reason is that both serrodyning and toggling should be in phase all the time, and if there is a mismatch in their periodicity, then these two modulation schemes will be in phase only for a given time interval. In our experiments, the toggle frequency is chosen to be 5 MHz, and serrodyne frequency has been chosen to be 1 MHz. This is the highest serrodyne frequency shift with an electronic sawtooth source ever reported.

When we add the two “half” waveforms (as in Fig. 3.4a) in the RF domain the desired waveform is constructed as given in figure 3.5. The resulting output gives us a serrodyne spur suppression of 43 dB, the highest suppression at 1 MHz frequency shift and this
method introduces an extra suppression of 7 dBs compared to ordinary serrodyne result (see Fig. 3.2b.).

Figure 3-5: The desired waveform in time and frequency domain

The drawback of this approach is the frequency spurs due to toggling. We are suppressing the spurs due to serrodyne but in the process we are adding spurs due to toggling. However, it turns out that suppressing the toggle spurs is much simpler than suppressing the serrodyne spurs. We have showed 54 dB suppression of the main toggle signal. Also the toggling frequency can be made larger such that the toggle spurs (at carrier frequency ± toggle frequency) will lie outside the RF region of interest.

By changing the serrodyne frequency arbitrarily one can tune the output microwave tone. The bandwidth where tuning is possible is given by the carrier frequency \((w_0 - w_p)\) ± maximum possible serrodyne frequency. In our experiment, the carrier frequency is chosen to be 99 MHz, and maximum serrodyne frequency is 1 MHz due to electronic limitations. As a result we can create with high suppression all of the frequencies between 98 MHz and 100 MHz, which is the RF band we are interested in. Since the
toggle frequency is 5 times larger than serrodyne frequency, toggle spurs will always lie outside of this range and can easily be filtered out.

### 3.3.2. Time Delay Method

Another method to realize serrodyne spur suppression by Time Division Multiplexing is called the Time-Shift Approach, and is shown in Figure 3.6, and uses only one phase modulator. Two frequency components from the phase locked spectrum are selected; one passes through the phase modulator and its frequency is shifted. Then these two frequencies are coupled in a 2x2 coupler. Arm B of the coupler has an extra 30 meters of fiber resulting in a delay of 150 ns relative to Arm A. The relative time delay splits the time coordinate of the phase discontinuity of Arm A and Arm B (see Figure 3.7a). This allows the light in Arm A to be turned off via an intensity modulator, whenever the signal has a phase discontinuity and employ Arm B instead.

![Experimental setup diagram](image)

**Figure 3-6**: Experimental setup: HF: Hyperfine filter, PM: phase modulator, VD: variable delay, IM: intensity modulator

Similarly, we can use Arm A while Arm B carries the phase discontinuity information. In other words, the phase discontinuity is eliminated by turning on and off
the signal with the two intensity modulators that work $\pi$ out of phase, and generate waveforms as in Figure 3.7b. Then the two waveforms are combined and the desired waveform is generated without a phase discontinuity i.e. no phase reset. This elimination of the phase discontinuity by switching the light can be seen in Figure 3.7.

![Figure 3-7](image)

Figure 3-7: (a) Arm A and Arm B after 150 ns relative delay and (b) waveforms after intensity modulators, note that phase discontinuity has been eliminated.

One thing to note however is the spurs due to intensity modulation of the signal. As discussed before, these can be eliminated both by intensity matching of the arms, and by using a switching rate faster than the serrodyne rate. By increasing the switching rate (i.e. increasing the frequency of the square wave intensity modulation), the corresponding spurs will be pushed further away from the carrier frequency, and they can simply be filtered out. There are two constraints on time division multiplexing, one is that the light must be fully extinguished during phase resetting, and the toggle frequency should be an integer multiple of serrodyne frequency.
Another point to consider is that two “half” waveforms should be phase matched when combined in the RF domain to have a stronger carrier power and suppress the toggle spurs. For these waveforms to be phase matched, it is necessary and sufficient that the time delay between the arms is equal to an integer multiple of the period of the desired signal. In order to realize the required fine-tuning in the time delay between the arms, a commercially available variable delay line has been used; for a 25 GHz signal the period is 1.2 cm, and a maximum delay tunability of 6 mm (half of the period) is enough to satisfy phase matching condition.

In our experiment, the serrodyne frequency was 1 MHz, the switching frequency was 5 MHz, and the frequency difference between optical modes (i.e. carrier frequency) was 25 GHz. An unshifted carrier tone and +1 MHz serrodyne shifted tone are given in Figure 3.8. We have shown 43 dB of spur free dynamic range resulting in 7 dB extra suppression as compared to an ordinary serrodyne scheme, which gives 36 dB of suppression as in Figure 3.2b. We have also shown continuous frequency tunability of the 25 GHz carrier up to +1 MHz offset. In Figure 3.8, only the frequency up-shift is shown, however by changing the slope of the serrodyne modulation from positive to negative, the down-shift frequency can be realized as well.
Figure 3-8: (a) Unshifted carrier, (b) 1 MHz shifted tone and (c) frequency tuning, this is the overlap of 5 graphs all having SFDR larger than 42 dB

It should be noted that the Time-Shift Approach is much more stable as compared to the Phase-Shift Approach, because the temperature fluctuations in the fiber affect the
phase of the optical wave and both Arm A and Arm B of the setup are affected similarly. As a result the combined signal does not suffer from relative phase fluctuations as in the Phase-Shift Approach where two “half” waveforms originate from different phase modulators and have different phase fluctuations.

3.4. Electronic Limitations in Optical Arbitrary Waveform Generation

Both Phase Delay Approach and the Time Delay Approach result in a spur free dynamic range of 43 dB. At this point, we began to analyze the origin of this 43 dB suppression floor and what factor or factors are limiting the spur free dynamic range. The most obvious imperfection in our system results from the finite bandwidth (100 MHz) of the electronic arbitrary function generator (EAFG) used in the experiments (Tektronix Model: AFG 3102). The frequency spectrum of a mathematically perfect sawtooth waveform (0% falltime) together with the spectra of the experimental sawtooth waveforms from a Tektronix EAFG and a Agilent EAFG is shown in Figure 3.9.
Figure 3-9: The RF Spectra of experimental spectra compared with a perfect sawtooth spectrum.

As seen in Fig. 3.9., the spectrum of the Tektronix sawtooth deviates from the ideal case, resulting in an experimental sawtooth with imperfections. Based on the experimental spectrum of the Tektronix EAFG, we have reconstructed the experimental sawtooth waveform in time domain and calculated the best possible serrodyne suppression ratio using this experimental waveform assuming everything else (amplitude modulation, toggling, and stitching of waveforms) is perfect. The serrodyne simulation with the experimental sawtooth waveform is given in Appendix B. According to the simulation, the maximum spur free dynamic range you can achieve with the reconstructed time domain sawtooth signal is only 45 dB. This shows that our method is currently limited by the nonlinearities in the electronic sawtooth signal from the Tektronix arbitrary function generator. Some simulation results are shown in Fig. 3.10.
Fig. 3.10a. Reconstructed sawtooth signal, together with a straight line, to show the nonlinearities of the signal.

Fig. 3.10b. Zoomed in version of Fig. 3.10a. The nonlinearities of the signal are visible.

Fig. 3.10c. Reconstructed sawtooth signal, and the resulting serrodyne signal, the glitch coincides with the falltime of the sawtooth signal.

Fig. 3.10d. Serrodyne signal after toggling and stitching. The signal no more suffers from the glitch due to falltime of the sawtooth signal.

Fig. 3.10e. The frequency spectrum of the final serrodyne signal shown in Fig. 3.10d. The spur free dynamic range is 45 dB.

Figure 3-10: Time and Frequency domain plots of reconstructed waveforms.
3.5. Discussion

A novel modulation technique based on time division multiplexing is demonstrated for shifting the frequency of an optical beat signal without suffering from low SFDR due to phase resetting with two different approaches (Phase Delay and Time Delay). These methods realize an effectively endless linear phase modulation from zero phase to infinite phase.

Our experiments showed that time delay approach is much more stable compared to phase delay method since we are operating in a fiber based medium. With this approach, a 25 GHz signal has been continuously shifted up to 1 MHz offset with 43 dB spur free dynamic range [15]. We have also showed that we are currently limited by the electronic function generators and further improvements in the electronic arbitrary function generators will also improve the spur free dynamic range of our method.

Our method is not system specific and can be incorporated with any experimental setup where serrodyne is used to create an optical beat signal, and improves the spur free dynamic range up to 7 dB, which increases the sensitivity of such systems.
CHAPTER 4: EXTENDED CHIRPED WAVEFORM GENERATION VIA PARABOLIC PHASE MODULATION

4.1. Introduction

In the previous chapter, we have shown how we have added tunability to the generated RF signal by serrodyne phase modulation, and improved its spur free dynamic range by a time division multiplexing technique. In this chapter, we will introduce a novel method to extend the frequency span and frequency sweep time of a chirped RF signal by incorporating parabolic phase modulation with time division multiplexing.

As discussed before, the time derivative of the phase of a signal determines its frequency. Therefore a parabolic phase modulation results in a linear increase (or decrease) in the frequency as shown in figure 4.1. The difference between the maximum and the minimum frequency values is called the frequency span of the chirp signal; the period of parabolic phase modulation is called the frequency sweep time; and the arithmetic average of the maximum and the minimum frequencies is called the center frequency. Chirped RF signals with long frequency sweep times and large frequency spans are desirable for range detection applications, since long sweep times improve the range and large frequency span improves the resolution of the system [29].
4.2. Principle of Operation

Our proposed experimental setup is same as before however the modulation scheme is different. The setup consists of parallel optical channels, where each channel corresponds to one frequency component of a mode-locked spectrum as shown in figure 4.2.

Every channel, except the base channel (Channel 0) is phase modulated with the same parabolic waveform. As a result, each channel has the same frequency span and the same frequency sweep time but different center frequencies. In addition, each channel’s phase modulation is shifted in time by half a period with respect to its adjacent channels. If the frequency span of each channel is equal to twice the channel spacing, then the resulting frequency versus time mapping of each channel will have the shape of a sawtooth (i.e. the time derivative of a periodic parabola) with the maximum and minimum frequencies.
equal to the center frequencies of its upper and lower adjacent channels respectively. The overlap of the frequency versus time mapping of all the channels is shown in figure 4.3.

The intensity modulators in the setup are used to apply the time division multiplexing concept. These modulators turn on and off the channels in such a way that at a given point of time only one channel is on, and all the other channels are off. Only channel 0 is on all the time, and beats with the channel that is in the “on” state, and shifts the chirp from the optical domain to the RF domain. When channel (n-1) is on, it will beat with channel zero resulting in a chirped
RF signal whose frequency is linearly increasing. While the frequency of this signal gets closer to the center frequency of channel (n), the intensity modulator of channel (n-1) will start to turn off, and simultaneously the intensity modulator of channel (n) will turn on. As a result, channel 0 will start to beat with channel (n). The key point to note is that even though channel 0 starts to beat with a different channel, the resulting chirped signal frequency will continue to increase linearly, because during the transition time channel (n) and channel (n-1) carry the same frequency. Similarly, as the frequency of channel (n) gets closer to the center frequency of channel (n+1), channel (n) is turned off and channel (n+1) is turned on. The resulting chirp signal continues to increase. The shadowed region in figure 4.3., shows the channels that are selected by the intensity modulators. As a result the combined chirped waveform has twice the frequency span and twice the sweep time compared to the chirp on a single channel. The working principle of the system in frequency domain is shown in figure 4.3., and the time domain picture is shown in figure 4.4.
Figure 4-4: After beating with the base channel, and intensity modulation, each channel gives parts of the desired chirp signal. When those waveforms add together the extended chirp signal is generated.

As seen in figure 4.4., each channel beats with channel 0 at different times and each of them generates chirped signals that are parts of a larger chirped signal. When these channels are added in time, the desired waveform is generated. It is clear to see that the frequency span, and the frequency sweep time of the resultant wave is limited by the number of channels; i.e. as you add more channels you can increase both the frequency sweep time and frequency span.

4.3. Experiment and Results

In order to demonstrate our approach we constructed a system with three channels (including base channel) as shown in figure 4.5. In these initial experiments, we have used a tunable laser as the light source, therefore all of the channels have the same wavelength at the input. The center frequency of channel 0 is shifted up 100 MHz by an
acousto-optic modulator. The center frequency of channel 1 is up shifted 5 MHz by serrodyne (sawtooth) phase modulation, and similarly the center frequency of channel 2 is down shifted 5 MHz by the same method. Note that the difference between the center frequencies of channel 1 and channel 2 is 10 MHz, as a result the parabolic phase modulation should create a frequency span of 20 MHz (from +10 MHz to -10 MHz). The required frequency span is satisfied by a parabolic phase modulation at the frequency of 5 MHz and a peak to peak voltage corresponding to $V_{\pi}$. After parabolic phase modulation, channel 1’s frequency ranges between -5 MHz and +15 MHz, while the frequency of channel 2 ranges between -15 MHz and +5 MHz, with respect to original optical input frequency. After beating with channel 0, the new frequency range of the upper arm becomes from 95 MHz to 115 MHz, and similarly the lower arm’s frequency range becomes from 85 MHz to 105 MHz. As discussed in the previous section, between these two channels there is a time delay equal to half of the modulation period. The two intensity modulators are modulated with the same square wave but at different DC bias levels, to make sure that they work contrary to each other (i.e. one turns off while the other one is turning on vice versa). The two channels illuminate separate photodetectors and the individual waveforms are added in the RF domain to construct the desired chirped signal. The resultant signal has a span of 20 MHz (from 90 MHz to 110 MHz), and a frequency sweep time of 200 ns (corresponding to 5 MHz modulation).
Figure 4-5: Experimental setup used to demonstrate the feasibility of our scheme. AOM: acousto-optic modulator, PM: phase modulator, IM: intensity modulator.

Although in figure 4.5., there are two phase modulators shown for both channel 1 and channel 2, in these experiments, we have used only one phase modulator for those channels. Since the phase modulation is a linear process, instead of modulating two different waveforms (a sawtooth for frequency shifting, and a parabola for chirp) with two phase modulators, we have used the arithmetic summation of the sawtooth function and the parabolic function to modulate only one phase modulator per channel. In other words, the phase modulators are modulated with skewed parabolas, which both shift the center frequency and chirp the signal.

It should be noted that, the resultant signal has the same frequency span and frequency sweep time as the channels 1 and 2. As it can be seen in figure 4.4., the real extension is due to channel (n+1) and channel (n-1). Channel (n) is present to ensure that the transition from one channel to other is as smooth as possible, without a discontinuity in the time domain. Therefore, one needs a minimum of four channels (including channel 0) in order to observe an extension in frequency span and sweep time. The fact that the neighboring channels overlap in time domain during transition from one channel to the next ensures a smooth transition but the payoff is that we need one more channel to
compensate the sweep time and frequency span we have lost because of this overlap. If one uses a three-channel system, as in our case, one will only shift the center frequency of the chirp signal. As discussed earlier, in our experimental setup the center frequency of the upper arm is 105 MHz, while the lower arm has a center frequency of 95 MHz. However, the resultant signal has a center frequency of 100 MHz, which is in fact arbitrary.

The RF spectrum of the extended chirp signal and the RF spectrum of an ordinary chirp signal with the same properties (frequency span of 20 MHz, frequency sweep time 200 ns, and the center frequency of 100 MHz), which is generated by the conventional methods, are shown together in figure 4.6.

![Figure 4-6: Straight line is the extended chirp spectrum generated with our experimental setup; the dotted line is the ordinary chirp spectrum. The frequency offset is intentionally introduced for visual aid.](image-url)
As seen in figure 4.6., the chirp signal generated by our method agrees very well with an ordinary chirp signal. The amplitude mismatch is less than 1.16 dB up to the third harmonic and less than 2 dB up to the fourth harmonic.

By assuming that all the harmonics of this RF spectrum have the same phase as the harmonics of the RF spectrum of a “perfect” chirp that is generated by a computer simulation, one can reconstruct the time domain picture of our extended chirp signal. The time domain reconstruction is shown in figure 4.7.

![Figure 4-7: Upper graph is the reconstruction of the time domain based on the amplitude of the RF spectrum of extended chirp, lower graph is the computer simulated “perfect” chirp.](image)

On figure 4.7., the lower graph is a “perfect” chirp in the time domain simulated on the computer, while the upper graph is the reconstructed extended chirp based on the experimental amplitude values shown in figure 4.6., assuming the relative phase is identical to the perfect case. As seen in figure 4.7., the amplitude mismatch we have observed results in a small amplitude modulation envelope at low frequency.
4.4. Discussion

We have demonstrated a novel method to extend the frequency span and the frequency sweep time of a chirped signal. Owing to its linear approach, our system is limited by the total number of channels used. Using our approach we have generated a RF chirp signal with the center frequency of 100 MHz, the frequency span 20 MHz, and frequency sweep time 200 ns. The RF spectrum of this signal agrees well with the RF spectrum of a conventional chirped signal with the same properties. Minor amplitude mismatch between two spectra shows itself as a low frequency amplitude modulation on the chirp signal [30].
5.1. Introduction

Short pulses with high repetition rate are critical for applications such as high speed communication [31] and analog to digital sampling [32]. By the use of rational harmonic modelocking, one can generate optical pulses at a repetition rate higher than the bandwidth of the RF synthesizer and/or that of which the modelocker allows [33].

Rational harmonic modelocking is based on the frequency detuning of the modulation frequency with respect to an integer multiple of the cavity fundamental frequency. The relationship between the cavity fundamental frequency and the modulation frequency for rational harmonic modelocking is given as:

\[
f_{\text{mod}} = n \cdot f_c \pm \frac{f_c}{m},
\]

where \( f_{\text{mod}} \) is the modulation frequency, \( f_c \) is the cavity fundamental frequency, and \( n \) is the order of harmonic modelocking. The ratio \( \frac{f_c}{m} \) is the detuning of the modulation frequency, and it will be shown that the parameter \( m \) is the repetition rate multiplication factor. If the parameter \( m \) is an integer number, then the output pulse repetition rate becomes \( m \cdot f_{\text{mod}} \), even though the modulation frequency is \( f_{\text{mod}} \). In other words, depending on the amount of the modulation frequency detuning with respect to the cavity fundamental frequency, different harmonics of the modulation waveform will coincide with the different multiples of the cavity modes as shown in Fig. 5.1. As a result the RF
frequency which is a harmonic of both the cavity and the modulation waveform will be supported by the cavity and will determine the pulse repetition rate. In the example given in Fig. 5.1., normalized frequency values are chosen to be as \( f_c = 6 \), and \( f_{\text{mod}} = 14 \), and from these values the parameters \( n \) and \( m \) are calculated to be 2 and 3, respectively. The resulting pulse repetition rate will be 42, the third harmonic of \( f_{\text{mod}} \).

Using this method Wu and Dutta have realized a \( m \) value of 22, which is the highest repetition rate multiplication factor yet reported to our knowledge [34]. However, as the \( m \) value of the system increases, the pulses start to suffer from large pulse amplitude fluctuations as seen in figure 5.2., which limit the application of these pulses.

![Diagram showing frequency domain picture of rational harmonic modelocking. Solid lines denote the harmonics of the cavity fundamental frequency, and the dashed lines denote the harmonics of the modulation frequency.](image)
Figure 5-2: This figure has been taken from reference [34]. The increase in pulse-amplitude fluctuation is shown as the repetition rate multiplication factor goes from 1 (a), to 3 (b), 12 (c), 15 (d), and 22 (e).

One can easily derive the time domain version of equation 2. The equation showing the time domain constraint for rational harmonic modelocking is given as:

\[ \tau_{\text{mod}} = m \cdot |\tau_c - n \cdot \tau_{\text{mod}}|, \]

where \( \tau_{\text{mod}} \) is the modulation period, \( \tau_c \) is the cavity roundtrip time, and the parameters \( n \) and \( m \) are as previously defined. The difference between the cavity roundtrip time (\( \tau_c \)), and the \( n \) times the multiple modulation period (\( n \cdot \tau_{\text{mod}} \)) is equal to the temporal delay experienced by a pulse after one roundtrip with respect to the modulation window. This relative delay will accumulate for each roundtrip. If \( m \) is an integer number, then after \( m \) roundtrips, the pulse returns to its original position relative to the modulation window,
satisfying the condition given in equation 3. As a result there are \( m \) sets of pulses under the transmission window, resulting in a pulse repetition rate of \( m \cdot f_{\text{mod}} \). The time domain picture of this repetition rate multiplication effect due to modulation frequency detuning is depicted in Fig. 5.3.

![Diagram of pulse repetition rate multiplication](image)

**Figure 5-3:** Time domain picture of conventional rational harmonic modelocking. After each roundtrip the pulse will be shifted to a new location with respect to the modulation window resulting in pulse-amplitude fluctuations.

In Fig. 5.3., the dotted sinusoidal wave denotes the modulation waveform with a period of \( \tau_{\text{mod}} \). A single period of this waveform is defined as the transmission window. The letters \( a, b, c, \) and \( d \) correspond to different transmission values at different time slots (\( a \) corresponds to the maximum, and \( c \) corresponds to the minimum transmission value). The pulse that occupies time slot \( A \), will be located at time slot \( B \) after one cavity roundtrip as shown in Fig. 5.3. Correspondingly, this pulse will be in time slot \( C \) after two roundtrips (not shown in Fig. 5.3.), and will return to the time slot \( a \) relative to the
modulation window at the end of fourth roundtrip. The time domain picture of rational harmonic modelocking clearly explains the nature of pulse amplitude fluctuations observed at higher $m$ values. Since the pulses are redistributed over the non-uniform modulation window, the output pulses will suffer from large amplitude modulation.

Several pulse-amplitude-equalization schemes have been reported. Li et al. have proposed a pulse-amplitude-equalization scheme based on nonlinear polarization rotation, and have reported a harmonic suppression of 30 dB at a repetition rate multiplication factor of 4 [35]. Kim et al. reported a harmonic suppression of 12 dB at a repetition rate multiplication factor of 5 by the use of a dual-drive Mach-Zehnder modulator [36].

In this chapter we propose a practical modulation technique which realizes high repetition rate multiplication with self pulse-amplitude stabilization where the bottleneck of the system is the bandwidth of the RF synthesizer rather than the bandwidth of the modelocker. With our approach we have generated 5 GHz, 10 GHz and 15 GHz pulses with more than 16 dB suppression via adding small frequency detuning to 1 GHz modulation.

**5.2. Theory**

Our method is based on a more uniform modulation waveform so that the delayed pulses see the same transmission throughout the modulation window. Our proposed modulation waveform is a negative impulse function as shown in Fig. 5.4a.
The frequency detuning results in the multiplication of the pulse train with pulse-amplitude-equalization since the modulation is flat with respect to time.

The transmission through the amplitude modulator is turned off with each impulse at the rate of modulation frequency. If one introduces a frequency detuning to this system, then the negative impulse which is imprinted on the light passing through the modulator will be delayed after each roundtrip as in the conventional rational harmonic modelocking. This results in a multiplied rate of the negative impulses, i.e. the zero transmission points, and the optical pulses in the cavity will be generated in between these zero transmission points. Since the modulation envelope is flat, the delayed pulses will not suffer from the amplitude fluctuations even at high repetition rate multiplication factors (Fig. 5.4b). The frequency domain picture of our approach is different than the classical rational harmonic modelocking scheme shown in Fig. 5.1. The main difference
is that the negative impulse waveform is generated by the comb generator and it includes in-phase RF frequency components at multiples of the modulation frequency. As a result, harmonics of the negative impulse modulation are stronger compared to the harmonics of a sinusoidal modulation.

5.3. Experiment

The setup used in the experiment is shown in Fig. 5.5. The laser uses a fiberized semiconductor optical amplifier as the gain medium. The isolator ensures unidirectional operation in the cavity and the filter stabilizes the spectrum. The cavity fundamental frequency is 12 MHz, the modulation frequency is chosen to be 1 GHz, and \( n \) is 84.

![Diagram](image.png)

A 1 GHz sinusoidal signal from the synthesizer is sent to the RF comb generator and is transformed into narrow RF impulses at 1 GHz. The applied D.C. voltage to the optical amplitude modulator reverses these RF impulses and realizes a transmission window as shown in Fig. 5.4a.

Modulated ASE from the gain medium after the first roundtrip and its RF spectrum are shown in Fig. 5.6a. and Fig. 5.6b., respectively. The ringing effect is due to bandwidth limitations in the comb generator and the amplitude modulator. Empirical results show the ringing effect on the modulation helps to suppress unwanted spurs in the RF domain resulting in better pulse-amplitude equalization. By setting the modulation frequency to 1.01164 GHz, such that the frequency detuning is around 2.4 MHz (=12 MHz / 5), we have successfully generated pulse trains at 5 GHz with 17 dB suppression (Fig. 5.6c & 5.6d). When the modulation frequency is set to 1.01046 GHz, where the frequency detuning is 1.2 MHz (=12 MHz / 10), then pulse trains at 10 GHz with 19 dB suppression are generated (Fig. 5.6e & 5.6f). Furthermore, we have generated a 15 GHz pulse train with 16 dB suppression when the modulation frequency was 1.01007 GHz, such that the frequency detuning is 0.8 MHz (=12 MHz / 15) (Fig. 5.6g & 5.6h). For optimum suppression in each case, the RF power and DC bias voltage to the modulator have been modified slightly (± 1 dB for the RF power, ±0.2 Vπ for the DC bias). The reason for some RF peaks being larger than others can be explained by the fact that the modulation waveform is not perfectly flat as seen in Fig. 5.6a. Therefore, the transmission window adds a small non-uniform amplitude modulation to the pulse train which shows itself as the remaining RF spurs.
Figure 5-6: Modulated ASE after the first roundtrip at 1 GHz and its RF spectrum [(a),(b)].

Generated 5 GHz pulse train and its RF spectrum [(c),(d)]. Generated 10 GHz pulse train and corresponding RF spectrum [(e),(f)]. Generated 15 GHz pulse train and its RF spectrum [(g),(h)].
5.4. Discussion

In conclusion, we have proposed and experimentally demonstrated a novel modulation method to improve rational harmonic modelocking in order to generate high repetition rate pulses with high suppression of unwanted amplitude modulation. Our method is applicable to systems where the pulse repetition rate is limited by the bandwidth of the synthesizer rather than the bandwidth of the amplitude modulator. Using this method we have generated a pulse train with a repetition rate up to 15 GHz with 16 dB suppression using a 1 GHz RF source, which is the highest suppression ratio at large multiplication factor to our knowledge [37].
CHAPTER 6: FREQUENCY SKewed OPTICAL PULSES FOR RANGE DETECTION

6.1. Introduction

Optical pulses generated by conventional methods such as Q-Switching [38], fundamental [38] or harmonic modelocking [39], or by direct modulation of CW light [40] are indistinguishable in terms of their frequency components and time domain characteristics. This results in an ambiguity on the range detection in lidar based applications. In this work we have examined different approaches to generate a frequency skewed pulse train where each pulse will have a different optical frequency making them distinguishable by an interferometric measurement. The two main approaches are 1) the external cavity frequency skewed pulse train generation, covered in Section 2, and 2) the intracavity modulation scheme, which is covered in Section 3. Both resulting pulse trains were experimentally verified in a ranging application.

6.2. External Cavity Frequency Skewed Pulse Generation

6.2.1. Direct Modulation

One can generate frequency skewed optical pulses by direct parabolic phase modulation outside the laser cavity. Since the frequency of a signal is defined by the time derivative of its phase, a parabolic phase modulation will result in a linear increase in the optical frequency of the pulse train as shown in Figure 6.1. The main limitation of this modulation scheme is that the amount of pulse to pulse frequency shift is limited by
the depth of phase modulation, the frequency of phase modulation, and also by the number of optical pulses per phase modulation cycle.

Figure 6-1: Solid lines denote the optical pulse intensity. The dashed lines denote the parabolic phase (on the left side) and the linear frequency increase of the optical pulses (on the right).

A parabolic phase modulation at a frequency of $f_m$, and a peak to peak phase change of $\pi$ for a phase modulation signal of $\Phi(t) = 4\pi f_m^2 t^2$ will result in a maximum frequency span of $4f_m$ on the optical pulse train [30]. If the input pulse repetition rate is $f_p$, then the pulse to pulse optical frequency difference will be given by $\frac{4f_m^2}{f_p}$. Due to technical limitations on the frequency and the amplitude of the parabolic phase modulation we can conclude that, direct parabolic phase modulation of an optical pulse train does not result in a large frequency shift when the optical pulse repetition rate is high, and therefore it is not a very efficient modulation scheme.

### 6.2.2. Frequency Skew Loop

Another approach to realize an optical frequency skewed optical pulse train is to use an optical loop as shown in Figure 6.2.
The optical frequency skew loop works very similarly to a pulse repetition rate multiplier. The input pulse splits in two by the 3 dB coupler. Half of the pulse directly goes to the output port, and the other half enters the shifting loop and is delayed in time by the round trip time of the loop. If the round trip time is equal to an integer multiple of the period of the input pulse train, then there will not be a multiplication in the repetition rate of the pulse train. However, if there is a difference between the roundtrip time and the input pulse period, then the delayed pulse will re-enter the pulse train in a different time slot than its original slot resulting in an increase in the pulse repetition rate as shown in Figure 6.3. In this figure $\tau_L$ represents the roundtrip time of the loop, $\tau_P$ represents the period of the input pulse train, and $\tau_D$ is the difference between $\tau_L$ and $\tau_P$, i.e. the detuning in the time domain.
Figure 6.3: Input pulses are separated by $\tau_p$, while the output pulses are separated by $\tau_D$, resulting in a multiplication of the pulse repetition rate.

If the period of the input pulse train ($\tau_p$) is an integer ($n$) multiple of the detuning ($\tau_D$) then the repetition rate of the output pulse train is multiplied by $n$, very similar to rational harmonic mode-locking [37]. After each roundtrip, all pulses acquire an additional delay of $\tau_D$. As a result, the relative position of a pulse is directly related to how many times the pulse has circulated through the loop. The phase modulator inside the loop shifts the frequency of the input pulses by a method called serrodyning [15],[25],[26],[27],[28], which is linear phase modulation by a sawtooth waveform. As a result, each time a pulse passes through the loop its frequency is shifted. Consequently, the output of the setup consists of a pulse train where each pulse has a shifted frequency from its neighbor and therefore is distinguishable from other pulses by an interferometric measurement. As seen in Figure 6.3, the pulse that is labeled as “a” and located at time slot “0” will be delayed by $\tau_L$, and re-enter the pulse train at time slot “1”. Its frequency will also be shifted by the phase modulator. To denote this frequency shift, its label is now “aI”. After the second roundtrip inside the loop, it will be delayed again and occupy slot “2”. Because its frequency is shifted once more, its label changes to “aII” to denote
that its frequency is shifted twice. In other words, the pulses at time slot “0” in Figure 6.3. are the pulses which have not entered the loop. The pulses at time slot “1” are the delayed versions of pulse “0”s, and therefore have circulated the loop once and their frequency is shifted once. Similarly, the pulses at time slot “2” are the delayed versions of pulse “1”s, and have circulated the loop twice, resulting in a frequency shift which is two times higher than pulse “1” frequency shift, etc. Since a given frequency shifted pulse is not adjacent to its unshifted “source pulse” (i.e. “aI”, “aII”, “aIII”, etc. are not adjacent to each other), the input pulses should be coherent (i.e. the pulses “a”, “b”, and “c” should be coherent), if one wants to use this output pulse train for ranging applications. This makes the use of a harmonically mode-locked laser ineffective as a pulse source, because the adjacent pulses in a harmonically modelocked laser are not coherent. However, every $n^{th}$ pulse is coherent where $n$ correspond to the order of modelocking [41].

The erbium doped fiber amplifier is introduced to compensate the losses inside the loop, and is biased just below the threshold in order to prevent the loop from lasing. As a result the loss of the loop is slightly bigger than the gain. As the pulses circulate inside the loop the loss will accumulate and result in a sawtooth like pulse train.
Figure 6-4: (a): The input pulse train at 2 GHz repetition rate; (b): The output pulse train at 10 GHz repetition rate.

The fundamental frequency of the loop is 11.979 MHz, corresponding to a time delay of 83.48 ns. The input pulse repetition rate is tuned slightly around 2 GHz, in order to realize a 5 fold multiplication in the output repetition rate resulting in a 10 GHz pulse train. The experimental input and output pulse trains are shown in Figure 6.4.

The phase modulator in the loop is serrodyne at a 1 MHz rate, resulting in a 1 MHz frequency shift in each pulse with respect to its neighboring pulses. However, the pulses are not attenuated completely after 4 roundtrips; they continue to circulate the loop until they share the same time slot with another pulse. As a result, in the time slot “0”, there coexist two pulses, one pulse with a frequency which is not shifted and another pulse with a frequency that is shifted 5 MHz. Similarly, at time slot “1”, there coexist two pulses (a 1 MHz shifted pulse and a 6 MHz shifted pulse). Due to the temporal overlap of two coherent pulses whose frequencies are separated by 5 MHz, the output pulse train always results in a 5 MHz beat tone and the interference effects makes the system very susceptible to small thermal and vibrational fluctuations in the environment.
6.2.3. Ranging Application

We have sent the frequency skewed pulse train to a Michelson interferometer where one arm represents the target as shown in Figure 6.5. The pulse train is split in two. One half reflects from the reference mirror and the other half reflects from the target mirror. Both reflected optical waves are recombined and the photodetected signal is analyzed by a RF spectrum analyzer.

![Diagram of range measurement setup]

**Figure 6-5:** Range measurement setup. Depending on the target position different pulses beat at the photodetector.

Since each pulse has a different optical frequency, depending on the target position different pulses will beat and generate a range dependent beat signal. As a result we can determine the target position as a function of the beat frequency. The RF spectra at different delays of the target are shown in Figure 6.6.
Figure 6-6: Resulting RF spectra for various target positions.

As seen in the above graphs, if the relative delay between the target arm and the reference arm is 1 pulse, then the 1 MHz offset tone is stronger. Similarly when the relative delay is 2 pulses, then the 2 MHz tone is enhanced. However, there are a couple of important points to note. First is that the 5 MHz offset tone is always present, even if one examines the single arm of the interferometer where there is no beat with the target signal. As discussed before, this 5 MHz offset tone originates from the fact that 5 times shifted pulses overlap with the unshifted pulses, and 6 times shifted pulses overlap with the 1 time shifted pulses, etc. Another important point to note is the instability of the system. The RF spectrum for 3 pulse delay shows how fluctuations affect the system performance. Finally, one should note that 1 pulse delay and 4 pulse delay give similar results, just like 5 pulse delay, 0 pulse delay and single arm give similar results. The reason for this is that all of the optical pulses are coherent and mathematically a +1 pulse delay is identical to a -4 pulse delay. Similarly 0 pulse delay is identical to +5 and -5
pulse delays. This fact limits the total measurable range of the system. Due to all of these factors, the frequency shifted pulses that are generated by the external frequency shifting loop are not fully optimized for ranging applications. Another method to realize frequency shifted pulses is the intracavity frequency shifted pulse generation.

**6.3. Intracavity Pulse Frequency Shifting**

Intracavity pulse frequency shifting has a similar approach to external cavity frequency shifting loop. This method consists of a composite cavity as shown in Figure 6.7, i.e. a cavity with a Mach-Zehnder type interferometer [42]. This structure also multiplies the repetition rate of the optical pulses by “split, delay and recombine” method.

![Figure 6-7: Setup of fiberized semiconductor optical amplifier ring laser with a composite cavity.](image)

The laser has two modulators, one amplitude modulator (AM) and one phase modulator (PM), which are located at each arm of the intracavity interferometer, and work in parallel. The two arms of the interferometer also define two distinct cavities. First cavity includes the amplitude modulator (AM cavity), while the second cavity includes the phase modulator (PM cavity). PM cavity has a variable attenuator which keeps the PM cavity below threshold and prevents it from lasing. On the other hand the AM cavity is above threshold and produces pulses via harmonic mode-locking. The mechanism of pulse repetition rate multiplication and pulse frequency shifting is detailed in Figure 6.8.

![Image of laser modulators and cavities](image)

Figure 6-8: Thick-line pulse enters the composite structure and splits in two. Phase modulator (P.M.) shifts its frequency by serrodyning. The thin-line pulse denotes the frequency shifted pulse. After phase modulation the thin-line pulse is delayed in time and recombined at the output of the composite structure.

When a pulse enters the intracavity interferometer, it splits in two. Half of the pulse passes through the amplitude modulator. The other half passes through the phase modulator and the variable delay. The phase modulator is serrodyned as in the previous
case and therefore shifts the frequency of the pulse. The variable delay following the phase modulator delays the pulse in time with respect to its other half that is passing through the amplitude modulator. At the output of the intracavity interferometer, half of the original input pulse is recombined with the time delayed and frequency shifted version of itself. As a result, the intracavity interferometer generates a coherent copy of the input pulse at a different time slot and at a different frequency. If the amount of frequency shift introduced by the phase modulator is equal to the AM cavity fundamental frequency, then the frequency shifted and time delayed pulses will be supported by the AM cavity as well.

When one disconnects the PM arm of the interferometer, the output of the laser consists of a pulse train at 5 GHz repetition as seen on Figure 6.9a. On the other hand, when the AM arm is disconnected, the cavity does not lase, since PM arm is kept below threshold. However, when both arms are connected, pulse repetition rate multiplication occurs and the output pulse train becomes 20 GHz because the absolute time delay between the two arms is 50 ps as seen in Figure 6.9b. The AM cavity fundamental frequency is 11 MHz; therefore the serrodyning frequency is chosen to be 11 MHz as well.

There are two major differences between the pulse trains generated by this method and the previous method. The first major difference is that a frequency shifted pulse is adjacent to its one less shifted coherent version unlike the previous method. In other words, the pulses “a”, “aI”, “aII”, “aIII” are adjacent to each other as seen in Figure 6.9. The second difference is that the pulse groups a, b, c, etc. are not coherent, therefore there is no cross beat terms among the pulse groups as it was the case in the previous
This fact improves the performance of the system in terms of both stability and the range resolution.

Figure 6-9: (a) Output pulses from AM only cavity. Pulses a, b, and c are not coherent. (b) Output pulses when the composite cavity is active. The pulse repetition rate is multiplied and each pulse group is coherent among its members.

6.3.1. Ranging Application

The input and output pulse train of the laser is sent to the Mach-Zehnder interferometer shown in Figure 6.5. The target has been delayed with respect to the reference mirror. The resulting RF spectra for a given target position are given in Figure 6.10.
As seen on the above graphs, this scheme does not result in a constant beat tone at the repetition multiplication rate. The pulse to pulse frequency difference is 11 MHz due to the fundamental frequency of the AM cavity. As a result the beat tone is at 11 MHz offset at 1 pulse delay, and 22 MHz offset at 2 pulse delay. One important thing to note is the result at 4 pulse delay. Even though the 44 MHz shifted pulse shares the same space with an unshifted pulse (i.e. “a<sup>IV</sup>” and “b”, or “b<sup>IV</sup>” and “c”), one can still observe the 44 MHz tone owing to the fact that each pulse group is coherent among its members (i.e. “a”, “a<sup>I</sup>”, “a<sup>II</sup>”, “a<sup>III</sup>”, “a<sup>IV</sup>”, etc. are coherent) and incoherent among the other pulse groups (i.e. “a”, “b”, “c”, etc. are not coherent). Since the fundamental frequency of the laser is 11 MHz, each pulse group has a coherent copy every 27 meters in free space. This allows us to realize high resolution ranging at a distance.
6.4. Discussion

Two distinct approaches have been realized and compared for ranging applications. The intracavity approach shows better performance in many respects, such as stability, and range ambiguity. Using the composite cavity structure we have generated a 20 GHz pulse train by 5 GHz amplitude modulation. The frequency of each pulse is 11 MHz shifted compared to its neighboring pulse. The resulting frequency skewed pulse train is used for ranging applications and the target position is measured based on the RF frequency beat tone between the interference of the reference signal and the target signal. Our results suggest a method for high resolution ranging at a distance [43].
CHAPTER 7: REAL TIME DATA MINING IN A BIT STREAM BY THE USE OF OPTICAL FREQUENCY COMBS

7.1. Introduction

As the data rates and the amount of data stored and/or transferred increases, data mining becomes more and more important. Data mining refers to locating a certain bit sequence (target) inside a stored or streaming data. In this chapter, we propose two different approaches to data mining using optical frequency combs. The first approach uses an optoelectronic logic gate which can act on many optical input signals simultaneously. The second approach uses orthogonal Hadamard codes to encode the phase of the optical frequency components and detects the target via coherent detection method.

7.2. Optoelectronic Logic Gate Approach

7.2.1. Introduction

Boolean exclusive OR (XOR) and exclusive NOR (XNOR) logic gates are very useful in applications such as label switching, parity checking, and pattern recognition [44]. Previous attempts on building all-optical logic gates make use of nonlinear loop mirrors [45] or nonlinearities in semiconductor optical amplifiers [44],[46],[47],[48]. All optical XOR gate based on nonlinear loop mirror suffers from low power efficiency [45], and the approaches based on semiconductor nonlinearities suffer from degradation in signal quality if the input signal has on-off keying (OOK) format, therefore they require alternative data formats [44]. Another optoelectronic approach is based on an
optoelectronic bistable switch consisting of a phototransistor and a LED [49]. However, all of these applications are wavelength dependent and cannot process multiple signals at different wavelengths at the same time. In this paper, we propose an optoelectronic logic gate that is wavelength independent. As a result, our proposed approach can act on many input signals simultaneously making use of the large bandwidth optical domain offers. We have experimentally built the setup and demonstrated target bit data mining in real time streaming input data, assuming that we have no prior knowledge about the data bit stream, except the data rate.

In the following subsection, we will introduce the optoelectronic logic gate, and its working principle. Section 8.2.3 explains the experimental demonstration of 8 bit target pattern extraction using the proposed logic gate. In section 8.2.4, we propose an improved setup of the logic gate which performs real time data mining in a bit stream, and experimentally demonstrate a system that is capable of 2 bit long pattern extraction. The final subsection summarizes and concludes the section 8.2.

### 7.2.2. Optoelectronic Logic Gate

The proposed optoelectronic logic gate consists of three 1x2 (one input, two output) interferometric switches (IS) as seen in Figure 7.1. If a voltage equal to $V_\pi$ is applied to an IS, all of the input light is directed to output port 1 (shown with bold line in Fig. 7.1.); similarly when the applied voltage is equal to $V_{2\pi}$, then all of the input light is directed to output port 2 (shown with double line in Fig. 7.1.). An electrical 0 bit is defined as a voltage value of $V_{2\pi}$ resulting in an optical 0 bit (i.e. minimum optical power) from output port 1. Similarly, an electrical 1 bit is defined as a voltage value of $V_\pi$ resulting in
an optical 1 bit (i.e. maximum optical power) from output port 1. The reader should note that, when there is a non-zero optical input to the interferometric switch, then the output from ports 1 and 2 of the IS are complementary of each other independent of the applied electrical bit.

The logic gate can be divided into two parts; first part consists of a single interferometric switch for single bit operation, referred to as the “data imprint stage”, since the electrical input data is imprinted onto optical signal. The second part consists of two interferometric switches in parallel and can be called the “comparator stage”, since in this stage the input data on the optical domain is compared with the target data on the electrical domain.

When an electrical data bit $x$, where $x$ is either 0 or 1, is applied to the first interferometric switch (IS-1), the output of the port 1 of IS-1 is given by $x$ and the output of port 2 is given by the logical negation of $x$ denoted as $\bar{x}$, assuming the optical input to the interferometric switch is an optical 1, as seen in Fig. 7.1.
Figure 7-1: Optoelectronic Logic Gate. Electrical bits are shown in red, optical bits are shown in black.

The second interferometric switch (IS-2) is driven electrically by the target bit $y$, and its optical input is the output port 1 of the IS-1. Similarly, the third interferometric switch (IS-3) is driven electrically by $\bar{y}$, negation of target bit $y$ and its optical input is the output port 2 of IS-1. In a real experiment, both IS-2 and IS-3 can be driven by the same electrical waveform but biased at different DC levels resulting in inverse modulations. Output ports 1 of both IS-2 and IS-3 are combined and is defined as output A of the system, correspondingly each output port 2 of both IS-2 and IS-3 are combined and is defined as output B of the system. As seen in Figure 7.1, when the input data bit is $x$ and the target bit is $y$, then the output A is given by $x \cdot y + \bar{x} \cdot \bar{y}$, and at the same time the output B is given by $x \cdot \bar{y} + \bar{x} \cdot y$. Figure 7.2 shows the truth table of outputs A and B. As seen in Figure 7.2, if $x=y$ ($x\neq y$), then the output A is 1 (0), which is identical to an exclusive NOR (XNOR) gate. Correspondingly, if $x\neq y$ ($x=y$), then the output B is 1 (0), which is identical to an exclusive OR (XOR) gate.
In order to use this optoelectronic logic gate for bit pattern extraction, one can monitor only the output A or only output B. Monitoring both outputs simultaneously will improve SNR of the system and reduce the probability of false positives. If one decides to use only output A, then IS-2 and IS-3 can be replaced with simple amplitude modulators without loss of generality, since the output of port 1 of any interferometric switch is identical to the output of an amplitude modulator under the same conditions. In other words, the “comparator stage” can consist of two amplitude modulators, resulting in an XNOR gate.

To detect an $n$ bit long target pattern inside the streaming input data, all bits of the target signal from $y_1$ to $y_n$ should match to input data bits $x_{m+1}$ to $x_{m+n}$ one by one resulting in $n$ consecutive “1” bits synchronized with the target signal at the output of A which will be counted electronically, confirming both the existence and the location of the target pattern in the input data stream. In other words, the comparator stage will be driven periodically by the target waveform consisting of bits from $y_1$ to $y_n$, and if the

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>A</th>
<th>B</th>
</tr>
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<tbody>
<tr>
<td>$(1,1)$</td>
<td>$x \cdot y + \bar{x} \cdot \bar{y}$</td>
<td>$x \cdot \bar{y} + \bar{x} \cdot y$</td>
</tr>
<tr>
<td>$(0,0)$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$(1,0)$</td>
<td>0</td>
<td>1</td>
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<td>$(0,1)$</td>
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<td>1</td>
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**Figure 7-2:** Truth table of outputs A and B.
target pattern matches to the input data stream then the output of the system will consist of \( n \) consecutive “1” bits. However, it is clear that with this approach the relative timing between the input data signal and the target pattern is vital. If \( y_1 \) does not overlap with \( x_{m+1} \) in time, the target pattern will not be detected. In order to ensure the timing between the input data and the target signal is correct, one must launch the input data to the comparator stage in all the possible relative timings.

### 7.2.3. Eight Bit Target Pattern Extraction

To experimentally demonstrate our method, we have built the setup as shown in Fig. 7.3. The setup acts as an XNOR gate using only the output A, and two interferometric switches have been replaced with amplitude modulators. The input streaming data used in the experiment is “01000110101001001001010100111101001100” which is the 40 bit long word “CREOL” written in binary code. The 8 bit long target pattern we have chosen was the letter “R” in binary, which is “01010010”. Since the target pattern is 8 bits long, the setup has to check all 8 possible relative timings between the input data and the target pattern consecutively.
Eight delayed versions of the input signal have been generated manually and sent to the comparator stage consecutively. For each delayed input, the output of the system has been observed with an oscilloscope used as the electronic counter. As seen in Fig. 7.3, only one of the delayed channels results in 8 consecutive “1” bits synchronized with the target pattern at the output which confirms the existence and the location of the letter R in the input data stream. The output of the system that detected the target pattern is shown in Fig. 7.4. As expected, all 8 bits of the target pattern match one by one to the corresponding bits in the input data stream. The on/off ratio of the resulting system was 11 dB. The data rate used in the experiment was 4 Mb/s and is limited by the electronic data generators used in the experiment.
Figure 7-4: Output of the system for 8 bit target pattern extraction. In the time slot, between 2 μs and 4 μs, there exist 8 consecutive “1” bits.

The setup depicted in Figure 7.3. can successfully detect any target pattern assuming the input data is recorded and can be sent to the setup consecutively with controlled delay. However, the proposed setup can be modified so that it can act on real time streaming input data instead of prerecorded data.

Because the proposed logic gate is independent of the input wavelength, one can send all the relative timing signals to the “comparator stage” simultaneously using a different wavelength for each channel. This is the main advantage of our logic gate compared to other approaches (electrical or all-optical), and also the reason that the setup can work for n bit pattern extraction for real time streaming data. For detecting an n bit long target pattern, there must be n optical input data channels where each one is time delayed by one bit with respect to its neighboring channels.
7.2.4. Bit Target Pattern Extraction in Streaming Data

Figure 7.5 shows the proposed logic gate that has been optimized for 8 bit long target pattern extraction. As discussed above, the 8 bit setup has 8 separate wavelength channels each modulated by corresponding interferometric switches. The wavelengths have been combined by the use of virtually imaged phased arrays (VIPA) [50]. As shown in Fig. 7.5, each wavelength carries the same input data but with an additional 1 bit time delay with respect to its neighboring channels. The required 1 bit time delay between the channels can be realized either in the optical domain (by delaying the modulated optical signals), or in the electrical domain (by driving the interferometric switches with delayed electrical waveforms). In our experiments, we have delayed the signals in the electrical domain. As a result, the input data stream in all the possible relative timings with respect to the target waveform enter to the “comparator stage” together, and all of them are checked for the target pattern simultaneously. At the output of the “comparator stage” wavelength channels are separated with a VIPA and sent to corresponding electronic counters, which will check for 8 consecutive “1”s synchronized with the target signal.
In order to demonstrate that the comparator stage can process many input signals simultaneously, we have built a 2 channel version of the proposed setup for detection of a 2 bit long target pattern as seen in Fig. 7.6. The main difference between the proposed setup and the experimental setup is that, the experimental setup uses regular fiberized couplers to combine the modulated channels instead of virtually imaged phased arrays. In this experiment we have used two wavelengths, $\lambda_1$ and $\lambda_2$, spaced by ~24 GHz. The 2 bit long target pattern chosen was “10”, and the periodic input data is chosen to be “11100” at 50 Mb/s. The two wavelengths have been spatially separated by the use of a virtually imaged phased array and the input data is imprinted on $\lambda_2$ and its 1 bit delayed version is imprinted on $\lambda_1$. 

![Figure 7-5: Proposed setup for 8 bit target pattern extraction.](image)
Figure 7-6: Experimental setup for simultaneous 2 bit target pattern extraction. O.C.S.: Optical Comb Source, VIPA: Virtually Imaged Phased Array, IS: Interferometric Switch, AM: Amplitude Modulator. The data plots in the figure are experimental data.

After data imprinting, two wavelengths have been combined with a simple fiberized coupler. The “comparator stage” processes both wavelengths (i.e. both input signals) simultaneously. After the comparator stage, the wavelengths are separated again with a VIPA and detected simultaneously with a two channel oscilloscope used as the electronic counter. As seen in Figure 7-7, the channel $\lambda_2$ has two consecutive “1” bits, while the other channel does not. The experimental setup has successfully detected and located the 2 bit long target pattern inside the real time streaming input data.
Figure 7-7: Output of the system for real time 2 bit pattern extraction. In the time slot, between 40 ns and 80 ns, there exist 2 consecutive “1” bits.

One important point about evaluating the output of the proposed setup is that having \( n \) consecutive “1” bits does not necessarily imply the existence of the \( n \) bit long target pattern. Since the target waveform is periodic, if the input data stream includes any cyclic permutation of the target waveform, it will also result in \( n \) consecutive “1” bits in one of the channels. Therefore the necessary and sufficient condition for confirming the existence of the target pattern inside the input data is having \( n \) consecutive “1” bits where the first “1” bit overlaps in time with the first bit of the target waveform \( y_1 \). In other words, the electronic counter should always start counting at the beginning of the target waveform.
The speed of the proposed setup is limited by the bandwidth of the interferometric switches and the separation of wavelength channels. In our experiments, the speed limitation was due to the limited bandwidth of the electronic waveform generators. With our approach data mining at 10’s of GHz rates should be possible using commercially available components. Another important point to note is that, the actual data processing speed of the optoelectronic logic gate will also linearly increase with the number of channels used in the system, since the logic gate operates on $n$ bits from different channels simultaneously.

The setup has been built by fiberized components; however it is possible to implement the same layout on the chip scale for improved channel number and stability.

7.2.5. Conclusion

We have theoretically and experimentally demonstrated an optoelectronic XNOR logic gate for target bit pattern extraction and data mining. The proposed setup is very simple and effective; it has successfully located and detected 8 bit and 2 bit long target patterns inside a prerecorded and a real time streaming input data respectively without requiring any prior knowledge about the input data stream other than the data rate.

7.3. Data Mining with Orthogonal Hadamard Codes

7.3.1. Introduction

Hadamard matrices are solutions to Hadamard’s Maximum Determinant Problem [51]. The problem simply asks what $n \times n$ matrix or matrices have the maximum absolute
value determinant with elements $|a_{ij}| \leq 1$. It has been shown that all Hadamard matrices satisfy the following equation [52]:

$$H \cdot H^T = n \cdot I_n,$$  \hspace{1cm} (7.1)

where $H$ is the $nxn$ Hadamard matrix, $H^T$ is its transpose and $I_n$ is the $nxn$ unity matrix. In 1867 Sylvester showed that assuming $H$ is a $nxn$ Hadamard matrix, then the $2nx2n$ matrix $G$ will also be a Hadamard matrix if $G$ is constructed as follows [53]:

$$G = \begin{bmatrix} H & H \\ H & -H \end{bmatrix}$$  \hspace{1cm} (7.2)

Starting from the fact that the $1x1$ unity matrix $I_1$ satisfies the equation (7.1) and therefore is a Hadamard matrix, one can construct the $2x2$ Hadamard matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ using equation (7.2). Similarly the $4x4$ Hadamard matrix constructed by Sylvester’s equation is given by: $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$. Hadamard matrices can also be shown visually as in the following figure. Black squares are 1’s, and white squares are -1’s [54]. This method of demonstrating Hadamard matrices is very useful for large values of $n$. 

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One important point to note is that Sylvester’s construction is not the only way to generate Hadamard matrices, and a $nxn$ Hadamard matrix is not unique. In other words, there are more than one solution to equation (7.1) for any given $n$. Hadamard matrices are used for error-correcting codes for long time even in 1969 when the spacecraft Mariner took pictures of Mars, it has used Hadamard codes for error correction while transmitting the pictures to earth [55]. If $H_n$ is a Hadamard matrix then $-H_n$ will also be a Hadamard Matrix. If one replaces the -1 elements with 0 in both $H_n$ and $-H_n$, then each row in both of these matrices are called as a Hadamard code. However in this thesis, a Hadamard code is defined as rows of Hadamard matrices without replacing -1’s with 0’s.

7.3.2. Proposed Experimental Setup

From equation (7.1), it can be seen that each row of a Hadamard matrix is orthogonal. This orthogonality property is what makes Hadamard matrices so attractive for data mining. Our proposed approach maps every $n$ bit long data word to a $2^n$ bit long
Hadamard code and uses a coherent detection method which will be explained in detail. If the target bit pattern is \( n \) bit long, it means that the target pattern is a member of a set with \( 2^n \) members; because the total number of all the possible \( n \) bit long patterns is \( 2^n \).

The proposed setup includes a field programmable gate array (FPGA) [56] that maps each \( n \) bit long section in the input data to the corresponding \( 2^n \) bit long Hadamard code as seen in the following figure. This mapping has to be one-to-one (injective) and onto (surjective). One example is shown in the figure below. The 4 bit long target bit pattern “0, 1, 1, 0” is arbitrarily mapped to 16 bit long Hadamard code “1, 1, -1, -1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1”.

![Diagram of proposed setup for data mining with Hadamard codes.](image)

Figure 7-9: Proposed setup for data mining with Hadamard codes.

The proposed experimental setup consists of an interferometer with a balanced detector. An optical frequency comb source with a total number of \( 2^n \) comb lines should be used as the light source. The frequency comb is split in two and sent to two separate
spatial light modulators (not shown in the figure 7.9.) where the corresponding Hadamard bit sequence is encoded to phase of the frequency components as “π” being “1” and “-π” being “-1” as shown in figure 7.9. The “local oscillator” (LO) arm has a static phase coding which corresponds to the target bit pattern, whereas the “signal” arm has a dynamic phase coding which changes as the input data is streaming. The arms are combined with a beam splitter where the same wavelengths overlap physically. Depending on the relative phase between each wavelength, they will constructively interfere on one of the output ports of the interferometer and destructively interfere on the other output, resulting. As a result, the wavelengths that are in phase (i.e. their bit values match) will exit from one port, while the wavelengths that are out of phase (i.e. their bit values do not match) will exit from the other port of the interferometer. Both output ports are photodetected and the difference in the photoelectric power is monitored as a function of time. As discussed above, when the phase encoding of the signal arm and the target arm match (i.e. target data pattern exists in the input signal), all of the optical power will exit from one port and no power will exit from the other arm resulting in a maximum photoelectric difference at the monitor. If the signal and target patterns do not match, then due to the orthogonality of the Hadamard codes, always half of the wavelengths will be in phase and the other half will be out of phase, resulting in equal photoelectric power in both arms, and therefore zero power at the monitor. Since we are using orthogonal codes to encode the phase of the wavelengths, monitor output will be either maximum (when the patterns match) or zero (when the patterns do not match), therefore we can claim that our proposed approach has mathematically infinite extinction ratio.
7.4. Discussion

Both methods for data mining introduced in this chapter have their advantages and disadvantages. The optoelectronic approach checks the bits one by one and the results are counted electronically. As a result, the bottleneck of this approach is the electronic counter and its ability to distinguish optical “0” bits and “1” bits. However, the proposed system can be improved by optical amplification, and the false positive probability (i.e. claiming the existence of the target pattern inside the data while in reality it does not exist. ) can be reduced.

Data mining using the Hadamard codes on the other hand suffers from scalability since it requires a very large number of optical frequency comb components. In order to detect a $n$ bit long pattern, the system needs to use a $2^n$ bit long Hadamard code for a complete orthogonality, which makes it less practical to detect long bit patterns. However, this can be solved by using semi-orthogonal codes which can be shorter than Hadamard codes and therefore require fewer optical frequency combs. Using semi-orthogonal codes will however reduce the dynamic range of the system.

In this chapter we introduced two novel approaches for data mining applications. First approach uses an optoelectronic logic gate which act upon many input optical signals simultaneously resulting in parallel processing. We have experimentally demonstrated 8 bit and 2 bit pattern recognition using this approach. The second approach maps the input and the target data into orthogonal Hadamard codes and compares them in the optical
domain with coherent detection. Due to the orthogonality of the system, the proposed setup will have a very low bit error rate, and also very low probability of false positive.
8.1. Introduction

Optical pulses at high repetition rate with low noise have many applications in TDM based communication [16], time domain based arbitrary RF waveform generation [57], and analog to digital conversion [9]. For most of these applications, the main limiting factor of performance is the timing and amplitude fluctuations of the optical pulses. There have been many theoretical and experimental work to investigate the phase and amplitude noise in modelocked lasers [58],[59],[60]. There are many methods to reduce the phase noise of an actively modelocked laser. These methods include using an intracavity etalon [4], optical injection into the laser cavity [61], and exploiting the dispersion effect inside the cavity [62].

In this chapter we use intracavity active phase modulation to reduce the phase noise of an actively modelocked laser based on the theory of Haus and Rana [63]. The theory predicts that phase noise can be reduced when certain conditions on intracavity phase modulation are satisfied depending on the amount of dispersion in the cavity and on the bandwidth of the spectrum.
8.2. Theory on Effects of Intracavity Phase Modulation

The Heisenberg-Langevin equation that governs the time evolution of a pulse in the slow time scale $T$ can be written as follows [63]:

$$\frac{d\hat{\phi}}{dT} = A\hat{\phi} + (B - jD)\frac{1}{2}\frac{\partial^2\hat{\phi}}{\partial t^2} + (a_M + jp_M)[\cos(2\pi f_M t) - 1]\hat{\phi} + \hat{F}$$

(8.1)

where $\hat{\phi}$ is the normalized operator that describes the slowly varying envelope of the optical pulse and is a function of both $t$, and $T$ which is the time variable to express the changes on the time scales longer than cavity round trip time, $A$ is the complex variable describing the phase, the gain and the loss that the optical pulse acquires in one cavity roundtrip, $B$ is the effect of intracavity filtering on the pulse, $D$ is group velocity dispersion, $a_M$, and $p_M$ are the depth of modulation for amplitude and phase modulations respectively, $f_M$ is the modulation frequency (same for both amplitude and phase modulation), and $\hat{F}$ is the Langevin noise operator that describe the noise from intracavity gain and loss, it is also a function of both $t$ and $T$.

According to the theory of Haus and Rana [63], the nonorthogonality of the eigenfunctions of Heisenberg-Langevin equation results in a change in pulse to pulse timing fluctuation in the presence of dispersion and/or phase modulation.

Starting from equation 8.1, Haus and Rana show that the jitter of the pulse train is given by:
\[
J \approx \sigma_a^2 \left( 1 + \tan^2 \left( \frac{\tan^{-1} \left( \frac{p_M}{a_M} \right) + \tan^{-1} \left( \frac{D}{B} \right)}{2} \right) \right)^{3/2} \\
\quad \left( 1 + \frac{p_M}{a_M} \tan \left( \frac{\tan^{-1} \left( \frac{p_M}{a_M} \right) + \tan^{-1} \left( \frac{D}{B} \right)}{2} \right) \right)
\]

(8.2)

where $\sigma_a^2$ is the pulse to pulse timing fluctuation when $D$ and $p_M$ parameters are zero.

Figure 8.1 shows how the timing fluctuation normalized to $\sigma_a^2$ changes as a function of $D/B$ for different values of $p_M/a_M$.

![Figure 8-1: Normalized Timing Jitter as a function of D/B for different $p_M/a_M$ values.](image)

As seen in Fig. 8.1, as the strength of phase modulation inside the cavity is increasing, the jitter reduces in the negative $D/B$ region. The figure 8.2 on the other hand, shows the change of the jitter as a function of $p_M/a_M$ parameter for a given $D/B$ ratio.
In Fig. 8.2, a negative $p_m/a_m$ ratio means that amplitude and phase modulations are out of phase, while a positive $p_m/a_m$ ratio means that they are in phase. As a result we can conclude that the relative phase is also a very important parameter to adjust and for a positive dispersion regime, the amplitude and phase modulations should be out of phase for noise reduction. However, the reader should note that the theory of Haus and Rana does not include any nonlinear effects such as gain saturation, but also the noise from the RF source is neglected.

### 8.3. Experimental Setup of AMPM Laser

In order to demonstrate the effects of intracavity phase modulation on the phase noise of a modelocked laser, we have build a fiberized ring laser with a semiconductor gain medium, and two lithium niobate modulators; one amplitude modulator as a modelocker,
and a phase modulator for controlling the amount of intracavity phase modulation. The laser setup is shown in Fig. 8.3.


10 meters of dispersion compensating fiber (-170ps/nm·km) has been added to the cavity to overcompensate the anomalous dispersion from ~10 meters of single mode fiber (+17ps/nm·km) that exist on the pigtails of the fiberized components and to ensure that the laser operates in the negative dispersion regime. The cavity fundamental frequency is around 10 MHz, and the modelocking rate is chosen to be 10.24 GHz.

Both the phase and the amplitude modulator have a variable RF attenuator to change the corresponding modulation strength and vary the $p_M/a_M$ ratio. Also the RF arm to the
amplitude modulator has a variable phase shifter to control the relative phase between the
two modulating waveforms and change the sign of the $p_M/a_M$ ratio.

The RF phase shifter is inserted to the amplitude modulator arm so that the relative
phase can be monitored as a timing delay in the sampling scope. Figure 4 shows how the
sampling scope trace and the supermode noise spurs are affected when the sign of the
$p_M/a_M$ ratio changes. Unless the intracavity dispersion is zero, changing the sign of $p_M/a_M$
ratio has a dramatic effect on the jitter of the laser as predicted by the theory (see Fig.
8.2.) and as demonstrated in Fig. 8.4. The peak power of the supermode noise spurs
increases approximately 10 dB.

Fig. 8.4a shows the RF spectrum of the photodetected pulse train at 10.24 GHz
repetition rate when the $p_M/a_M$ ratio is positive (i.e. the modulation waveforms are in
phase), and Fig. 8.4b shows how the RF spectrum changes when the $p_M/a_M$ ratio is
negative (i.e. the modulation waveforms are in phase). In both cases the absolute value of
$p_M/a_M$ ratio is 1.6. The change in the sign of the $p_M/a_M$ ratio is coupled to the change in
the relative timing of pulse train in the sampling scope and can be monitored directly as
seen in Fig. 8.4c.
Figure 8-4: The RF spectrum when $p_M/a_M$ ratio is equal +1.6 (a), and when $p_M/a_M$ ratio is equal to -1.6 (b) Resolution bandwidth is 30 kHz, and frequency span is 50 MHz. Plot c shows the averaged sampling scope trace for both cases.

In the absence of an intracavity bandpass filter, the optical spectrum changes as the strength of phase modulation increases. As a result both the $D/B$ and $p_M/a_M$ ratios will change at the same time. In order to fix the $D/B$ parameter we have inserted an intracavity bandpass filter with a 3 dB bandwidth of 0.3 nm. The optical spectra for different $p_M/a_M$ ratios are shown in Fig. 8.5.
Figure 8-5: Optical spectra for different $p_M/a_M$ ratios.

Even though there are still some changes in the optical spectrum, we can claim that we have decoupled the $D/B$ and $p_M/a_M$ ratios up to some level. For every spectrum corresponding to a different $p_M/a_M$ ratio, the single side band noise spectrum is measured. Single side band noise spectrum shows how much spectral power the phase noise of the signal has and its integration gives the total pulse to pulse timing fluctuation as given in equation 8.3 [64].

$$\Delta t = \frac{1}{2\pi f_{\text{rep}}} \sqrt{\frac{1}{2} \int_{f_{\text{min}}}^{f_{\text{max}}} L(f) df}, \quad (8.3)$$

where $L(f)$ is the single sideband noise spectrum, in units of dBc/Hz, $f_{\text{min}}$ and $f_{\text{max}}$ are the lower and upper limits of the integral, and $f_{\text{rep}}$ is the pulse repetition frequency and in this case is equal to $f_M$ (the modulation frequency). In order to obtain the total pulse to pulse timing fluctuation the upper integration limit must be the Nyquist frequency, and the
lower integration limit depends on the time duration in which the pulse to pulse fluctuations are measured.

The single sideband noise spectra together with the integrated jitter for different values of $p_M/a_M$ ratio are given in Fig. 8.6.

![Figure 8-6: Single Sideband Noise Spectra and corresponding integrated jitter for different $p_M/a_M$ ratios.](image)

As seen in figure 8.6, as the $p_M/a_M$ ratio increases, the spectral power of the single sideband noise between 1 kHz and 200 kHz, together with super mode noise spurs clearly reduce. In order to obtain a more quantitative comparison, we have extrapolated the $L(f)$ up to the Nyquist frequency and calculated the pulse to pulse fluctuations integrated from 1 Hz to Nyquist frequency (5.12 GHz). The resulting jitter values as a function of $p_M/a_M$ ratio is given in the following figure together with the theoretical expectation for a large negative $D/B$ parameter.
Figure 8-7: Integrated jitter for different $p_M/a_M$ ratios. The theoretical expectation for large $D/B$ ratio (a); and the experimental results (b).

Since the $D/B$ parameter is slightly different for each data point in the experiment and the theory does not take into account effects such as gain saturation, or noise from the synthesizer, no attempt has been made to fit a theoretical line to the data points. However, it is important to note the similarity in the trend of how the integrated jitter changes as a function of $p_M/a_M$ ratio in both the experiment and theory.

In order to reduce the jitter further down, we have removed the bandpass filter, and let the spectrum move freely with increasing phase modulation strength. In this case, the spectrum change is much more apparent compared to the case with the intracavity filter as seen in Fig. 8.8.
The single sideband noise spectrum for each case is measured and the integrated timing jitter extrapolated up to the Nyquist frequency is calculated. The results are shown in Fig. 8.9.

Figure 8-8: Optical spectra for $p_m/a_M$ being equal to 0 and 1.6.

Figure 8-9: Single sideband phase noise for $p_m/a_M$ being equal to 0 and 1.6.
Again in this case, the major noise reduction is in the frequency range from 1 kHz to 200 kHz, and also in the power level of supermode noise spurs. The integrated jitter from 1 Hz to 5.12 GHz is 304 fs for the case when there is no intracavity phase modulation, and it reduces to 150 fs when the $p_M/a_M$ ratio is +1.6. As a result, the jitter has been reduced by 50% through using intracavity active phase modulation.

**8.4. Conclusion**

In this work, we have experimentally verified the theory of Haus and Rana on the effects of intracavity phase modulation on the jitter of the output pulses from a modelocked laser. The intracavity phase modulation clearly reduces the pulse to pulse timing fluctuations. By introducing active phase modulation to our laser, we have reduced the jitter from 304 fs to 150 fs.
CHAPTER 9: CONCLUSION AND FUTURE WORK

In this work, we have showed our results on generation of stable optical combs, arbitrary waveform generation and novel intracavity modulation schemes for rational harmonic mode-locking and ranging applications.

By incorporating serrodyne phase modulation with a time division multiplexing approach we have realized an endless phase modulation scheme, which is essential for arbitrary waveform generation. Using this method we have generated a continuously tunable RF signal at 25 GHz with 43 dB spur free dynamic range [15].

We have also realized extended RF chirp generation by parabolic phase modulation together with time division multiplexing and generated a RF chirp signal with the center frequency of 100 MHz, the frequency span 20 MHz, and frequency sweep time 200 ns [30].

By using a novel intracavity modulation, called “negative impulse modulation”, we have multiplied the pulse repetition rate of modelocked laser by rational harmonic modelocking without suffering from pulse-amplitude fluctuations which is associated with the standard rational harmonic modelocking mechanisms. We have realized a repetition rate multiplication factor of 15 with 16 dB suppression [37].

Frequency skewed pulse generation has been realized inside the laser cavity, resulting in an extremely practical pulse source for ranging applications [43].

We have built and operated an optoelectronic logic gate for data mining. the experimental setup has successfully detected and located an 8 bit long target pattern inside a streaming input data.
By using active intracavity phase modulation, we have experimentally verified the theory of Haus and Rana and demonstrated reduction of phase noise by 50% from 304 fs to 150 fs.

Our future work will focus on experimentally demonstrating the pattern recognition approach based on Hadamard codes as discussed in chapter 7. We will also continue to investigate the effects of intracavity active phase modulation on the noise characteristics of harmonically modelocked lasers. Previous experiments, we have completed are done where the phase and amplitude modulations have the same frequency. In the future experiments, we will investigate the effects of having phase modulation at the cavity fundamental, whereas the amplitude modulation is done at an integer multiple of the cavity fundamental frequency.
APPENDIX A: MATHEMATICA CODE FOR OPTIMIZING THE PARAMETERS FOR DUAL SINE WAVE PHASE MODULATION
(* Each Color represents a harmonic of the sinusoidally phase modulated optical signal. The graph shows the relative amplitude versus the modulation depth *)

Plot[{10*Log[10, 0.001 + Abs[BesselJ[0, t]]], 10*Log[10, 0.001 + Abs[BesselJ[1, t]]], 10*Log[10, 0.001 + Abs[BesselJ[2, t]]], 10*Log[10, 0.001 + Abs[BesselJ[3, t]]], 10*Log[10, 0.001 + Abs[BesselJ[4, t]]]}, {t, 0, 10}, PlotRange -> {-35, 0}, PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 0, 1], RGBColor[0, 0, 1], RGBColor[0, 1, 0], RGBColor[0, 0, 0]}]

(* This function returns the corresponding amplitude of nth harmonic of an optical signal that is phase modulated by 2 sine waves at different frequencies (f and m*f) and at different depth of modulations (A and B respectively)
Also relative phase is considered and included to equation as the variable PH *)

\[
\begin{align*}
\text{Amplitude of } e^{i\pi n ft} (\text{nth harmonic}) = & \sum_{k=-\infty}^{\infty} \text{BesselJ}[k, A] * e^{i2 \pi k f t} * e^{i2 \pi k \phi} \\
& + \sum_{l=-\infty}^{\infty} \text{BesselJ}[l, B] * e^{i2 \pi l m f t}
\end{align*}
\]

Defining the Error Function

(* This version takes into account the following components: 0,+-1,+-2,+-3,+-4,+-5 *)

Sidebanddifferencefunctionupto5[A_, B_, m_, PH_] := Max[{-10*Log[10, 0.001 + Abs[Nthordercomponent[0, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[1, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-1, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[2, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-2, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[3, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-3, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[4, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-4, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[5, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-5, m, A, B, PH]]]}, -Min[{-10*Log[10, 0.001 + Abs[Nthordercomponent[0, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[1, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-1, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[2, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-2, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[3, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-3, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[4, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-4, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[5, m, A, B, PH]]], 10*Log[10, 0.001 + Abs[Nthordercomponent[-5, m, A, B, PH]]]}]
Sideband difference function up to 4 \( A, B, m, \Phi \) := Max\{10 \times \log_{10} (0.001 + Abs[Nthordercomponent[0, m, A, B, \Phi]])\}, 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[1, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-1, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-2, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-3, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-4, m, A, B, \Phi]])\]}

(* This version takes into account the following components: 0, +1, +2, +3, +4 *)

Sideband difference function up to 3 \( A, B, m, \Phi \) := Max\{10 \times \log_{10} (0.001 + Abs[Nthordercomponent[0, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[1, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-1, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-2, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-3, m, A, B, \Phi]])\], 10 \times \log_{10} (0.001 + Abs[Nthordercomponent[-4, m, A, B, \Phi]])\]}

(* This version takes into account the following components: 0, +1, +2, +3 *)
$10^\text{Log}[0.001+\text{Abs}[\text{Nthordercomponent}[-2,m,A,B,PH]]], 10^\text{Log}[0.001+\text{Abs}[\text{Nthordercomponent}[3,m,A,B,PH]]], 10^\text{Log}[0.001+\text{Abs}[\text{Nthordercomponent}[-3,m,A,B,PH]]]$

General View up to 3rd order, different freq ratios

(* m=3 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto3}[A,B,3,1],{A,0,10},{B,0,10},PlotPoints->40,Mesh->False,PlotRange->{0,1}]

(* m=3 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto3}[A,B,3,0.5],{A,0,10},{B,0,10},PlotPoints->40,Mesh->False,PlotRange->{0,1}]

(* m=3 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto3}[A,B,3,0.25],{A,0,10},{B,0,10},PlotPoints->40,Mesh->False,PlotRange->{0,1}]

General View up to 4th order, different freq ratios

(* m=3 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto4}[A,B,3,0.25],{A,0,10},{B,0,10},PlotPoints->40,Mesh->False,PlotRange->{0,1}]

(* m=3 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto4}[A,B,3,0.25],{A,1.25,1.75},{B,1.25,1.75},PlotPoints->40,Mesh->False,PlotRange->{0,5}]

(* m=5 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto4}[A,B,5,0.25],{A,0,10},{B,0,10},PlotPoints->40,Mesh->False,PlotRange->{0,1}]

General View up to 5th order, different freq ratios

(* m=3 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto5}[A,B,3,1],{A,0,10},{B,0,10},PlotPoints->40,Mesh->False,PlotRange->{0,1}]

(* m=3 *)
Plot3D[1/\text{Sidebanddifferencefunctionupto5}[A,B,3,0.5],{A,0,10},{B,0,10},PlotPoints->40,Mesh->False,PlotRange->{0,1}]

(* m=5 *)
Plot3D[1/Sidebanddifferencefunctionupto5[A,B,3,0.25],{A,0,10},{B,0,10},PlotPoints-&gt;40,Mesh-&gt;False,PlotRange-&gt;{0,1}]

General View up to 7th order, different freq ratios

(* m=3 *)
Plot3D[1/Sidebanddifferencefunctionupto7[A,B,3,1],{A,0,10},{B,0,10},PlotPoints-&gt;40,Mesh-&gt;False,PlotRange-&gt;{0,1}]

(* m=3 *)
Plot3D[1/Sidebanddifferencefunctionupto7[A,B,3,0.09],{A,0,10},{B,0,10},PlotPoints-&gt;40,Mesh-&gt;False,PlotRange-&gt;{0,1}]

(* m=4 *)
Plot3D[1/Sidebanddifferencefunctionupto7[A,B,4,0.5],{A,0,10},{B,0,10},PlotPoints-&gt;40,Mesh-&gt;False,PlotRange-&gt;{0,1}]

Selected Peak Points for Flat Spectra
First Flat Point: A is around 5.1265 and B is around 2.4921, relative phase =0
(2x5+1=11 components) m=3

Plot3D[1/Sidebanddifferencefunctionupto5[A,B,3,0],{A,0,10},{B,0,10},PlotPoints-&gt;40,Mesh-&gt;False,PlotRange-&gt;{0.35,0.9}]

Plot3D[1/Sidebanddifferencefunctionupto5[A,B,3,0.01],{A,0,10},{B,0,10},PlotPoints-&gt;40,Mesh-&gt;False,PlotRange-&gt;{0.78,0.8},ViewPoint-&gt;{0,0,1}]

Sidebanddifferencefunctionupto5[5.1265,2.4921,3]

B=2.4921;

Plot[{10*Log[10,0.001+Abs[Nthordercomponent[0,3,A,B,0]]],10*Log[10,0.001+Abs[Nthordercomponent[1,3,A,B,0]]],10*Log[10,0.001+Abs[Nthordercomponent[2,3,A,B,0]]],10*Log[10,0.001+Abs[Nthordercomponent[3,3,A,B,0]]],10*Log[10,0.001+Abs[Nthordercomponent[4,3,A,B,0]]],10*Log[10,0.001+Abs[Nthordercomponent[5,3,A,B,0]]],10*Log[10,0.001+Abs[Nthordercomponent[6,3,A,B,0]]],{A,0,10},PlotRange-&gt;{-35,0},PlotStyle-&gt;{RGBColor[1,0,0],RGBColor[0,1,0],RGBColor[0,0,1],RGBColor[1,0,1],RGBColor[0,0,0]}}

(* Spectrum as a result of ONLY A=5.1265 modulation *)

M=5.1265;
ListPlot[{{-8,10*Log[10,0.001+Abs[BesselJ[-8,M]]]},{-7,10*Log[10,0.001+Abs[BesselJ[-7,M]]]},{-6,10*Log[10,0.001+Abs[BesselJ[-6,M]]]},{-5,10*Log[10,0.001+Abs[BesselJ[-5,M]]]},{-4,10*Log[10,0.001+Abs[BesselJ[-4,M]]]},{-3,10*Log[10,0.001+Abs[BesselJ[-3,M]]]},{-2,10*Log[10,0.001+Abs[BesselJ[-2,M]]]},{-1,10*Log[10,0.001+Abs[BesselJ[-1,M]]]},{0,10*Log[10,0.001+Abs[BesselJ[0,M]]]}}]
\text{ListPlot[\{\{-24,10*Log[10,0.001+Abs[BesselJ[-8,M]]]\},\{-21,10*Log[10,0.001+Abs[BesselJ[-7,M]]]\},\{-18,10*Log[10,0.001+Abs[BesselJ[-6,M]]]\},\{-15,10*Log[10,0.001+Abs[BesselJ[-5,M]]]\},\{-12,10*Log[10,0.001+Abs[BesselJ[-4,M]]]\},\{-9,10*Log[10,0.001+Abs[BesselJ[-3,M]]]\},\{-6,10*Log[10,0.001+Abs[BesselJ[-2,M]]]\},\{-3,10*Log[10,0.001+Abs[BesselJ[-1,M]]]\},\{0,10*Log[10,0.001+Abs[BesselJ[0,M]]]\},\{3,10*Log[10,0.001+Abs[BesselJ[1,M]]]\},\{6,10*Log[10,0.001+Abs[BesselJ[2,M]]]\},\{9,10*Log[10,0.001+Abs[BesselJ[3,M]]]\}],PlotJoined->True,PlotStyle->PointSize[0.02],PlotRange->{0,-25}]

(* Spectrum as a result of ONLY B=2.4921 modulation *)

M=2.4921;
ListPlot[\{\{4,10*Log[10,0.001+Abs[BesselJ[4,M]]]\},\{5,10*Log[10,0.001+Abs[BesselJ[5,M]]]\},\{6,10*Log[10,0.001+Abs[BesselJ[6,M]]]\},\{7,10*Log[10,0.001+Abs[BesselJ[7,M]]]\},\{8,10*Log[10,0.001+Abs[BesselJ[8,M]]]\}\},PlotJoined->True,PlotStyle->PointSize[0.02],PlotRange->{0,-25}]

(* Combined Spectrum *)

A=5.1265;
B=2.4921;
ListPlot[\{\{-24,10*Log[10,0.001+Abs[Nthordercomponent[-8,3,A,B,0]]]\},\{-21,10*Log[10,0.001+Abs[Nthordercomponent[-7,3,A,B,0]]]\},\{-18,10*Log[10,0.001+Abs[Nthordercomponent[-6,3,A,B,0]]]\},\{-15,10*Log[10,0.001+Abs[Nthordercomponent[-5,3,A,B,0]]]\},\{-12,10*Log[10,0.001+Abs[Nthordercomponent[-4,3,A,B,0]]]\},\{-9,10*Log[10,0.001+Abs[Nthordercomponent[-3,3,A,B,0]]]\},\{-6,10*Log[10,0.001+Abs[Nthordercomponent[-2,3,A,B,0]]]\},\{-3,10*Log[10,0.001+Abs[Nthordercomponent[-1,3,A,B,0]]]\},\{0,10*Log[10,0.001+Abs[Nthordercomponent[0,3,A,B,0]]]\},\{3,10*Log[10,0.001+Abs[Nthordercomponent[1,3,A,B,0]]]\},\{6,10*Log[10,0.001+Abs[Nthordercomponent[2,3,A,B,0]]]\},\{9,10*Log[10,0.001+Abs[Nthordercomponent[3,3,A,B,0]]]\},\{12,10*Log[10,0.001+Abs[Nthordercomponent[4,3,A,B,0]]]\},\{15,10*Log[10,0.001+Abs[Nthordercomponent[5,3,A,B,0]]]\},\{18,10*Log[10,0.001+Abs[Nthordercomponent[6,3,A,B,0]]]\},\{21,10*Log[10,0.001+Abs[Nthordercomponent[7,3,A,B,0]]]\},\{24,10*Log[10,0.001+Abs[Nthordercomponent[8,3,A,B,0]]]\}\},PlotJoined->True,PlotStyle->PointSize[0.02],PlotRange->{0,-25}]

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Second Flat Point: A is around 1.4347 and B is around 1.4347, relative phase = $\pi/2$
(2x4+1=9 components) m=3

\[
\text{Plot3D}\left[\frac{1}{\text{Sidebanddifferencefunctionupto4}[A,B,3,0.25]}\right],\{A,0,10\},\{B,0,10\},\text{PlotPoints}->40,\text{Mesh}->\text{False},\text{PlotRange}->\{0.35,0.9\}\]

\[
\text{Plot3D}\left[\frac{1}{\text{Sidebanddifferencefunctionupto4}[A,B,3,0.25]}\right],\{A,1.3,1.6\},\{B,1.3,1.6\},\text{PlotPoints}->40,\text{Mesh}->\text{False},\text{PlotRange}->\{0.0,4\},\text{ViewPoint}->\{0,0,1\}\]

\[
\text{Sidebanddifferencefunctionupto4}[1.4347,1.4347,3,0.25]
0.344572
\]

B=1.4347;
\[
\text{Plot}\left[\{10\times\log_{10}(0.001+\abs{\text{BesselJ}[8,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[7,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[6,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[5,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[4,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[3,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[2,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[1,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[0,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[1,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[2,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[3,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[4,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[5,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[6,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[7,M]}),10\times\log_{10}(0.001+\abs{\text{BesselJ}[8,M]})\}\right],\{A,0,10\},\text{PlotRange}->\{-35,0\},\text{PlotStyle}->\{\text{RGBColor}[1,0,0],\text{RGBColor}[0,1,0],\text{RGBColor}[0,0,1],\text{RGBColor}[1,0,1],\text{RGBColor}[0,0,0]\}\]

(* Spectrum as a result of ONLY A=1.4347 modulation *)

M=1.4347;
\[
\text{ListPlot}\left[\left\{-8,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-8,M]}),\right\}+\left\{-7,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-7,M]}),\right\}+\left\{-6,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-6,M]}),\right\}+\left\{-5,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-5,M]}),\right\}+\left\{-4,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-4,M]}),\right\}+\left\{-3,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-3,M]}),\right\}+\left\{-2,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-2,M]}),\right\}+\left\{-1,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-1,M]}),\right\}+\left\{0,10\times\log_{10}(0.001+\abs{\text{BesselJ}[0,M]}),\right\}+\left\{1,10\times\log_{10}(0.001+\abs{\text{BesselJ}[1,M]}),\right\}+\left\{2,10\times\log_{10}(0.001+\abs{\text{BesselJ}[2,M]}),\right\}+\left\{3,10\times\log_{10}(0.001+\abs{\text{BesselJ}[3,M]}),\right\}+\left\{4,10\times\log_{10}(0.001+\abs{\text{BesselJ}[4,M]}),\right\}+\left\{5,10\times\log_{10}(0.001+\abs{\text{BesselJ}[5,M]}),\right\}+\left\{6,10\times\log_{10}(0.001+\abs{\text{BesselJ}[6,M]}),\right\}+\left\{7,10\times\log_{10}(0.001+\abs{\text{BesselJ}[7,M]}),\right\}+\left\{8,10\times\log_{10}(0.001+\abs{\text{BesselJ}[8,M]}),\right\}\right],\text{PlotJoined}->\text{True},\text{PlotStyle}->\{\text{PointSize}[0.02],\text{PlotRange}->\{0,-25\}\}
\]

(* Spectrum as a result of ONLY B=1.4347 modulation *)

M=1.4347;
\[
\text{ListPlot}\left[\left\{-24,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-8,M]}),\right\}+\left\{-21,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-7,M]}),\right\}+\left\{-18,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-6,M]}),\right\}+\left\{-15,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-5,M]}),\right\}+\left\{-12,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-4,M]}),\right\}+\left\{-9,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-3,M]}),\right\}+\left\{-6,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-2,M]}),\right\}+\left\{-3,10\times\log_{10}(0.001+\abs{\text{BesselJ}[-1,M]}),\right\}+\left\{0,10\times\log_{10}(0.001+\abs{\text{BesselJ}[0,M]}),\right\}+\left\{1,10\times\log_{10}(0.001+\abs{\text{BesselJ}[1,M]}),\right\}+\left\{2,10\times\log_{10}(0.001+\abs{\text{BesselJ}[2,M]}),\right\}+\left\{3,10\times\log_{10}(0.001+\abs{\text{BesselJ}[3,M]}),\right\}+\left\{4,10\times\log_{10}(0.001+\abs{\text{BesselJ}[4,M]}),\right\}+\left\{5,10\times\log_{10}(0.001+\abs{\text{BesselJ}[5,M]}),\right\}+\left\{6,10\times\log_{10}(0.001+\abs{\text{BesselJ}[6,M]}),\right\}+\left\{7,10\times\log_{10}(0.001+\abs{\text{BesselJ}[7,M]}),\right\}+\left\{8,10\times\log_{10}(0.001+\abs{\text{BesselJ}[8,M]}),\right\}\right],\text{PlotJoined}->\text{True},\text{PlotStyle}->\{\text{PointSize}[0.02],\text{PlotRange}->\{0,-25\}\}
\]
15,10*Log[10,0.001+Abs[BesselJ[-5,M]]],{-12,10*Log[10,0.001+Abs[BesselJ[-4,M]]]},{-9,10*Log[10,0.001+Abs[BesselJ[-3,M]]]},{-6,10*Log[10,0.001+Abs[BesselJ[-2,M]]]},{-3,10*Log[10,0.001+Abs[BesselJ[-1,M]]]},{0,10*Log[10,0.001+Abs[BesselJ[0,M]]]},\{3,10*Log[10,0.001+Abs[BesselJ[1,M]]]\},\{6,10*Log[10,0.001+Abs[BesselJ[2,M]]]\},\{9,10*Log[10,0.001+Abs[BesselJ[3,M]]]\},\{12,10*Log[10,0.001+Abs[BesselJ[4,M]]]\},\{15,10*Log[10,0.001+Abs[BesselJ[5,M]]]\},\{18,10*Log[10,0.001+Abs[BesselJ[6,M]]]\},\{21,10*Log[10,0.001+Abs[BesselJ[7,M]]]\},\{24,10*Log[10,0.001+Abs[BesselJ[8,M]]]\},PlotJoined->True,PlotStyle->PointSize[0.02],PlotRange->{0,25}

(* Combined Spectrum *)

A=1.4347;
B=1.4347;
ListPlot[\{(\-8,10*Log[10,0.001+Abs[Nthordercomponent[-8,3,A,B,0.25]]]),\{-7,10*Log[10,0.001+Abs[Nthordercomponent[-7,3,A,B,0.25]]]),\{-6,10*Log[10,0.001+Abs[Nthordercomponent[-6,3,A,B,0.25]]]),\{-5,10*Log[10,0.001+Abs[Nthordercomponent[-5,3,A,B,0.25]]]),\{-4,10*Log[10,0.001+Abs[Nthordercomponent[-4,3,A,B,0.25]]]),\{-3,10*Log[10,0.001+Abs[Nthordercomponent[-3,3,A,B,0.25]]]),\{-2,10*Log[10,0.001+Abs[Nthordercomponent[-2,3,A,B,0.25]]]),\{-1,10*Log[10,0.001+Abs[Nthordercomponent[-1,3,A,B,0.25]]]),\{0,10*Log[10,0.001+Abs[Nthordercomponent[0,3,A,B,0.25]]]),\{1,10*Log[10,0.001+Abs[Nthordercomponent[1,3,A,B,0.25]]]),\{2,10*Log[10,0.001+Abs[Nthordercomponent[2,3,A,B,0.25]]]),\{3,10*Log[10,0.001+Abs[Nthordercomponent[3,3,A,B,0.25]]]),\{4,10*Log[10,0.001+Abs[Nthordercomponent[4,3,A,B,0.25]]]),\{5,10*Log[10,0.001+Abs[Nthordercomponent[5,3,A,B,0.25]]]),\{6,10*Log[10,0.001+Abs[Nthordercomponent[6,3,A,B,0.25]]]),\{7,10*Log[10,0.001+Abs[Nthordercomponent[7,3,A,B,0.25]]]),\{8,10*Log[10,0.001+Abs[Nthordercomponent[8,3,A,B,0.25]]]),PlotJoined->True,PlotStyle->PointSize[0.02],PlotRange->{0,25}
APPENDIX B: MATHEMATICA CODE FOR SERRODYNE SIMULATION WITH THE EXPERIMENTAL SAWTOOTH WAVEFORM
(* This program uses the experimental values of the measured harmonic powers to reconstruct the time domain picture of the sawtooth waveform *)
(* This program takes the first 290 harmonics into consideration.

The amplitudes are transformed from dBm scale to mW and then from mW to mVRms (assuming 50 ohm system) and then from mVRms to mVpp

Based on the vector analyzer results all harmonics are considered in phase.
*)

ListofHarmonicsversion2dataset1={-13.16,-19.33,-22.83,-25.5,-27.33,-29,-30.5,-31.66,-32.66,-33.66,-34.5,-35.16,-36,-36.66,-37.33,-37.83,-38.83,-39.33,-39.83,-40.33,-40.83,-41.16,-41.66,-42.16,-42.66,-43,-43.5,-43.83,-44.16,-44.33,-44.83,-45,-45.16,-45.5,-45.66,-46,-46.16,-46.5,-46.66,-46.83,-46.83,-47,-47.16,-47.33,-47.5,-47.66,-48,-48.16,-48.33,-48.83,-49.16,-49.33,-49.66,-50,-50.16,-50.5,-50.83,-51.16,-51.5,-51.83,-52,-52.33,-52.5,-52.83,-53,-53.33,-53.5,-53.83,-54,-54.16,-54.33,-54.66,-54.83,-55.16,-55.5,-55.83,-56.16,-56.5,-56.83,-57.16,-57.66,-58,-58.33,-58.83,-59.16,-59.33,-59.5,-59.83,-60,-60.16,-60.5,-60.66,-60.66,-60.83,-60.96,-60.5,-60.5,-60.33,-60.33,-60,-60,-59.5,-59.66,-59.5,-59.33,-59.33,-59.33,-59.5,-59.66,-60,-60.33,-60.66,-61.33,-61.66,-61.83,-62,-62.66,-63.16,-63.66,-63.66,-64.16,-64.5,-64.83,-65.16,-65.33,-65.33,-65.5,-65.66,-65.83,-65.5,-65.66,-65.66,-65.83,-65.66,-66.16,-65.16,-65.66,-64.66,-64.66,-64.16,-64.33,-64.66,-64.83,-65.16,-65.83,-66.83,-67.16,-67.66,-67.83,-67.83,-68,-69,-69.33,-69.16,-69.33,-69.66,-69.5,-70.33,-70,-70.33,-70.33,-70.66,-70.66,-70.66,-70.16,-70.16,-70.33,-70,-70.5,-71.16,-71.66,-71.66,-71.33,-72.16,-72.66,-73,-73.5,-74,-74.16,-74.5,-75,-75.5,-75.5,-75.16,-75.66,-75.66,-75.83,-75.83,-75.66,-76.16,-76.66,-76.83,-77.16,-77.5,-77.5,-77.5,-77.5,-77.83,-77.83,-78.16,-77.16,-77.66,-78.16,-77.66,-78.83,-78.83,-78.83,-78.83,-78.83,-79.33,-79.33,-79.33};
Length[ListofHarmonicsversion2dataset1]
290

x=ListPlot[ListofHarmonicsversion2dataset1,PlotJoined-> True];
\[ A1[n_] := \text{ListofHarmonicsversion2dataset1}[n]; \]
\[ (\ast \text{dBm to mW}) \]
\[ \text{AmW1}[n_] := 10^{0.1}A1[n]; \]
\[ (\ast \text{mW to mVrms assuming 50 ohms}) \]
\[ \text{AmVrms1}[n_] := 1000 \sqrt{\frac{\text{AmW1}[n]}{1000}} \ast 50; \]
\[ (\ast \text{mVrms to mVpp}) \]
\[ \text{AmVpp1}[n_] := \text{AmVrms1}[n] \ast 2 \ast \sqrt{2}; \]
\[ \text{RecTime}[t_, f_, a_, x_\_] := a \cdot \sum_{n=1} x \left( \frac{\text{AmVpp1}[n]}{2} \right) \ast \sin[n \ast 2 \pi t + \pi]; \]

Plot[{RecTime[t, 1, 1/200, 290], 0.5, -0.5}, {t, 0, 2}, PlotRange -> All]

Plot[{RecTime[t, 1, 0.93/200, 290], t - 0.5}, {t, 0, 1}, PlotRange -> {-0.5, 0.5}]

Defining the Sawtooth Waveform

(* RSawtooth is the reconstructed time domain sawtooth waveform based on the measured RF spectrum *)

\[ \text{RSawtooth}[t_, f_, a_, x_\_] := \text{RecTime}[t, f, a, x]; \]

(* Period is 20 *)

Sawtooth Phase Modulation

(* This simulation assumes heterodyne serrodyne with optical frequency spacing as 50, serrodyne frequency as 1, toggling frequency as 5*)

(* Everything except sawtooth waveform is assumed perfect *)

Phase Modulated Arm

\[ \text{EArm1}[t_, f_, \text{switch}_-, x_-, a_\_] := \text{Exp}[\pi (2 \pi 503 t + \text{switch} \ast 2 \pi \text{RSawtooth}[t, f, a, x])]; \]

\[ \text{EArm2}[t_] := \text{Exp}[\pi (2 \pi 500 t)]; \]

Beat Signal

\[ \text{EBeat}[t_-, f_-, \text{switch}_-, x_-, a_\_] := \text{Re}[\text{EArm1}[t, f, \text{switch}, x, a] \ast \text{Conjugate}[\text{EArm2}[t]]]; \]
Plot[{{EBeat[t, 1, 1, 290, 0.93/200] - 1.2), RSawtooth[t, 1, 0.93/200, 250] - 3.2}, {t, -0.5, 1.5}, PlotRange -> All]

The RF Spectra of the Beat Signal before Toggling Sampling of the signals
EBeatSample = Table[EBeat[t, 1, 1, 290, 0.92/200], {t, 1.5 + (3/2048), 1.5, 3/2048}];
Length[EBeatSample]
2048
EBeatReference = Table[-Cos[2 \[Pi] t 4], {t, 1.5 + (3/2048), 1.5, 3/2048}];
ListPlot[EBeatSample]

EBeatfreq = Fourier[EBeatSample];
EBeatReffreq = Fourier[EBeatReference];
Chop[EBeatfreq];
Chop[EBeatReffreq];
EBeatfreqrms = \[\sqrt{2}\] Abs[EBeatfreq];
EBeatReffreqrms = \[\sqrt{2}\] Abs[EBeatReffreq];
EBeatfreqpower = EBeatfreqrms^2 / 50 * 100
EBeatReffreqpower = EBeatReffreqrms^2 / 50
EBeatfreqdBm = Log[10, EBeatfreqpower]
EBeatReffreqdBm = Log[10, EBeatReffreqpower]
EBeatfreq2 = Take[EBeatfreqdBm, 100]
EBeatReffreq2 = Take[EBeatReffreqdBm, 100]
x = ListPlot[EBeatfreq2, PlotJoined -> True, PlotRange -> {45, -50}]

Max[EBeatfreq2]
36.9694
Max[Take[EBeatfreq2, 12]]
4.42569

y = ListPlot[EBeatReffreq2, PlotJoined -> True, PlotRange -> All]

Show[x, y]

Toggling
(* We will only take a perfect square waveform into account *)
(* m is the ratio of toggling to serrodyning and will be taken as 5 *)
(* T is the period of sawtooth NOT the square waveform *)
(* d is the relative delay*)

Toggle[T_, m_, t_, d_] := Which[
  -T/(2 m) <= Mod[t - d - T/(2 m), T/(2 m)] < -T/(4 m), 0,
  -T/(2 m) <= Mod[t - d - T/(2 m), T/(2 m)] < -T/(4 m), 1,
  T/(2 m) <= Mod[t - d - T/(2 m), T/(2 m)] < T/(2 m), 0,
  T/(2 m) <= Mod[t - d - T/(2 m), T/(2 m)] < T/(4 m), 1
]

Plot[{Toggle[1, 5, t, -0.1]*(0.5 + 0.5*RSawtooth[t, 1, 0.51/30, 113])}, {t, -1, 1}]
Beat Signal after Toggling
Plot[[EBeat[t,1,1,290,0.93/200]-1]*Toggle[1,2,t,-0.25],RSawtooth[t,1,0.93/200,290]-4,EBeat[t,1,1,290,0.93/200]-2],[t,-0.5,1.5],PlotRange-> All]

Relative Delay and Recombine
Plot[[EBeat[t,1,1,290,0.93/200]-1]*Toggle[1,2,t,-0.25]+1,(EBeat[t-0.75,1,1,290,0.93/200]-1)*Toggle[1,2,t-0.75,-0.25]],[t,-0.5,1.5],PlotRange-> All]

Plot[[2+(EBeat[t,1,1,290,0.93/200]-1)*Toggle[1,2,t,-0.25]+(EBeat[t-0.25,1,1,290,0.93/200]-1)*Toggle[1,2,t-0.25,-0.25]],[t,-0.5,1.5],PlotRange-> All]

EOutputsignal[t_,amp_]:=2+(EBeat[t,1,1,290,amp]-1)*Toggle[1,2,t,-0.25]+(EBeat[t-0.75,1,1,290,amp]-1)*Toggle[1,2,t-0.75,-0.25]
Plot[[EOutputsignal[t,0.93/200]-1,-Cos[2 t]],{t,0,1}]
The RF Spectra of the Output Signals
Sampling of the signals
ESample = Table[EOutputsignal[t, 0.92/200] - 1, {t, -1.5 + (3/2048), 1.5, 3/2048}];
Length[ESample]
2048
ERefference = Table[-Cos[2 \[Pi] t 4], {t, 1.5 + (3/2048), 1.5, 3/2048}];
ListPlot[ESample]
Efreq = Fourier[ESample];
EReffreq = Fourier[ERefference];
Chop[Efreq];
Chop[EReffreq];
Efreqrms = Abs[Efreq]/\[Sqrt]2;
EReffreqrms = Abs[EReffreq]/\[Sqrt]2;
Efreqpower = Efreqrms^2/50 * 100;
EReffreqpower = EReffreqrms^2/50;
EffreqdBm = Log[10, Efreqpower];
EReffreqdBm = Log[10, EReffreqpower];
Efreq2 = Take[EfreqdBm, 100];
ListPlot[Efreq2, PlotJoined -> True, PlotRange -> {45, -50}];
(* Resulting Spur Free Dynamic Range is 45 dB *)
Max[Efreq2]-Max[Take[Efreq2,12]]
45.1385

x=ListPlot[Efreq2,PlotJoined-> True,PlotRange-> All];

y=ListPlot[EReffreq2,PlotJoined-> True,PlotRange-> All];

Show[x,y]

(* Serrodyne with Perfect Sawtooth *)
(* Perfect Sawtooth *)

PSawtooth[T_, f_, a_, t_]:= Which[-[T
2
] ≤ T
2
Mod[t - T
2
, T] - T
2
< T
2
-T f,

= a
2
f
2
- T f ≤ t
2
Mod[t - T
2
, T] - T
2
< T
2
, - a
2
f
2
- T f ≤ [Mod[t - T
2
, T] - T
2
] < T
2
, - a
2


Serrodyne Phase Modulation
(* This simulation assumes heterodyne serrodyne with optical frequency spacing as 50, serrodyne frequency as 1, toggling frequency as 5*)
(* Everything including sawtooth waveform is assumed perfect *)

Phase Modulated Arm
EArm1[t_, f_, switch_] := Exp[2 \[Pi] 503 t + switch \[Pi] 2 \[Pi] PSawtooth[1, 0.01, 1, t - 0.5]]
EArm2[t_] := Exp[2 \[Pi] 500 t]

Beat Signal
EBeat[t_, f_, switch_] := Re[EArm1[t, f, switch]*Conjugate[EArm2[t]]]

Toggling
(* We will only take a perfect square waveform into account *)
(* m is the ratio of toggling to serrodyne and will be taken as 5 *)
(* T is the period of sawtooth NOT the square waveform *)
(* d is the relative delay *)

Toggle[T_, m_, t_, d_] := Which[-T // 2 m \[LessEqual] \[Mod][t - d - T // 2 m, T // m] - T // m \[LessEqual] - \[Mod][t - d - T // m, T // m] - \[Mod][T // m, T // m] < -4

Plot[{Toggle[1, 5, t, -0.1]*(0.5 + 0.5*PSawtooth[1, 0.01, 1, t - 0.5])}, {t, -1, 1}]
Beat Signal after Toggling
Plot[{{EBeat[t,1,1]-1)*Toggle[1,2,t,-0.25],
PSawtooth[1,0.01,1,t-0.5]-4,EBeat[t,1,1]-2},{t,-0.5,1.5},PlotRange-> All]

Relative Delay and Recombine
Plot[{{EBeat[t,1,1]-1)*Toggle[1,2,t,-0.25],
(EBeat[t-0.25,1,1]-1)*Toggle[1,2,t-0.25,-0.25]},{t,-0.5,1.5},PlotRange-> All]

EOutputsignal[t_]:=2+(EBeat[t,1,1]-1)*
   Toggle[1,2,t,-0.25]+(EBeat[t-0.25,1,1]-1)*Toggle[1,2,t-0.25,-0.25]
Plot[{EOutputsignal[t]-1,-Cos[2\[Pi] t 4+-,,t,0,1-+]},{t,0,1}]

The RF Spectra of the Output Signals
Sampling of the signals
ESample=Table[EOutputsignal[t]-1,{t,-1.5+(3/2048),1.5,3/2048}];
ERefERENCE=Table[-Cos[2 \[Pi] t 4],{t1.5+(3/2048),1.5,3/2048}];
ListPlot[ESample]
Efreq=Fourier[ESample];
EReffreq=Fourier[EReference];
Chop[Efreq];
Chop[EReffreq];

\[ \text{Efreqrms} = \sqrt{\text{Abs}[\text{Efreq}]} \]
\[ \text{EReffreqrms} = \sqrt{\text{Abs}[\text{EReffreq}]} \]

\[ \text{Efreqpower} = \frac{\text{Efreqrms}^2}{50} \]
\[ \text{EReffreqpower} = \frac{\text{EReffreqrms}^2}{50} \]

EfreqdBm=Log[10,Efreqpower]*10;
EReffreqdBm=Log[10,EReffreqpower]*10;
Efreq2=Take[EfreqdBm,100];
EReffreq2=Take[EReffreqdBm,100];
x=ListPlot[Efreq2,PlotJoined-> True,PlotRange-> All]

y=ListPlot[EReffreq2,PlotJoined-> True,PlotRange-> All]
Show[x,y]
REFERENCES


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