Inventory Management Problem for Cold Items with Environmental and Financial Considerations

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INVENTORY MANAGEMENT PROBLEM FOR COLD ITEMS WITH ENVIRONMENTAL AND FINANCIAL CONSIDERATIONS

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Industrial Engineering and Management Systems in the College of Engineering and Computer Science at the University of Central Florida
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Jennifer Pazour (Chair)
Dima Nazzal (Co-Chair)
ABSTRACT

The overarching theme of this dissertation is analytically analyzing the cold supply chain from a financial and environmental perspective. Specifically, we develop inventory policy models in the cold supply chain that consider holding and transportation unit capacities. The models provide insights for the decision maker on the tradeoff between setting order quantities based on the cost or the emission function.

In Chapter 2, we review two major bodies of literature: 1) supply chain design, and 2) sustainability in supply chain design. We benefit from this literature review to map the current body of research on traditional supply chain for further comparison with the cold supply chain. Sustainability in supply chain network design is often measured by the carbon footprint; other sustainability metrics such as water footprint and sustainable energy are not included. Literature on supply chain design can be further broken down into its three major components: 1) facility location/allocation, 2) inventory management, and 3) facility location/allocation combined with inventory management.

In Chapter 3, we study and present an overview of the cold chain. In accordance to the three levels of supply chain management decision making, the study is divided into the following three sections: (1) strategic level, (2) tactical level, and (3) operational level. Specifically, we capture how these decisions will impact the three main components of sustainability: economic, environmental, and social components. In addition, we explain how these components are different in the cold chain, in comparison to the traditional supply chain, and why such unique differences are worth studying. The intent of this chapter is to provide an overview of cold
chains and to identify open areas for research. Examples from industrial cases, in addition to data and information from white papers, reports and research articles are provided.

In Chapter 4, the cold item inventory problem is formulated as a single-period model that considers both financial and emissions functions. A new formulation for holding and transportation cost and emission is proposed by considering unit capacity for holding and transportation. This model applies to cold items that need to be stored at a certain, non-ambient temperature. Holding cold items in a warehouse is usually done by dividing the warehouse into a set of cold freezer units inside rather than refrigerating the entire warehouse. The advantage of such a design is that individual freezer units can be turned off to save cost and energy, when they are not needed. As a result, there is a fixed (setup) cost for holding a group of items, which results in a step function to represent the fixed cost of turning on the freezer units, in addition to the variable cost of holding items based on the number of units held in inventory. Three main goals of studying this problem are: 1) deriving the mathematical structure and modeling the holding and transportation costs and environmental functions in cold chains, 2) proposing exact solution procedures to solve the math models, and 3) analyzing the tradeoffs involved in making inventory decisions based on minimizing emissions vs. minimizing cost in cold chains.

This problem demonstrates the tradeoff between the cost and the emission functions in an important supply chain decision. Also, the analytical models and solution approaches provide the decision maker with analytical tools for making better decisions.

In Chapter 5, we expand the developed model from Chapter 4 to include multiple types of products. We consider a group of products that share capacities as a family of products. According to the problem formulation, we have two types of decision variables: (1) determining if a product is a member of a family or not, and (2) how much to order and how frequently to
order for products within each family. We propose a solution procedure in accordance with the
decision variable types: (1) a procedure for grouping (partitioning) the products into different
families, and (2) a procedure to solve the inventory problem for each family. A set of
experiments are designed to answer a number of research questions, and brings more
understandings of the developed models and solutions algorithms.

Finally, the conclusions of this dissertation and suggestions for future research topics are
presented in Chapter 6.
TO MY PARENTS:
RAHIM BOZORGI AND NAHID NAFISI
AND MY SISTERS:
ATEFEH& ELIKA
FOR ALWAYS BEING THERE, IN THE BEST POSSIBLE WAY
ACKNOWLEDGMENTS

I would like to express the deepest appreciation to my advisors Dr. Jennifer Pazour and Dr. Dima Nazzal, for showing me the research path, discussing each and every detail in my work to make it as clear and good quality as possible, and helping me to improve my critical thinking. Without their supervision and constant help this dissertation would not have been possible.

Besides my advisors, I would like to thank the rest of my PhD committee, Professors Karwowski and Zheng for their helpful career advice and suggestions in general.

In addition, a thank you to Dr. Mansooreh Mollaghasemi, who helped me in my early days in the US, and was always there as a mentor and an elder friend.

I can’t think of what I have learned in UCF, without thinking of Dr. Jose Sepulveda, as I was his Research Assistant for almost three years. I have learned work discipline, ethics, sharpness and paying attention to details, in addition to several technical skills and knowledge. He was also a good friend for me, kind, caring and passionate about me, my country, etc. I also am thankful to Dr. Serge Sala-Diakanda for his kind and help through the course of a Health Care project that we were working on for two years.

I am known to be a social person, and I think it’s because of having so many wonderful people around me. I am thankful to them for providing support and friendship that I needed. Specially, I would like to thank Mr. (s): Ali Maknuni Nejad, Araz Feizi, Shervin Ahmadbeigi, Mohammadreza Baharnemati, Reza Vaezi, Kaveh Moghaddam, Mubarak Banisaker, Robert Kantor and Ms. (s): Leila Zia, Hoda Parvin, Tannaz Hosseinzadehadeh, Catherine Cum and Brittany Villalard.

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1 INTRODUCTION

This introductory chapter is organized as follows. Supply chain management is defined in Section 1.1. The general motivation for sustainability is explained in Section 1.1, and in Section 1.2 quantification of the environmental function is discussed. In section 1.3 we discuss how sustainability has been considering in the supply chain management literature. Finally, in Section 1.5 we provide a summary of this dissertation.

1.1 Supply Chain Management

A Supply Chain (SC) is a system of suppliers, manufacturers, warehouses, transportation centers, distributors, and retailers. The main purpose of this system is to transform raw materials to final products, supply those products to customers, through a portfolio of resources in order to make profit for its entities.

A supply chain network is a set of facilities and related links that connect the facilities together to bring a product from one layer of the supply chain to the other. These layers may be from a producer to a warehouse, from a producer to a retailer, or from a warehouse to a retailer (Figure 1: Supply Chain Network).

Supply chain management (SCM) is a set of approaches concerned with the efficient integration of suppliers, manufacturers, warehouses and stores in order to produce and distribute the products in the right quantities, to the right locations, at the right time while minimizing system wide costs, and satisfying customer service level requirements\(^1\). One way to organize SCM decisions is by separating them into strategic, tactical, and operational decisions. Strategic

\(^1\) [http://www.amazon.com/Introduction-Supply-Chain-Management-Perspective-ebook/dp/B00BACR6D6](http://www.amazon.com/Introduction-Supply-Chain-Management-Perspective-ebook/dp/B00BACR6D6)
decisions focus on the design of the SC, such as the location of its facilities, and whether to outsource or perform a SC function in-house.

Figure 1: Supply chain network
Tactical decisions have a time-frame of a quarter to a year and they usually focus on planning of supply chain functions such as which markets will be supplied from which locations, and inventory policies to be followed, among others. Operational SC decisions have a time horizon of a week or day and focus on making decisions regarding individual customer orders, such as generating pick lists at a warehouse or setting delivery schedules for trucks.

The topic area of supply chain management has received significant attention during the last two decades; yet, many interesting new aspects are still open research problems. For example, environmental issues, which are a result of the industrialized era are becoming more important and have received global recognition (Linton, Klassen, & Jayaraman, 2007).
The handling, holding and transportation of temperature-sensitive products along a supply chain is known as the cold supply chain (or cold chain). It applies to a broad range of items that are required to be maintained in a specific temperature range. Examples of cold chain items are deep freeze items (-28 to -30 Celsius) such as seafood, frozen items (-16 to -20 Celsius) such as meat, chill items (2 to 4 Celsius) such as fruit, vegetables and fresh meat, and pharmaceutical items (2 to 8 Celsius), such as medications and vaccines (Rodrigue, Comtois, & Slack, 2013).

In this dissertation, we will study supply chain management with a focus on environmental considerations. We will consider a specific subset of supply chain networks that carry temperature sensitive items, and are referred to as cold chains. The research topic is illustrated in Figure 2.

1.2 Sustainability

During the 1960’s-1990’s, industries focused on higher throughput rates and having products with better quality. Since then global attention has turned toward higher level issues, such as the quality of the work environment, the environmental impacts of a product or process, and the social impacts of different technologies and policies. Due to the interdependencies among these issues, the attention shifted to bringing ideas and solutions that satisfy these issues as a system,
rather than individually. The word sustainability is commonly used to refer to a system with environmental and social considerations. Literally, the word sustainability means: a system that can remain the same and function in the same manner as time goes by\(^2\).

In this section, we present some facts and data to show the importance of sustainability, as well as a few examples to demonstrate the global movement toward sustainability. These facts are presented to highlight the importance of sustainability in general on the current research in the supply chain management area. As the main focus of this dissertation is on the cold chain, we will provide specific data and motivation on the importance of sustainability in cold chains in Chapter 3.

To emphasize the importance of sustainability, we briefly present a few case studies. We obtain the data on the examples of Commonwealth Edison and Anderson Corporation from a study by EPA in 2000 (US EPA, n.d.). The cases of IKEA, Sharp Electronic, Office Depot, and Interface are from a brochure by the Smartway Company\(^3\).

1.2.1 Wal-Mart Case

Announced in October 2005 by its president and CEO, Wal-Mart was to establish a sustainability program to reduce the company’s environmental effects. Their goals were to be entirely supplied by renewable energy, eliminating the waste to reach the zero-waste state, and to sell sustainable products (Denend, Plambeck, & Business, 2007). In 1989, the company changed its policies and tried to move toward recyclable, degradable packaging, at no additional costs for customers. This program in 2005 was another step toward that goal. At the end of 2006, the company evaluated the program’s performance and the results showed that in its first year the program brought a

\(^2\) http://www.merriam-webster.com/dictionary/sustainability

\(^3\) http://www.nysanet.org/pdf/smartway-shipper-brochure.pdf
benefit of several “super centers” for the company, in addition to a better, more sustainable social face.

1.2.2 The Coca Cola Case

The Coca Cola Company (TCCC) has established a sustainability program, with different parts and goals. Table 1 shows two of their sustainability goals.

<table>
<thead>
<tr>
<th>Energy Efficiency and Climate Protection</th>
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<tr>
<td><strong>Goal</strong></td>
</tr>
<tr>
<td>Grow the business but not the systemwide carbon emission in our manufacturing operations through 2015 compared with our 2004 baseline</td>
</tr>
<tr>
<td>Reducing emissions from Coca Cola manufacturing operations in developed countries by 5% by the end of 2015, compared to the 2004 baseline.</td>
</tr>
</tbody>
</table>

TCCC has established “sustainable packaging” and “healthy communities” programs as well.

Based on the latest report of TCCC, here are some relevant facts on its involvement in sustainability issues, exactly quoted:

“In 2010, we reduced our global greenhouse gas emissions from manufacturing by 2 percent compared to 2009—from 5.33 million metric tons to 5.20 million metric tons—even as our unit case volume increased 5 percent. Though we are trending in the right direction, we still have a lot of work to do to get back to 2004 levels. Our total emissions in 2010 were 9 percent higher than our 2004 baseline of 4.79 million metric tons. The good news? Our productivity is

4 This table is borrowed from Coca Cola sustainability program annual report 2010
improving. Our global product volume in 2010 was 25.5 billion unit cases—29 percent more than in 2004—and our greenhouse gas intensity (per liter of product) have improved 14 percent since 2004.” ⁵

And for energy efficiency, TCCC reports claim that:

1. “In developed countries, we have made steady progress toward our goal of reducing emissions from manufacturing by 5 percent compared to 2004 levels by 2015. In 2010, we reduced emissions in developed countries by 1 percent compared to 2009 and 6 percent compared to 2004, our baseline year.

2. In another facility at Ballina Beverages, they implemented a new combined heat and power plant with which, they could reduce the carbon dioxide (CO₂) by 17 percent.”

Table 2 reports a high level summary of the emission indicators for the Coca Cola Company. The entire table is borrowed from the Coca Cola Company website⁶.

---

### Table 2: Coca Cola sustainability program performance

<table>
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</thead>
<tbody>
<tr>
<td>Direct greenhouse gas emissions for the Coca-Cola system, measured in million metric tons carbon dioxide equivalent (MMt CO₂e)</td>
<td>1.90MMt CO₂e</td>
<td>1.94MMt CO₂e</td>
<td>2.00MMt CO₂e</td>
<td>1.98MMt CO₂e</td>
<td>2.00MMt CO₂e</td>
</tr>
<tr>
<td>Indirect greenhouse gas emissions from electricity purchased and consumed (without energy trading) by the Coca-Cola system</td>
<td>3.30MMt CO₂e</td>
<td>3.45MMt CO₂e</td>
<td>3.20MMt CO₂e</td>
<td>2.98MMt CO₂e</td>
<td>2.97MMt CO₂e</td>
</tr>
<tr>
<td>Total greenhouse gas emissions for the Coca-Cola system</td>
<td>5.20MMt CO₂e</td>
<td>5.33MMt CO₂e</td>
<td>5.28MMt CO₂e</td>
<td>5.05MMt CO₂e</td>
<td>5.12MMt CO₂e</td>
</tr>
<tr>
<td>Total megajoules of energy used by the Coca-Cola system</td>
<td>58.8B</td>
<td>57.9B</td>
<td>58.5B</td>
<td>57.5B</td>
<td>57.9B</td>
</tr>
<tr>
<td>Energy use ratio (efficiency), defined as megajoules of energy used per liter of product produced by the Coca-Cola system</td>
<td>0.45</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.49</td>
</tr>
<tr>
<td>Total electricity purchased by the Coca-Cola system, measured in megawatt hours (MWh)</td>
<td>6,596,462 MWh</td>
<td>6,425,507 MWh</td>
<td>6,162,180 MWh</td>
<td>5,714,036 MWh</td>
<td>5,565,379 MWh</td>
</tr>
<tr>
<td>Number of HFC-free refrigerated coolers and vending machines placed in markets each year</td>
<td>162,000</td>
<td>72,000</td>
<td>31,400</td>
<td>8,100</td>
<td>2,500</td>
</tr>
</tbody>
</table>

#### 1.2.3 ComEd case

The electric utility company, Commonwealth Edison (ComEd), has practiced innovative accounting that has reduced their costs and environmental burdens significantly. In early 90’s, ComEd recognized that the total cost of managing materials and equipment was higher than the initial acquisition cost. In specific, the associated costs of environmental management were found to be usually overestimated. This acknowledgment by the managers resulted in the first step of a life cycle management actions by the company, which reduced the chemical inventories of the company at the generating stations. These reductions, in addition to some other successes

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8 [http://www.epa.gov/sustainability/analytics/supply-chain.htm](http://www.epa.gov/sustainability/analytics/supply-chain.htm)
in other areas led the ComEd to plan for a Life Cycle Management (LCM), two years later in 1995. After that, a group of dedicated staff from ComEd, established an effective collaboration among different divisions of ComEd to assess the life cycle costs and benefits, systematically. This set of actions resulted in $50 million financial benefit, while reducing the waste volume.

1.2.4 Progressive Commitment to Environmentally Sound Transport: IKEA
In addition to innovative shipping practices such as flat pack technologies and new pallet technology, which saves packing space, weight, and wasted material, IKEA encourages its transportation service providers to join SmartWay. Today, IKEA ships virtually 100% of its freight with SmartWay Carrier and Logistics Partners⁹.

1.2.5 Meeting Fuel-Efficiency Challenges at Facilities: Sharp Electronics
Sharp Electronics requires logistics companies to use SmartWay Carriers to ship Sharp products, prohibits truck idling at facilities, and ships 15% to 18% of their shipments by rail. Other strategies include using electric forklifts and keeping terminals open during the night to reduce idling¹⁰.

1.2.6 Demonstrating Social Responsibility: Office Depot
Office Depot reduced its transportation footprint by shifting some freight to intermodal transport, introducing battery-operated forklifts at facilities, and purchasing nearly 300 ultra-low-emission local distribution trucks that are 40% more fuel efficient than larger, conventional trucks for a savings of over 4.5 million gallons of fuel annually¹¹.

---

1.2.7 Greening the Light-Duty Fleet: Interface

Creating a greener and cleaner light-duty fleet is an important aspect of Interface’s sustainability goals. To meet this challenge, Interface now requires all future vehicles leased by the company for their sales force to be SmartWay or SmartWay Elite certified vehicles\(^\text{12}\).

1.3 Quantifying the Environmental Function

Despite the global attention toward sustainability and all the programs and budgets assigned to this important topic, decision making that incorporates sustainability is still in its early ages. In order to be usable by practitioners, sustainability needs to be well defined for any specific problem, be quantified and measurable. However, as it is a new and complex topic, quantification of environmental impacts in supply chain decisions has not been thoroughly studied.

Pollution from producing a product or offering a service to customers is difficult to measure, especially if one wants to capture all, exact impacts on the environment. From “cradle to grave”, a term that is used often by researchers in the field, is so broad and sometimes immeasurable that makes it almost impossible to be used in practice. Instead, the practical approach would try to cut unnecessary, small details to access at least a good estimation of the environmental impacts.

For this purpose, a number of approaches have been developed as well. Life Cycle Assessment (LCA) and Life Cycle Inventory (LCI) are examples of these quantifying methods.

Having a quantified measure of pollution, one may want to optimize the pollution amount. This may occur through two different approaches:

• Consider sustainability as a constraint that needs to be satisfied, for which, governmental regulations are a good example.

• Approach sustainability as an opportunity to improve, and consider it as an objective function, which might be improved, rather than just meeting a pre-defined level.

Cruze and McRae (1989) show that having the environmental function as an objective function can lead to better solutions, rather than modeling it as a constraint to be satisfied.

1.4 Considering Sustainability in Supply Chain Management

During the last decade, the problem of integrated supply chain has received significant attention from researchers and several aspects of the problem have been studied while a number of good solutions have been also developed (Shen, Coullard, & Daskin, 2003).

Traditionally, the goal of research in supply chain management is to achieve greater corporate competitiveness. This goal is achieved by defining strategies that would increase customer satisfaction, on time delivery, production rate and production quantity.

The researchers considered different types of products, tried to solve the problem for multi-period horizons, having different number of decision levels and considering stochasticity in their models. They considered different objective functions, which were mainly: cost related functions, customer satisfaction functions, risk function, and sustainability functions.

It is just recently that the researchers began including sustainability into supply chain modeling (Seuring & Müller, 2008; Seuring, 2013).

One reason for the interest in including sustainability into SCM is the global attention to environmental and social problems that has been raised during the recent years (Utting, 2002).

We can see more related terms in the current date, than any other period of time before, the terms such as: sustainability, global warming, environmental effects, green products, and so on, are a
sample of those and can be translated as the global movement toward this direction. Increasing attempts of activists and Non-Governmental Organizations (NGOs) on these topics bring knowledge and society awareness to these topics. This could affect a company’s market share, if the company does not consider social-environmental issues.

In the supply chain area, sustainability may be applied to different stages: manufacturing, transportation, processes, raw material extraction & refining, waste collection, recycling, reusing, and waste treatment.

In the field of Supply chain design, due to the high complexity of the problem and several features to consider, sustainability typically considers environmental emissions (Seuring, Sarkis, Müller, & Rao, 2008). According to Seuring & Müller (2008), there were only 20 articles out of 191 total reviewed articles that studied social aspects, while there were 140 out of 191 studying the environmental issues. More specific, it is mostly carbon footprint that is taken into consideration, as all the emissions can be transformed into the Carbon Dioxide Equivalent (CO2e) (Chaabane, Ramudhin, & Paquet, 2012). Consequently, there has been less focus on the impact of SCM to the water footprint, energy sustainability. Table 3 is a simple illustration of the SCM decisions and how they relate to sustainability considerations. The main context of Table 3 is borrowed from Bevilacqua, Ciarapica, & Giacchetta (2012). We conduct a more comprehensive analysis on the related literature, as represented in Chapter 2.
Table 3: Relation between different components of supply chain and sustainability problems

<table>
<thead>
<tr>
<th>Facility Location</th>
<th>Environmental Impact</th>
<th>Sustainable Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Negative consequences of sitting on natural habitants (ecosystems) and habitat destruction</td>
<td>Reduce the total travel distances of staff and material, to and from the company</td>
</tr>
<tr>
<td></td>
<td>Negative effects on humans and animals, from increased noise pollution and energy consumption; contamination of air and water</td>
<td>Avoid runoff from construction activity and new pavement.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Abate noise pollution.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reduce the effects of air pollution on the community.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material Flow</th>
<th>Environmental Impact</th>
<th>Sustainable Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Modes of transportation used to move materials have significant effects on pollution, energy usage and traffic congestion.</td>
<td>Reduce the number of shipments.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Source locally.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Strategically locate warehouses.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Consolidate shipments.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Select transportation modes wisely.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inventory Control</th>
<th>Environmental Impact</th>
<th>Sustainable Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Noise pollution, higher energy consumption, higher traffic of vehicles</td>
<td>Minimize the total movement of material, from and to the facility using consolidation of delivery and accepting to carry larger quantities of inventory.</td>
</tr>
</tbody>
</table>

1.5 Dissertation Problem Statement

In this dissertation we consider supply chain management decisions associated with the cold supply chain that consider both financial and environmental objectives. In Chapter 2, we review two major bodies of literature: 1) supply chain design, and 2) sustainability in supply chain design. This has been done to bring a basic knowledge about the traditional supply chain. Based on Chapter 2, we study the cold chain in specific in Chapter 3. In addition, we discuss the difference between the traditional supply chain, and the cold chain in Chapter 3. This relation between Chapter 2 and Chapter 3, and the organization of these two Chapters are illustrated in Figure 3.
In Chapter 4, we consider one type of product only, and develop a new inventory model that considers both cost and emission of the transportation and holding of the inventory. In Chapter 5, we expand the problem modeled in Chapter 4, for the case of having multiple products.

In our case, we encounter two different types of objective functions: (1) cost function and (2) emission function. We approach each problem, by optimizing each objective separately, and studying the tradeoff between the two objective functions.

The research objectives for the first problem (cold items inventory problem-single product) are:

- Study the structure of a new inventory cost-environmental objective function
- Formulate the new inventory problem mentioned above and propose a robust inventory policy
- Develop a solution method, based on the structure of the problem
• Conduct computational studies to support method efficiency

Thereafter, we would be able to answer the following questions:

• Do the financial and environmental functions have similar shapes?
• If they are not similar, do the dissimilarities affect the optimal solution for the problem?
• What is the trade-off between the environmental and cost functions?

The research objectives for the second problem (cold items inventory problem-multi-product) are:

• Study the structure of the new inventory model with multiple products
• Formulate the new inventory problem mentioned above and propose a robust inventory policy
• Develop a solution method, by decomposing the variables into two different sets, which is aligned with the structure of the problem formulation
• Conduct computational studies and analyze the results to obtain a better understanding of the model features

The rest of this document is organized as follows: in Chapter 2, we review two major bodies of literature: 1) supply chain design, and 2) sustainability in supply chain design. Literature on supply chain design can be further broken down into its three major components: 1) facility location/allocation, 2) inventory management, and 3) facility location/allocation combined with inventory management. These problems will later be referred to in Chapter 3.

In Chapter 3, we study and present an overview of the cold chain. In accordance to the three levels of supply chain management decision making, the study is divided into the following three sections: (1) strategic level, (2) tactical level, and (3) operational level. Specifically, we capture
how these decisions will impact the three main components of sustainability: economic, environmental, and social components. In addition, we explain how these components are different in the cold chain, in comparison to the traditional supply chain, and why such unique differences are worth studying. The intent of this chapter is to provide an overview of cold chains and to identify open areas for research. Examples from industrial cases, in addition to data and information from white papers, reports and research articles are provided.

In chapter 4, the cold item inventory problem is formulated as a single-period model that considers both financial and emissions functions. A new formulation for holding and transportation cost and emission is proposed by considering unit capacity for holding and transportation. This model applies to cold items that need to be stored at a certain, non-ambient temperature. Three main goals of studying this problem are: 1) deriving the mathematical structure and modeling the holding and transportation costs and environmental functions in cold chains, 2) proposing exact solution procedures to solve the math models, and 3) analyzing the tradeoffs involved in making inventory decisions based on minimizing emissions vs. minimizing cost in cold chains. In chapter 5, we further extend the model developed in Chapter 4, to consider multiple types of products. By relaxing the assumption of having only one type of product, we formulate the inventory problem having multiple cold items, and discuss the solution approach for the developed model. A set of numerical experiments are also presented, for further analyzing the problem and finding new features of the model. In Chapter 6, we provide a summary of our research contributions, insights from our research, and discuss further research directions.
1.6 References


2 LITERATURE REVIEW

2.1 Introduction

In this chapter, we aim to provide a general introduction to some of the main problems in the supply chain management research area. Although the focus of this dissertation is on the cold chains, we start with a broader field of research on supply chain. The reason is that in order to compare the cold chain to the traditional supply chain (which is the focus of Chapter 3), it is important to understand supply chain design and its associated literature.

Literature on supply chain design can be further broken down into its three major components: 1) facility location/allocation, 2) inventory management, and 3) facility location/allocation combined with inventory management.

The literature research approach in this chapter is carried in the following way: we first perform a wide search to find a set of closely related articles using keywords that include: sustainability, inventory management, supply chain, emissions, and variations of these keywords. Once we find an article, we check both its references and works that cite the article, which enable us to understand a string of research that extends the initial idea further. This method is efficient, because as we will show, research in this domain is extended in a serial format. Once an initial idea is developed, several extensions are developed that extends the work further. After finding the papers in the string (which is similar to local search, or cross over in Genetic Algorithm (GA)) we explore the literature for another article (as mutation in GA) and start doing the local search for the related string on the newly find paper. In the rest of this chapter, we will review the literature on supply chain problems with focus on the ones that develop quantitative approaches in 2.2, followed by the literature of the researches that apply sustainability into the supply chain in 2.3. We summarize our findings in 2.4.
2.2 Supply Chain Design

The supply chain network is a set of facilities and related links that connect the facilities together to bring a product from one level of the supply chain to the other. These levels may be from producer to warehouse, producer to retailer, or warehouse to retailer. These levels are also called echelons in the supply chain research literature; a two-echelon supply chain may consist of demand points, retailers and warehouses, or retailers, warehouses and producer. A supply chain has several components and there are several approaches and studies under the name of the supply chain. Our scope is to formulate and solve a supply chain network design with two objectives: economic and environmental. This would distinguish this research from managerial studies that focus only on how to operate an existing supply chain. To study the literature of supply chain design, we will review the location-allocation problems, inventory management problems, and the works that consider location and inventory problems at the same time.

2.2.1 Location-allocation problem

Facility location is a well-known problem in the literature. As a general description, the facility location problem can be defined as: given a set of demands (which can be continuous or on discrete points, deterministic or stochastic), find one or more locations to locate a facility on (reference needed for definition of facility location). Based on the overall problem characteristics and formulation perspective, a model can be categorized into some major groups:

- Locating undesirable facilities: in this formulation, we have a multi-objective model to both minimize the cost and maximize the distance from living communities simultaneously. Example applications of the undesirable facility location problem are to locate nuclear facilities or waste disposal sites.
• Locating emergency facilities: in this category, the objective is to minimize the travel time, and cost minimization might not be the first objective to satisfy. That is due to the special purpose of these emergency facilities, such as: fire departments, hospitals, medical service bases, etc.

• Locating industrial facilities: in this category, we consider different performance measures, and formulate the problem based on that. The performance measures might be: minimum travelled distance, minimum travel time, minimum travel cost, maximum customer satisfaction, minimum locating cost. Manufacturing sites, warehouses, are examples of this group of facilities.

To solve the formulated problems, several approaches are investigated, used and tested against each other to find the best approach for each type of problem. The solution approaches are mainly categorized into: 1) exact solutions (analytical models), and 2) meta-heuristic solutions. Analytical models are used if the problem is tractable (i.e., the candidate locations were reduced to a reasonable size, with no complex constraint or objective function.

Meta-heuristic methods, on the other hand, are mainly utilized for complex or large-sized problems. The following meta-heuristics are applied to the facility location problem in the literature: Genetic Algorithm (GA) (Hadj-Alouane & Bean, 1997, Alp, Erkut, & Drezner, 2003, Jaramillo, Bhadury, & Batta, 2002, Zhang & Rushton, 2008), Simulated Annealing (SA) (Pishvaee, Kianfar, & Karimi, 2010, Hassan-Pour, Mosadegh-Khah, & Tavakkoli-Moghaddam, 2009), Ant Colony Optimization (ACO) (Arnaout, 2013; C. K. Y. Lin, 2009), etc. There are also some research that has investigated the efficiency of these algorithms and has compared them to each other (Bashiri & Karimi, 2010).
Leon Cooper (1963) mathematically shows that including the allocation problem into the facility location and solving the two problems simultaneously would lead to better results, since they are tightly related. After that, most of the research in this field includes both problems together. Allocation problem is basically an assignment problem, which is solved after the number and position of facilities are determined. The allocation problem seeks to minimize the transportation cost by assigning the facilities in an optimum way.

A summary of recent works are presented in the following. Due to its importance and vast areas of application, there are numerous articles in this field, and several review papers are proposed in the literature. We discuss a chronological series of proposed works in the literature, but with no mean of doing a thorough survey. Interested reader is referred to the current review papers in the field (El-Shaieb, 1978, Erlenkotter, 1981, Fleischmann et al., 1997, Owen & Daskin, 1998, Rahman & Smith, 2000, Hale & Moberg, 2003, ReVelle & Eiselt, 2005a, ReVelle & Eiselt, 2005b, Caunhye, Nie, & Pokharel, 2012).

We begin our review with the seminal work and highlight research achievements. We choose literature that illustrates the characteristics of the location-allocation problem. For example, location-allocation problems began as a single objective model, and then are extended to include multiple objectives, multiple periods, etc. In addition, since our main focus is not on pure location-allocation problem, we present a flavor of highlights in the field from the beginning to the current time, and our focus is on the aggregated models that includes inventory management problems, as well. Having this said, the papers published since 2000 are explained in more detail than the ones published before 2000.
2.2.2 Inventory management problem

The inventory management problem is a well-studied important problem in the supply chain literature. The Economic Order Quantity (EOQ) model is a basic inventory model for deterministic demand and makes several assumptions. The assumptions of the EOQ model are:

- Constant ordering cost
- Known, evenly spread demand over time
- Fixed lead time
- Fixed purchasing price, no discount
- Instantaneous replenishment, receiving the whole order size at once
- Only one product is considered

The EOQ model with all the assumptions may not represent most inventory environments, but works well to bring good intuitions and understanding of more complex, realistic models. There are several articles and review papers of the EOQ and its several variations (Wu, Ouyang, & Yang, 2006), Maddah & Jaber (2008), Kiesmüller, de Kok, & Dabia, (2011), Jaber, Bonney, & Moualek (2009).

In the EOQ model, an optimization model balances the two most important components of inventory cost function: 1) ordering (also named setup) cost, and 2) holding cost. These two are the two main components of the inventory cost function that are considered in inventory management articles. We will refer to these two functions further in this chapter and will analyze the role of each in some of the reviewed papers.

As mentioned earlier, the EOQ is the foundation of several inventory formulations. Relaxing each (or any combination of) the EOQ model assumptions result in new sets of models, with

The inventory models may vary by their application area as well. Perishable inventory models are a set of problems that discuss issues about items that have certain period of time to be sold (or out of the warehouse) (Nahmias, Perry, & Stadje, 2004) (Broekmeulen & van Donselaar, 2009). This group mainly consists of food, cold beverages, blood, and fashion items, which may not be sold after a certain period of time with the same price as the beginning of the period. In this group of models, the researchers mainly try to come up with the optimal mixture of products, the right time to break down the price, and when to replenish with new items to maximize the profit. Due to the high volume of research on this area, we refer to some of the most recent works in the literature. ((Ravichandran, 1988)(Karaesmen, Scheller–Wolf, & Deniz, 2011)(Lian & Liu, 1999).

Another research stream for perishable items inventory models is to consider the item quality over time. These works deal with policy making for pricing and inventory turnover as the item starts deteriorating. This is applicable for fashion items, as well.

2.2.3 Integrated models of inventory with location-allocation

In this section, we discuss the body of literature on supply chain that specifically focuses on integrated models of inventory with location-allocation problems. Since the word supply chain design (and/or supply chain modeling) is a general name, a lot of different articles with different
objectives and decision variables lie under the name. We focus our review on the models that consider the location-allocation and inventory problems together.

We focus on network design and cost optimization models, ignoring articles that are not directly related to our problem (i.e., manufacturing, process and design study).

Modeling the inventory problem with the location-allocation problem is a relatively new line of research (beginning in 1996). Due to the relatively few numbers of articles and as this literature forms the direct basis of our proposed work, we explain each paper in more detail. We present this research in chronological order, except for the related papers that are extension of an initial idea in a paper. For extension articles, we explain further expansions to the model rather than jumping into the next in chronological order.

The logic behind considering the joint problem is that the isolated location-allocation problem enforces a fixed setup cost for the inventory problem that has to be solved more often during the scheduling horizon. This can greatly affect the inventory problem if the location-allocation problem results in different solutions. Solving the location-allocation and inventory problem independently results in an optimal solution for the isolated problem. By integrating the two problems, it is possible to find a global optimal that can reduce the total cost for both location-allocation and inventory problems. This solution may not be the optimal for individual problems, but it is optimal if both are considered simultaneously, to minimize the total cost.

Nozick & Turnquist (1998) are one of the first researchers that integrate the inventory problem into the location-allocation problem. In their work, Nozick & Turnquist (1998) propose a location-allocation model for an automotive manufacturing company. As a case study for the proposed model, Nozick & Turnquist (2001) investigate the automotive manufacturing cost in more detail and provide some computational results.
Jayaraman (1998) is also among the first researchers that formulate a location-allocation model considering inventory issues. Jayaraman names the proposed model the Facility Location, Inventory, Transportation NETwork or FLITNET, and considers inventory, location and transportation costs as the main components of cost objective function.

Sabri & Beamon (2000) introduce a multi-objective model that optimizes supply chain costs and flexibility simultaneously. In the supply chain field, flexibility is the ability to change the produced products in each level to fulfill the customer demand. The proposed model consists of two sub-models. The strategic sub-model is related to strategic level decisions, such as location of facilities and the assignment of customer demand to different facilities. The operational sub-model, on the other hand, focuses on the operational level to maximize the flexibility.

In the early 2000, Daskin, Coullard, & Shen (2002) introduce a joint inventory location model for a blood bank supply chain. This research is based on a real world case for a blood bank logistic network in a multicounty region in Chicago, IL. The problem is described as: given a set of current hospitals that can be considered as demand point for blood, how many and where to locate a number of Distribution Centers (DCs) to satisfy the demand, how to assign the hospitals to DCs and to determine the inventory level of each DC.

As an extension to their previous work, Shen et al., (2003) formulate the same problem as a Set Covering problem. The Set Covering formulation facilitates the modeling, but due to the high number of columns, a Column Generation approach is taken to solve the Set Covering Problem. This work is extended by Shu, Teo, & Shen, (2005). Although the model is the same as with previous work, a general approach is presented that is applicable beyond the two special cases of the previous work. Special structure of the set-covering model of the problem leads to a branch-and-price sub problem, or simply named “pricing problem” to be solved.
Snyder, Daskin, & Teo, (2007) study the same problem, with the Mixed Integer- Non-Linear Programming (MINLP) formulation, but with stochasticity in demand. The demand is considered to be stochastic, but the probability function is assumed to be fixed along the periods.

In another extension, Max Shen & Qi, (2007) include the routing problem into the same problem as in Shen et al., (2003). They focus on the transportation cost, by considering it not as a simple linear function of distance, but by solving a Vehicle Routing Problem that produces a non-linear cost function.

In another work, Teo & Shu (2004) propose a model for locating DCs and assigning the retailers to the DCs, to minimize the location, inventory and transportation costs, in one aggregated model. Their work is an extension to the One Warehouse Multi-Retailer (OWMR) problem by Perl & Daskin (1985), which includes transportation and location costs as well. After defining the cost function of each element, they formulate the problem as a set covering problem.

Ambrosino & Grazia Scutellà (2005) propose a number of supply chain network design models that include: location, allocation, inventory and transportation costs. One set of proposed models are based on a previous work by Perl & Daskin (1985). Perl & Daskin propose a model for location-allocation of warehouses in a supply chain, but without considering inventory or transportation costs.

A risk pooling, two-echelon production-inventory-distribution system is developed by Vidyarthi, Çelebi, Elhedhli, & Jewkes (2007). The model includes safety stock inventory of retailers at DCs, and by doing so, named risk pooling in the literature, total cost of the supply chain is reduced. The model includes multi-product, plant location, and DC location, safety stock to fulfill a predefined service level, production and inventory capacity. Considering the fixed cost of location both for plant sites and DCs, the objective function also includes the transportation
cost from plants to DCs and from DCs to retailers, for each product, in addition to the holding
cost of safety stock, but not the whole inventory cost.

In another approach, Chen, Lin, & Yih (2007) model a supply chain network design problem
from a key point decision makers’ point of view. They consider a supply chain network design
that includes: location, allocation, inventory, production and transportation costs. Their model
considers production capacity of each production facility, but not the inventory capacity of
warehouses.

Liao & Hsieh (2009) propose a capacitated, multi-objective version of the model first introduced
by Shen et al. (2003) and M.S et al. (2002). In their work, Liao, Hsieh, & Lai (2011) consider
service level (or customer satisfaction) as an additional objective function to the cost objective
function. They use a modified version of a Genetic Algorithm (GA), called Non-dominated
Sorting Genetic Algorithm (NSGA2) to solve the multi-objective, non-linear programming
model. Later on, Liao et al. (2011) use this model as a part of a bigger vendor-managed
inventory problem. In a vendor-managed inventory problem, the supplier is responsible to
monitor and decide for inventory level of its customers.

Another location-inventory model is formulated by Park, Lee, & Sung (2010). The main
contribution in this paper is considering the lead time between supplier and DC, which is ignored
in the previous works. Having a three-level supply chain network, the authors are to decide upon
the number and location of both suppliers and DCs, to minimize the summation of three cost
functions: location, inventory and safety stock, and transportation.

Keskin, Üster, & Çetinkaya (2010) formulate a vendor selection problem, with consideration of
capacity constraints. It includes both warehouse capacity constraints and dispatching capacity
constraints, which limits the order quantity for any retailer from DC. Referring to an article by
Weber & Current (1993) which compared vendor selection with facility location problem and mentioned the similarities between the two, the authors formulate the problem as a facility location problem, aggregated with inventory policy selection.

2.3 Sustainability

Sustainability is a general concept that may be used by researchers from several disciplines, each with different meanings and specific interpretations for their particular area. To define our scope we focus on the sustainability in designing supply chain networks. As we will see in the next sections, sustainability in supply chain network design is often measured by the carbon footprint; other sustainability metrics such as water footprint and sustainable energy are not included. This is true especially for the works conducted by researchers from the Industrial Engineering and Operations Research field.

In this section, we analyze current related works on sustainability in supply chain design. Specifically, we study sustainable location allocation problems in 1.3 and sustainable inventory management problem in 2.3.2.

2.3.1 Sustainable location-allocation problem

Considering sustainability and environmental impacts in Supply Chain Network Design (SCND) problem has become more frequent. This is common in the chemical engineering field, as the impacts that chemical products have on the environment are significant and require great attention.

A sustainable supply chain design for chemical products is modeled by Hugo & Pistikopoulos (2005). This work is an integration of location-capacity expansion-technology selection problem and sustainability. A multi-objective Mixed Integer Linear Programming (moMILP) model is
developed to formulate the problem. Net Present Value (NPV) with tax and depreciation considerations is used to describe the economical function and the environmental function is formulated using eco indicator-99, an extension of Life Cycle Assessment (LCA). LCA is a method to capture the emission of different parts of an activity, such as manufacturing, inventory, transportation, recycling, etc. The model is solved with a parametric optimization, to discretize the solution space. Solving the discretized problems results in a set of frontier solutions, among which the decision maker can choose one based on further decision factors. The supply chain design decisions considered in Hugo & Pistikopoulos (2005) model are location-allocation, transportation and technology selection; the inventory costs are included. Later, Bojarski, Laínez, Espuña, & Puigjaner (2009) extend this work by introducing the carbon cap-and-trade concept into the model. In a carbon (or emission, in general) cap-and-trade model, a firm is allowed to produce certain amount of carbon (or any other emission), and also permitted to go beyond that by paying the other firms with lower carbon than their limit. In this manner, a firm also may sell its extra carbon producing limit, for extra earning. The authors model cap-and-trade and include a balance constraint between generated, bought and sold carbon amounts. In addition, the model allows having different prices for different time periods which is not considered by Hugo & Pistikopoulos (2005).

Frota Neto, Bloemhof-Ruwaard, van Nunen, & van Heck (2008) use a Multi-Objective Programming (MOP) model to balance cost and environmental objective functions. They use Data Envelopment Analysis (DEA) to solve the proposed MOP and find a satisfying solution for both objectives.

Van der Vorst, Tromp, & Zee (2009) introduce a simulation modeling software for the food supply chain design. Providing no mathematical formulation, the paper is mostly an explanation
of software and the necessity of developing such software. The developed software considers the food quality in the model, which is not considered by most of the researchers.

A more detailed research on calculating the exact amount of emission of transportation is done by Sundarakani, de Souza, Goh, Wagner, & Manikandan (2010). In their research, the authors use the long-range Lagrangian transport method and also the Eulerian transport method to calculate the emission of transportation. They consider the angle of wind from three dimensions, the temperature, and heat exchange for calculating the emissions. The proposed model is useful for research that aims to compare the transportation emission of different routes.

A sustainable supply chain design with social impacts is proposed by Dehghanian & Mansour (2009). In this work, the three objectives: economical, environmental and social are defined and considered in a mathematical programming optimization model. The supply chain elements are location-allocation, but the environmental impacts are limited to transportation and processing at the sites, and do not include the impacts of different locations. The proposed multi-objective model is solved using a Genetic Algorithm.

In all the above mentioned works (except for Hugo & Pistikopoulos (2005)), the solution procedure is “a priori”. A priori approach is designed in a way to automatically omit the inferior solutions and come up with a single, so called “optimal” solution. In a different manner, Zamboni, Bezzo, & Shah (2009) use “a posteriori” approach, that brings a set of non-inferior solutions to the decision maker for further analysis. The advantage of this approach is that all the possible solutions are analyzed and investigated with a higher level of criteria. They use a biofuel case study from northern Italy to show the proposed method’s efficiency. This work is the extension to their previous article (Zamboni, Shah, & Bezzo, 2009). In the first article, the supply chain problem is considered without an environmental objective function. Therefore, there is no
multi-objective optimization and consequently, no conflicting objectives and no trade-off between the two objectives.

Corsano, Vecchietti, & Montagna (2011) introduce the plant design problem into the sustainable location-allocation network design for a biofuel supply chain. The proposed model includes plant design constraints, which are mostly for flow balance for each process that takes place inside a plant, in addition to a location-allocation networks design problem. Therefore, they have a new layer of decision making in their modeling. They also consider disposal cost of wastes in modeling the cost function, which is not considered by other researchers.

2.3.2 Sustainable inventory management

To the best of our knowledge, it is in 2010 that Benjaafar, Li & Daskin (2010) for the first time, study quantitative tradeoffs in the lot sizing problem with carbon footprint considerations. In their white paper, Benjaafar, Li & Daskin (2010) start this line of research by using simple models and basic mathematics to get insights from their simple models. The paper has three main sections: 1) single firm with carbon footprint caps, which considers a lot sizing problem that minimizes the summation of fixed and variable ordering costs, holding costs and backordering costs. They consider a similar structure for the carbon footprint function that is a linear function of the quantity of order, inventory size and a fixed part. The total generated carbon footprint has a cap, and therefore the model has a constraint not to violate the carbon cap. 2) Single firm with carbon tax, cap-and-trade and offsets. The authors consider different policies to treat the firm and its generated carbon footprint. They consider three cases: a) paying tax for each unit of carbon footprint, which puts a cost in the objective function to be minimized, b) having a cap-and-trade policy, and c) considering carbon offsets. In the cap-and-trade case, certain number of firms are considered with a cap for each firm. Inside the group, the firms are allowed to sell their
extra cap or buy more caps, in case they are not able to meet their own limit. This may cause profit for seller firms and extra cost for buyers, which is reflected in the objective function. In the third section, they study: 3) multiple firms with and without collaboration. In this case, they consider a group of firms with a fixed limit of carbon footprint cap. The firms may or may not collaborate to reach the limited cap, and benefit from collaboration or by enforcing others to match their share. After studying these cases, the authors state some insights from the numerical examples of proposed models.

In a later publication, Chen, Benjaafar & Elomri (2011) proposed a model for carbon-constrained EOQ. In this work, the authors use the same strategy of the previous paper to study the trade-off in carbon constrained EOQ model. As in their previous paper, they consider similar carbon footprint function for the cost function, and derive the optimal order quantity with regard to cost and emission functions. At this point, they start defining different scenarios for different cases of the cost function and emission function coefficients, and its impact on the optimal order quantity of the cost and emission function. To avoid special cases, they analyze the cases based on parameters and ratios, to come up with more robust results and insights. The analysis shows that the solution is more sensitive to emission function than to the cost function, so that any deviation from optimal order quantity obtained by cost function in the total cost function may be less than the same amount of deviation from optimal order quantity obtained by emission function. In the third section of this paper, the authors investigate the trade-offs in facility location problem and emission function. In this part, they reach the same results: deviating from the optimal number of facilities that minimize the cost function causes less effect on optimal solution than deviation from the optimal solution for emission function.
Song & Leng (2011) study four carbon emission policies and their effects on the firm’s total profit. In this work, Song & Leng (2011) use four policies, which are initially proposed by the Congressional Budget Office of the US Congress, in a single period production planning problem, to find the optimal quantity of production. Since they consider single period and the way they formulate their problem, it has the same structure as the newsvendor problem in the inventory management research. The four policies are namely: carbon cap, carbon tax, cap-and-trade and carbon offset. Through analytical results and with numerical examples, they investigate each policy and its effects on the total profit. As conclusion, the authors come up with some suggestions for policy makers in the field and also for manufacturing managers to maximize their profit under emission policies. As for the future research, they suggest a number of ideas: 1) relaxing the single period assumption and considering the multi-period problem, 2) considering the pricing problem for the firms along with the production planning problem, and 3) investigating a method to find out which policy works best for a firm.

In all of the above mentioned articles, there are similar policies on carbon emissions imposed. In addition, they all consider single-period models. The cap may get tighter over time by governmental legislations, or there might be a roll-over policy for unused emission credits which single period models may not be able to formulate. In addition, for a single period model, investment in new technologies may not be optimal, while this is an option for long term models.

Absi, Dauzere-Peres, Kedad-Sidhoum, Penz and Rapine (2011) analyze a multi-period lot sizing problem. Considering deterministic demand, the authors formulate a lot sizing problem with inventory holding costs, fixed and variable production and transportation costs with carbon emission capacity constraint. They consider four types of constraints: 1) on a single period cap,
2) cumulative carbon emission cap, 3) rolling carbon cap and 4) global carbon emission constraints.

2.3.3 Sustainable supply chain design
In this sub-section, we discuss the limited work on combining inventory with location-allocation problems with sustainability considerations.

In their pioneering work, You & Grossmann (2008) integrate sustainability in the supply chain network design that includes location-allocation and inventory management. Considering stochastic demand, they build their model as an extension to a work proposed by Shen, Coullard, & Daskin (2003). In their work, (Shen et al., 2003) consider a set of existing locations and formulate a model to minimize the total cost of inventory, location and transportation by choosing from current facilities to be also distribution centers for the other facilities. You & Grossmann (2008) use a similar idea but with environmental considerations. They formulate environmental impacts as a second objective function, and develop a multi-objective solution method for the proposed model.

Guillén-Gosálbez & Grossmann (2009) formulate a sustainable chemical supply chain design problem considering inventory costs. The proposed model consists of several constraints, among those the inventory limitation, warehouse capacity expansion and material flow. The model is similar to the one proposed by Hugo & Pistikopoulos (2005), with added inventory cost and constraints, and assuming stochasticity of the environmental impact.

In Guillén-Gosálbez & Grossmann (2010), the authors include another source of stochasticity in an extended model. They develop a new solution procedure for this new model using chance constraint concepts and solution methods. In both of these works, however, the source of stochasticity is considered to be the environmental function and not the demand.
Recently, Chaabane, Ramudhin, & Paquet (2012) propose a sustainable supply chain design. They consider a multi-objective optimization model that considers inventory costs into location-allocation, with economical and environmental objectives. Carbon emission is used as a representation of environmental impacts, and the problem is solved using Lingo software package.

### 2.4 Summary

In this chapter, we reviewed the research on joint works of supply chain and sustainability. The literature review included a formal definition of supply chain design focusing on the location-allocation, location-inventory, and location-allocation inventory models. Several extensions of these problem were studied, and some were synthesized in 2.2.

In parallel, by increasing attention to environmental issues, another branch of research focuses on considering environmental impacts of a supply chain and designs a supply chain that has less harm to the environment. This category also starts with combining environmental impacts into the supply chain problems. Several models and solution procedures are developed to solve the sustainable supply chain problem. We tried to provide an overview and introduction to the main problems in the field of sustainability in supply chain modeling, in 2.3. In Chapter 3, we will study and present an overview of the cold chain. Specifically, we capture how these decisions will impact the three main components of sustainability: economic, environmental, and social components.

### 2.5 References


distribution model for the dynamic location and allocation problem with safety stock


model with Weibull deterioration distribution, shortage and ramp-type demand.

the environmentally conscious design of chemical supply chains under uncertainty in the


Solving a Multi-Objective Multi-Depot Stochastic Location-Routing Problem by a


3 OVERVIEW OF THE COLD SUPPLY CHAIN

3.1 Abstract
This chapter is about introducing the cold chain, its different components and how these components affect different decisions in supply chain management. In particular, special attention is paid to capture the three main components of sustainability: economic, environmental, and social components. In addition, we explain how these components are different in the cold chain, in comparison to the traditional supply chain, and why these differences are worth studying. The intent of this chapter is to provide an overview of cold chains and to identify open areas for research.

3.2 Introduction
The handling, holding and transportation of temperature-sensitive products along a supply chain is known as the cold supply chain (or cold chain). It applies to a broad range of items that are required to be maintained in a specific temperature range. Examples of cold chain items are deep freeze items (-28 to -30 Celsius) such as seafood, frozen items (-16 to -20 Celsius) such as meat, chill items (2 to 4 Celsius) such as fruit, vegetables and fresh meat, and pharmaceutical items (2 to 8 Celsius), such as medications and vaccines (Rodrigue et al., 2013).

Numerous studies in recent years focus on the emissions resulting from the cold supply chain (Calanche et al., 2013; Dekker, Bloemhof, & Mallidis, 2012; James & James, 2010; Wang, Chen, Lee, & Tsai, 2013). These studies consider the emission from refrigerated trucks and transporters, cold warehouses, packaging$^{13}$ and other components in the supply chain.

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$^{13}$ http://www.ngpharma.eu.com/article/Sustainability-within-the-cold-chain/
The cold chain represents an important and substantial part of the supply chain. According to a survey by United States Department of Agriculture (USDA) in 2012, there is a total gross refrigerated storage capacity of 3.96 billion cubic feet in the United States out of which, 3.22 billion cubic feet is the usable refrigerated storage capacity\textsuperscript{14}. The global market of just deep-freeze foods in 2009 was estimated to be $165.4 billion, which is expected to reach $199.5 billion in 2014\textsuperscript{15}. Due to the large volume of the cold chain, any inefficiency can cause significant amounts of waste. Food and Agriculture Organization of the United Nations (FAO) reports\textsuperscript{16} that about ten percent of all fruit and vegetable waste in the North America occurs during the distribution process, which is approximately $4 billion.

The cold chain is responsible for approximately 1\% of the emission in the world (James & James, 2010). For the UK this value is reported to be somewhere around 3.5\% (Garnett, 2007). According to a report by the Center for Sustainable Systems\textsuperscript{17}, there are some greenhouse gases in the atmosphere only because of industrial activities. HFC, the main refrigeration coolant currently used, is one of these gases, which has a large Global Warming Potential (GWP) indicator (14800). Leakage of the refrigerators is also an important contributor to emission, which results in huge costs for firms\textsuperscript{18} in addition to the emission to the environment.

According to World Health Organization (WHO), food logistics and cold chains play an important role in bringing safe, clean food to the end user\textsuperscript{19}. Infection that can spread through the food chain can cause disease and even death among human beings. For example, Salmonella is a

\textsuperscript{14}http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1034
\textsuperscript{15}Deep-Freeze: Optimizing Efficiency in Deep-Freeze Warehouses. Food Logistics.
\textsuperscript{17}http://css.snre.umich.edu/css_doc/CSS05-21.pdf
\textsuperscript{18}http://www2.epa.gov/sites/production/files/documents/IOR_ReducingRefrigerantEmissions.pdf
\textsuperscript{19}http://www.who.int/foodsafety/fs_management/retail/en/index.html
bacteria that has killed tens of thousands of people in the 1990’s (“WHO | Salmonella (non-typhoidal),”). To prevent these damages, WHO started a program to study food safety in 2000(“WHO | Food safety,”).

Among all the factors, the cold chain plays an important role in the wastage rate and improvements of the vaccine supply chain can result in reduction in wastage rate. Vaccine wastage is an important issue for health and vaccination program and in some cases the vaccine wastage rate is about 39% (Assi et al., 2011). This wastage not only affects a vaccination program’s efficiency, but also causes cost increases in two ways: 1) the purchasing (manufacturing) cost of the wasted vaccines that need to be replaced, 2) the cost of vaccine disposal. As modeled and considered in several related articles, the disposal cost of wasted vaccines is high enough that it needs to be considered in cost minimization models (Lee et al., 2010; Assi et al., 2011).

All the above mentioned data and facts have one point in common: they all deal with the cold chain. Beamon (2008) mentioned two specialty supply chains that need more attention in the near future: 1) Food supply chain, and 2) Relief supply chain. We believe that there is a third group of supply chains that require extra attention: the cold chain. As briefly illustrated in this section, the cold chain plays an important role in everyday life, and this importance is expected to increase in the future.

In summary, there are several reasons to study the cold chain:

- Its cost function has a different shape from the traditional supply chain’s cost function
- Its emission function is different from the traditional supply chain’s
- It has a direct impact on human health
This chapter studies the definition of the cold chain from a supply chain decision modeling point of view. This chapter will depict the differences between the cold chain and the traditional supply chain, will identify research gaps in the current body of literature where traditional supply chain models are not applicable to the cold supply chain and new cold chain models are required to be developed. Different decision making levels of the cold chain are investigated: 2 for Strategic level decisions, 3.3.2 for Tactical level decisions, and 3.3.3 for Operational level decisions. The effects of cold chain on the main three segments of sustainability, which are: economic, environmental, and social are discussed for each level. Research gaps and guidelines for future research are discussed in 3.4. The authors aim to start a new research trend for cold chains, which is in some cases, different from the traditional supply chains.

To aid the discoveries in this chapter, we searched through different journals, including: Vaccine, Food Control, and Food Research International. Data and statistics are obtained from federal websites, such as: USDA, or legislating and/or standard organizations, such as: Hazard Analysis Critical Control Points (HACCP). Food and health statistics and data are also obtained from the WHO and UNICEF websites through the published white papers and reports. Discussions and interviews were conducted with industry experts affiliated with Council of Supply Chain
Management Professionals-Central Florida Roundtable (CSCMP-CFL) experts and Orlando chapter of the Association for Operations Management (APICS)\(^2^0\).

### 3.3 The Cold Chain versus the Traditional Supply Chain

Decision making in supply chain management is usually divided into three main levels (Gebennini, Gamberini, & Manzini, 2009). These levels are tied to the planning horizon:

- **Strategic level** (long term decisions)
- **Tactical level** (mid-term decisions)
- **Operational level** (short term planning)

This study analyzes the cold chain along two dimensions: 1) the level of the supply chain decision making, and 2) the three components of sustainability: economic, environmental, and social. Figure 4 depicts this structure.

![Figure 4: Structure of the study regarding to different decision levels, and different sustainability impacts of each level of decision](image)

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\(^{20}\) American Production and Inventory Control Society
3.3.1 Strategic Decisions

The strategic decisions in supply chain management are usually tied with long term planning. These types of decisions are not made very often, and their effect lasts for a long period of time, e.g. 5-10 years. Facility location is a famous example of strategic decisions (Owen & Daskin, 1998b). Once a facility is chosen for operations (either by renting the facility and acquiring the equipment, or by building it, or buying) it remains operational for a long period of time. Therefore, special attention is needed for this level of decisions. For the cold chain, due to the extra costs associated with its facilities (including freezers, facility design, location, and installed equipment, etc.), the importance of this decision is even higher.

3.3.1.1 Economic dimension

The cost of having a warehouse or Distribution Center (DC) for cold items is significantly higher than for traditional supply chains.

This is due to several influential factors to the total cost:

- For cold items, the cooler/freezer is needed, in addition to warehouse space for regular items
- For loading/unloading docks, special types of doors/dividers are needed, to reduce the temperature change caused by moving the items

As a result, the managers and decision makers are not willing to expand the facilities, and want to keep the currently operated facilities.21

This is true for the cold transportation infra-structure as well. As an example, the cost of a 40-feet reefer is about $30,000, while it costs about $5,000 for a regular 40-feet container.22

21 Personal interview with CSCMP experts in the Central Florida Roundtable (CSCMP-CFL)
Another extra cost for cold items in comparison to regular items is different regulations and certificates that might be applied for cold items. As an example, in the United States, there are regulations on cold warehouses and they should be certified\(^\text{23}\) by: HACCP\(^\text{24}\), USDA\(^\text{25}\), FDA\(^\text{26}\) or even USDC\(^\text{27}\), which might add additional costs to bring the facility to the certifiable level.

### 3.3.1.2 Environmental dimension

For the cold items, the warehouses and DCs have freezers, refrigerators, coolers, and cold containers. Most of the currently using cooling systems are producing Green House Gases (GHG), and other sort of emissions (Denholm & Kulcinski, 2004). An article by the EPA addresses how to calculate the emission and the Green House Gases generated by the refrigerators\(^\text{28}\). Therefore, the environmental effects related to the cold-item facilities are significant and decisions should take into consideration the environmental impact.

On the other side, choosing new technologies and investing in the initial cost of construction can result in significant cost and emission savings. For example, a recently (2011) opened Walmart facility in Canada costs $15 million, for only 400,000 square feet, but would save 60% on energy costs, approximately $4.8 million over a course of five years. Their usage of hydrogen fuel cells for all of the 71 lift trucks of the facility not only reduces the annual emission by 55% or almost 530 tons annually, but also brings a $1.3 million cost reduction as well (Doherty, 2011).

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23 [http://www.rdmwarehouse.com/facilities.htm](http://www.rdmwarehouse.com/facilities.htm)

24 Hazard Analysis and Critical Control Points

25 United Stated Department of Agriculture

26 US Food and Drug Administration

27 United Stated Department of Commerce

3.3.1.3 **Social dimension**

The cold chain facilities also have social aspects that are different than the social features of the traditional supply chains.

As normally involved with human food or health items (vaccine, medicine, etc), there are different regulations for cold facilities that may not be required for traditional supply chain facilities. For example, there are regulations on having the facility clean, hygienic (depending on the products in the cold chain), pest control and sanitation (Government of Canada, 2012). These facilities should also be designed to be appropriate facilities for workers who work under the cold environment (which is considered as a harsh condition) (“Get Regulatory and Legislative Advice,”).

3.3.2 **Tactical Decisions**

Tactical decisions are the type of decisions that are related to mid-term planning in supply chain management. These can be varied from vendor selection and allocation of demand points to warehouses/ DCs, to production planning and maintenance scheduling.

3.3.2.1 **Economic dimension**

This section includes the economic aspects of the tactical decisions in cold chain, and explains the features that are different for cold chain rather that the traditional supply chain. We take one further step into details for the tactical level problems, and will investigate the three problems of:

(a) Inventory management problem, (b) Transportation Problem, and (c) Vendor selection problem.
3.3.2.1.1 Inventory

For cold items, inventory holding cost is not just the capital investment opportunity, but various operational costs that are not usually included for regular items, such as: electricity cost for the freezers (cold containers), inspection costs, wastage cost associated to the expired items, etc. In some cases, holding cost function for cold items has a different structure than the regular supply chains. In order to reach optimality for this decision, a new production model is developed and solved. This model, named CICEM is explained in details in chapter 4.

As most of the cold items are perishable, a different research flow is developed to capture this feature. There are inventory models that consider the expiration date limitation within the regular inventory problem (Vaughan, 1994). This may limit the number of items to be kept in the same holding unit, and as a result, is effective on determining how large a warehouse needs to be. The perishability characteristic and expiration date also results in lowering the price for items near their expiration date. “First-Expiring-First-Out” is an example for an unloading inventory policy, originated from the perishable nature of the cold items (Manzini & Accorsi, 2013).

The items in the cold chain are usually sensitive to the environmental factors. Smell and light of the environment are among the common features that require attention (Singh & Singh, 2005). For example, milk or yogurt may not be kept in a warehouse that has meat or seafood in it, as the milk might get scented by the presence of the meat/seafood in the container. These limitations would not only enforce additional costs to the firms to always maintain the environment as required, but also introduce additional constraints on holding capacity sharing decisions and make the inventory management more complex.

A product that is labeled that it was produced in the same facility as common allergic items, such as peanuts, wheat, etc. may encounter market share reduction (Guru & Horne, 1999). This
possible loss of market share is because the customers are sensitive to the labeled food, and even more sensitive to negative information on the labels (Tegene, Huffman, Rousu, & Shogren, 2003; Uchida, Onozaka, Morita, & Managi, 2014). The market share increase for allergen-free food is not negligible, and is predicted to grow further as $2.6 billion of only gluten-free food has been sold in the US. This number is predicted to reach $5 billion by 2015. In order not to lose the market share, certain products may not share transportation or holding capacity, which may increase cost.

3.3.2.1.2 Transportation and allocation

Transportation scheduling and allocation decisions are two other important tactical level decisions. While the essence of routing and allocation problem is the same for both cold chain and traditional supply chains, there are some features of the cold chain that distinguish the modeling approach for the two named supply chains. These differences between the cold chain and regular supply chain may end up in some modification and changes in the current models, or the requirement for developing new models.

One important difference is the transportation capacity sharing. There are many examples for delivery that share transportation capacity, as known in the literature as: LessThan Truckload (LTL). LTL is commonly used in real world cases by practitioners for regular items supply chain. But for cold chain, LTL might not be used so often and if used, different constraints should be considered due to several reasons:

- The most important reason is that the environment (and in specific, the temperature) of holding and delivery for different cold items might be different, which makes the LTL...
infeasible. While most of the vaccines are kept and transported in the temperature range of 2-8°C, the frozen food is kept below the freezing temperature, which is harmful for vaccines. As another example, a fresh fruit delivery truck might not be able to share capacity with a meat delivery chain, or ice cream transporter might not be able to share the capacity with a banana supplier.

- Many products that require the cold chain are edible products and there are restrictions on which edible products can be transported with other edible products due to the effect on each other’s taste and odor. For example, a shipment of fish may not be delivered in the same space and near a shipment of ice cream, or a dairy product, which may be affected, be the scent of the fish.

- For liquid items (both in transportation and inventory), different products may not share capacity as a routine practice. Apparently, one may not use a tanker to carry both orange juice and apple juice at the same time. Even if both items are orange juice, one with pulp and another without pulp, may not share transportation or holding capacity.

- Another limiting criterion for cold items is the lead time for delivery. In specific, for fresh items, or for items near their expiration date, there is a limitation on the time waiting at a transportation hub or being sent through longer routes, for cost reduction purposes, which may cause expiration of the items.

- Food allergy is another reason that makes the LTL and sharing of capacity for some products, as is explained in 3.3.2.3.

As a result, the transportation cost might be higher for these items, and the LTL and capacity sharing models may not be applicable in this field.
3.3.2.1.3 Vendor selection

Vendor selection might be considered as strategic or tactical level decision. In this work, we consider the vendor selection as a tactical level decision. One of the most threatening issues for the cold chain is the “cold chain breaks”, which is any failure to maintain the temperature within the recommended range. Long breaks or several short breaks may damage the cold items and even cause them to go bad. Cold chain break may have different reasons:

1. Shortage in the transporter fuel
2. Electricity loss in a cold warehouse
3. Technical problem with cold sensors and technology, both in warehouses or transportation means

3.3.2.2 Environmental dimension

As explained in 3.3.2.1, there are environmental components in the cold chain that do not exist, or are less important for traditional supply chains. As an example, the emission from the electricity usage by the freezers/refrigerators, the leakage of the coolant of the fridges, and the waste from expired items are significant in cold chain, but not in traditional supply chains. The transportation of cold items has not only the common emission of fossil fuel, but the additional emission from the refrigeration units installed on the transporters to carry the cold items in the appropriate temperature. Tassou, De-Lille, & Ge (2009) study the emission of food transporters, which are higher for lower-temperature food items. Other studies on the emission of food transportation include: (Rosenthal, 2008; Wakeland, Cholette, & Venkat, 2012).

30 http://www.logisticsmgmt.com/article/cold_chain_best_practices_innovations
On the other hand, since the LTL approach may not be regularly applicable for cold items; there would be less holding and transportation capacity available, which results in increase in the emission from holding and transporting cold items. The fact of not being able to use the LTL approach adds more complexity to the supply chain problems, such as transportation or holding. We will discuss these issues in Chapter 5, on developing a model for multi-product inventory management problem.

3.3.2.3 Social dimension

The main customers of cold chain are agri-food products and vaccine/medical items, which can be categorized as the essences of living. As a result, the cold chain requires a higher fill rate, or demand satisfaction rate, as it is related to human’s daily needs (as food) or health (vaccine, medical items, blood). Having limited back orders or shortages should be considered when planning for cold chain items, which introduce higher costs to the system. Hunger results as a lack of food, or any shortcoming through the food chain that may not bring the basic foods. The hunger phenomena is discussed in health topics, and there are some programs under the name of “hunger reduction” that are currently being run in poor countries (Masset, 2011; Muthayya et al., 2013; te Lintelo, Haddad, Leavy, & Lakshman, 2014). Controlling and reducing mal nutrition is another goal for which, cold chain is a role-playing factor (Mejía Acosta & Haddad, 2014). But this has not been studied from supply chain and logistics point of view.

As the recent changes in research interests show, there is a slight change from the goal of peer focus on financial, toward more attention to social and environmental aspects of each industry (“Creating Shared Value,”; Tate, Ellram, & Kirchoff, 2010). Cold chain, as a main stream of flow of food and health related items become more important and requires more attention to
reach the social and environmental goals. Having enough food might be an essential goal, but the quality of food, freshness of food and products, are also important. Bringing fresh fruits, vegetables, food, and also considering the quality, does increase the cost of delivery (holding, transportation, maintenance) which might not be in the favor of the customer (The Global Food Supply Chain, 2013; Schröder, 2003).

The cold chain also plays an important role on the side of health, prevention and treatment. Recently, more consideration is given to holding and delivery of medicine, vaccine, blood, and many other health related goods (“Vaccination,” 2011). According to USAID, cold chain is: “An Essential Part of Safe and Effective Vaccination Programs” (“Immunization basics - Snap Shots Volume 8”).

Considering social aspects of the cold chain as an objective is sometimes in conflict with the environmental and financial objectives. The best case for vaccine delivery, for example, is the fastest way from production to the end user, which might be air delivery. But the associated cost of air delivery for vaccine makes it almost unreachable for mid and lower class of the society. On the other hand, carrying batches of vaccine through cheaper methods such as ground shipping may cause vaccine expiry or damage, and also a longer lead time for the end user. There are recently some research articles that study the trade-off between financial and social aspects (Assi et al., 2011; Lee et al., 2012).

Cold chains can help on preventing the food allergy, by not exposing food products with the allergic products. As reported, between 2004 and 2006, there were approximately 9,500 annual hospital discharges, related to food allergy31.

The cold chain for food and medicine/vaccine should be arranged in such a way to make recalls (call outs) easy, fast, and as cheap as possible. A product may be recalled for several reasons: expiration, an issue found in an item, contamination, substitution, etc. The sold items are required to be collected from the customers, as well as collecting all the inventory from the warehouses, in transit, and in stores. There are numerous reports on the effect of food or medicine contamination which were recalled, and their negative effect on human health and life. As an example, 37 people killed as a result of an outbreak of a rare form of E. coli, and 3000 more were sickened. €210 million (nearly $268 million) is assigned in emergency aid for vegetable farmers affected by crisis, which was approved by the European Union. Foodborne diseases are estimated to cause approximately 76 million illnesses and 5,000 deaths each year, in the US only (Mead et al., 1999). There are similar reports on the recalls for pharmaceutical products, with an issue regarding the supply chain of the items. The recall also has a negative effect on a company’s reputation, and may cause reduction in market share. According to Food Safety Magazine, the effect of media and chats between the allergic customers is not negligible.

3.3.3 Operational Decisions
Operational decisions deal with short term planning. The time scale for these types of decisions is daily or weekly/biweekly.

3.3.3.1 Financial aspect
For cold items, the operational costs are different (and usually higher) than for the regular items.

32 http://www.supplychain247.com/article/wood_pallets_cited_as_cause_for_mcneils_tylenol_recall/packaging
Before being sent to the retail stores, the items are normally inspected to make sure that the item is in the appropriate shape, temperature, and is not deteriorated, and so on. This is usually being done with the use of tracking technologies, such as RFID\textsuperscript{34}s (Kelepouris, Pramatari, & Doukidis, 2007; Kim, Jeong, & Park, 2012; Prakash, Renold, & Venkatalakshmi, 2012). The result of the inspection might end in a level degrade for fruits, or near expiration date for perishable items, which would cause price reduction or some other policies to reduce the loss. This inspection and also loss of quality are costly factors for the firms (Antle, 1999; Traill & Koenig, 2010).

Packaging for cold items is another costly part of their delivery system. The cold items need to be packaged in a way not to easily get damaged/ harmed, be protected against small temperature changes, and depending on different items and industries, the special care that is required. It is also required, for food items, that the packaging preserves the quality of the food and prevents deterioration (Bottani, Montanari, Vignali, & Guerra, 2011). For examples, for the case of fresh fruit, let us say peaches, it is of favorite that they are not packed or jammed in their boxes. This will increase the space taken by each item, which would make the transportation capacity limited and more expensive, in addition to the packaging cost.

As a large part of the cold items are food, beverages and health related items, the cleanliness of the facilities and the equipment is of an especial importance. It is not only required by law and several regulations to build clean& hygienic facilities, but they should be regularly cleaned up and sanitized to avoid disease spread. For example, a tanker delivered orange juice needs to be washed and cleaned before the next load. This is more critical for medicine, vaccine or blood

\textsuperscript{34} Radio Frequency IDentification
banks that require a high level of cleanness. This cleanup process is a time and money consuming process.

In some cases, the dryness of the environment or not being exposed to direct light is among the requirements (Raghav & Gupta, 2003). These considerations require adjustments into the facilities and equipment which are time consuming and also impose additional costs for the companies.

According to a report by Modern Material Handling website, the operational cost in cold warehouses may be higher than for the regular warehouses. For electrical lift trucks, their battery performs 20%-50% less efficient in cold warehouses\textsuperscript{35}. It is true for the buttons on electrical devices, as they may lose their functionality in cold circumstance. Although special buttons are designed to be bigger and more reliable for cold temperature, their battery life reduces 40%-50%.

3.3.3.2 Environmental aspect

The emission of handling the cold item is higher in comparison to the emission of regular items. This include the emission from the waste (expired or defected items), which may requires special treatments (e.g. an expired vaccine or medicine may not be easily dumped into a garbage can, as it contains chemicals and substances that are harmful for environment).

3.3.3.3 Social aspect

The temperature of the work environment is lower in cold chains than in traditional supply chains. Intuitively, and according to studies, working in a cold environment is not desirable and may cause back pain and knee pain, and other sort of disorders (Bang et al., 2005; F. Chen, Li, Huang, & Holmér, 1991; Parsons, 2002; Pienimäki, 2002). The rate of injury is also higher in

\textsuperscript{35} http://mmh.com/images/site/MMH1201_BestPrac_ColdStorage.pdf
cold environments, and the worker’s performance decreases (Maakiné & Hassi, 2009). As a result, workers may be scheduled for shorter shifts, or more frequent break times during a shift, to alleviate the effect of a cold work environment. Consequently, the work schedule, shift hours, shipment, etc. would be influenced. There are several cases of fine claims by the workers against cold facilities, due to exposure to cold-related chemicals, such as ammonium\textsuperscript{36}, or not having an appropriate work environment\textsuperscript{37}.

Transparency of information and traceability of the food items, as a significant part of cold items, is another specific characteristic for cold chain. Food labeling is required by domestic and international organizations such as WTO (World Trade Organization) and FAO (food and Agriculture Office). The companies should have the food products not only for the legislations, but to be able to track their products and being transparent on their supply chain((Nel) Wognum, Bremmers, Trienekens, van der Vorst, & Bloemhof, 2011). Food labeling would also bring information for the consumers and customers about the origin of the food and its nutrition facts. This aspect of labeling and having a transparent supply chain, distinguishes the cold chain from the traditional supply chain.

\section*{3.4 Analysis, Conclusion, and Future Research Topics}

The handling, holding and transportation of temperature-sensitive products along a supply chain is known as the cold supply chain (or cold chain). It applies to a broad range of items that are required to be maintained in a specific temperature range. The cold chain represents an important and substantial part of the supply chain.

\textsuperscript{36}http://www.ishn.com/articles/96916-neb-workers-exposed-to-ammonia-at-food-storage-facility
\textsuperscript{37}http://www.nrtw.org/en/blog/americold-ufcw-discrimination-charge-12312012
This chapter contributed to the field of cold chain by highlighting the differences between the cold chain and the traditional supply chain. Different decision making levels of the cold chain are investigated: 2 for Strategic level decisions, 3.3.2 for Tactical level decisions, and 3.3.3 for Operational level decisions. The effects of cold chain on the main three segments of sustainability, which are: economic, environmental, and social are discussed for each level.

We conclude that there are several additional factors on cost, emission and social aspects for cold chain that do not exist for the traditional supply chains. In some limited fields, such as perishability of the cold items, several studies have been done. Yet, there are still a number of topics for cold chain that requires new models, or adjustments of current models.

3.4.1 Future research topic
In this section, we present a list of future research topics in the field of cold chain. In the author’s perspective, there is a need for developing new mathematical models for different components of the supply chain, as there are few currently developed.

1. **Strategic level problems:** the cost of cold facilities is on average, higher than their counterparts in traditional supply chain. They are also required to follow different additional regulations on the safety and hygienic sides, which makes them even more expensive. This raises the sensitivity of the decision on locating cold facilities. In our perspective, the facility location models for cold items should consider the following criteria, in addition to the common criteria followed in the traditional supply chain: (1) considering the availability by the end user in specific industries, such as food warehouses, vaccine and medical items warehouses, etc. these
2. **Tactical level problems:** In cold chains, the tactical decisions such as routing and allocation problems are often more complicated and different than traditional supply chains, and thus require development of new decision models. This complexity was brought to attention from all the three standpoints of economic, environmental and social aspects.

We have developed an inventory model for cold items that consider holding costs and emission for cold items, as well as transportation costs and emission in a single inventory model. This model will be presented in Chapter 4. We also have developed the multi-product version of this model, in Chapter 5. There are yet essential and interesting research topics that might be done in this field, such as: (1) the assignment/routing problem for cold items (which would be discussed in 5.2.1), (2) considering the perishability of the items, in the inventory model proposed in Chapter 4, (3) finding the Safety Stock (SS) level for perishable items, considering the shelf life time and other limitations in the model.

Research on product recall for cold items is another interesting topic. In some cases, the items needs to be returned in cold, while in some other cases, the product is required to be only collected from the customers/retailers and dumped. Designing a network of cold/ non-cold partitions for the two cases would be a problem to be solved.

In addition, considering all of these aspects (strategic-tactical decision problems, such as location-allocation, or location-inventory problems) in cold chains and considering these all in a decision making model is another missed part of this puzzle.
3.5 References


4 A NEW INVENTORY MODEL FOR COLD ITEMS THAT CONSIDERS COSTS AND EMISSIONS

4.1 Abstract

A new inventory model that considers both cost and emission functions is proposed for environments where temperature-controlled items need to be stored at a certain, non-ambient temperature and to do so modular temperature-control units are used. Transportation unit capacity and storage unit capacity are considered, which results in non-linear, non-continuous cost and emissions functions. A set of exact algorithms are developed to find the optimal order quantity based on cost and emission function minimization, and the mathematical proof of the optimality of the solutions are presented. Using a variety of parameter ratios, a set of experiments are run to show the effectiveness of the proposed model compared to the current model in the literature and to provide managerial insights into the cold item inventory problem. Optimum order quantity for cost function optimization and emission function optimization are compared against each other and the tradeoff between the functions is analyzed to provide insights.

4.2 Introduction

According to a study by the University of Michigan\textsuperscript{38}, the top two contributors to the Green House Gas (GHG) emissions are the electric and transportation sectors. In the electric sector, refrigerants are the second highest contributor. In the transportation sector, small and heavy duty trucks together form more than 50\% of the GHG emissions. Thus, policies that attempt to reduce

\textsuperscript{38} http://css.snre.umich.edu/css_doc/CSS09-05.pdf
emissions from transportation or refrigerant utilization have the potential to make an impact in the reduction of GHG emissions.

The handling, holding and transportation of temperature-sensitive products along a supply chain is known as the cold supply chain. Cold chain items are items that are required to be maintained in a specific temperature range. Examples of cold chain items are deep freeze items (-28 to -30 Celsius) such as seafood, frozen items (-16 to -20 Celsius) such as meat, chill items (2 to 4 Celsius) such as fruit, vegetables and fresh meat, and pharmaceutical items (2 to 8 Celsius), such as medications and vaccines (Rodrigue et al., 2013).

Numerous recent studies have focused on the emissions resulting from the cold supply chain (Calanche et al., 2013; Dekker et al., 2012; James & James, 2010; Wang et al., 2013). These studies consider the emissions from refrigerated trucks and transporters, cold warehouses, packaging and other components in the supply chain.

4.2.1 Problem statement
This paper examines an inventory model for cold items that considers temperature-controlled unit capacities associated with holding and transporting the cold items in a supply chain. In addition, both the cost and emission functions for such an environment are analyzed.

We model the environment where temperature-controlled items need to be stored at a certain, non-ambient temperature and to do so modular temperature-control units are used. Modular temperature-controlled units are found in industrial applications in the form of segmented industrial freezers, multiple walk-in coolers, or temperature-controlled rooms that are partitioned in the warehouse\textsuperscript{39}. Given the significant costs and emissions associated with creating a

\textsuperscript{39} \url{http://www.innovativecold.com/press_042010.htm}
temperature-controlled environment, many distribution centers operate using segmented or modular temperature-controlled units rather than a single temperature-controlled unit for the entire holding area. The power required for cooling is directly proportional to the size of the freezer, therefore, rather than refrigerating the whole area, instead, a number of temperature-controlled units inside the larger warehouse are used to keep the items cold. The advantage of such a design is that individual temperature-controlled units can be turned off to save cost and energy when they are not needed. Only when one unit reaches its capacity will the next unit be “turned on.” As a result, there is a fixed (setup) cost for holding a group of items, which results in a step function to represent the fixed cost of turning on temperature-controlled units, in addition to the variable cost of holding items based on the number of units held in inventory. Consequently, a linear holding cost and emission function is not applicable to model this environment.

The purpose of this paper is three-fold: 1) derive the mathematical structure and model the holding and transportation costs and emission functions in the described cold chain environment, 2) propose an exact solution procedure to solve the mathematical models, and 3) analyze the tradeoffs involved in making inventory decisions based on minimizing emissions versus minimizing cost in the described cold chain environment.

The remainder of this paper is organized as follows. Section 2 surveys the literature on inventory models that consider cost and sustainability factors. Section 3 introduces and discusses the different components of the problem, which are the holding and transportation cost and emission
functions, and derives the Cold Items Cost and Emission Minimization (CICEM) mathematical model. Section 4 presents the solution approaches for the developed models, and Section 5 presents our numerical examples. Section 6 provides a summary of our research and Section 7 offers suggestions for extending this research.

4.3 Literature Review

The inventory management problem is a well-defined, well-studied problem in the literature and interested readers are referred to the following review papers (Bijvank & Vis, 2011; Karaesmen et al., 2011; Khan, Jaber, Guiffrida, & Zolfaghari, 2011; Li, 2010; Paterson, Kiesmüller, Teunter, & Glazebrook, 2011; Qin, Wang, Vakharia, Chen, & Seref, 2011). The Economic Order Quantity (EOQ) was first presented by (Harris, 1913) and is the foundation for developing several research contributions in the field of inventory management.

The inventory problem has been studied considering other objective functions, in addition to, or instead of, the cost function (Franca, Jones, Richards, & Carlson, 2010; Rezaei & Davoodi, 2011; Tsai & Yeh, 2008; Rosič & Jammernegg, 2013). However, only recently has emission been considered in addition to the cost for the inventory problem. (Cano-Ruiz & McRae, 1998) illustrate that considering the environmental function as an objective function can lead to better solutions, rather than considering an environmental constraint to be satisfied.

(Benjaafar, Li, & Daskin, 2013) study the lot sizing problem considering the inventory policies impact on the carbon footprint. They study a single firm with carbon footprint caps with the objective to minimize the summation of the fixed and variable ordering costs, holding costs and backordering costs. They consider a linear function of the order quantity for the carbon footprint constraint. The authors model four environments: 1) cap on the carbon footprint, 2) carbon tax model, 3) cap-and-trade policy on carbon footprint, and 4) carbon offset model.
(X. Chen, Benjaafar, & Elomri, 2013) propose a carbon-constrained EOQ model that considers a carbon footprint, and model it as a constraint for the EOQ model. They derive the optimal order quantity with regard to the cost function and the emission function as a constraint. They explore the impacts of considering emissions as a constraint on the optimal order quantity for different scenarios and find that the solution is more sensitive to the changes in the emission function (constraint) than to the cost function (objective function). They also study the facility location problem with a constraint on the emission and find similar results.

(Song & Leng, 2012) study four carbon policies proposed by the Congressional Budget Office in a single period production planning problem to find the optimal production quantity. The problem has the same structure as the newsvendor problem, in addition to having a constraint on the emission. The four policies considered are carbon cap, carbon tax, cap-and-trade and carbon offset.

(Absi, Dauzère-Pérès, Kedad-Sidhoum, Penz, & Rapine, 2013) analyze a multi period lot sizing problem considering deterministic demand with inventory holding costs and fixed and variable production and transportation costs. They model the emission function as a constraint and define four different scenarios: 1) Periodic carbon emission constraint, 2) Cumulative carbon emission constraint, 3) Global carbon emission constraint, and 4) Rolling carbon emission constraint.

In all the above mentioned works, the emission function is considered as a constraint, and the cap for the associated emission is assumed to be given by a regulation. In contrast, (Bouchery, Ghaffari, Jemai, & Dallery, 2012) consider emission factors in the objective function of an inventory model. To the best of our knowledge, this is the only work that considers the emission function of holding as an objective function for the EOQ problem, which is optimized, along with the cost function. The authors identify a set of efficient frontier solutions with the goal of
minimizing the cost and emission functions. They use a posteriori analysis method to help the decision maker choose among the provided solutions by using past knowledge that reveals the decision maker’s utility functions. In another study, (Bonney & Jaber, 2011) examine some possible environmental consequences of common activities and suggest that an environmental aspect of all functions within the product life cycle including inventory planning and control should be considered. A simplified model is proposed to demonstrate how one could determine inventory parameters in an environmental context. In this model, the emission and costs associated with transportation are represented in the objective function. In addition to the inventory problem, the emission function has begun to be considered in other parts of the supply chain, including the transportation section. There are several research papers that are interested in minimizing the emission of transportation (Bastani, Heywood, & Hope, 2012; Grahn, Azar, & Lindgren, 2009; Ross Morrow, Gallagher, Collantes, & Lee, 2010; Safaei Mohamadabadi, Tichkowsky, & Kumar, 2009; Zahabi, Miranda-Moreno, Patterson, Barla, & Harding, 2012).

Concerning the transportation and emissions problems in a supply chain, (Ülkü, 2012) studies transportation from the perspectives of economics and the environment. His mathematical model includes the emission of packaging, the effects of load weight and traffic on fuel consumption (and hence, emission) of delivery vehicles. (Ji, Gunasekaran, & Yang, 2013) also consider transportation emission and recommend larger order sizes to reduce the cost and emission of packaging, but do not propose any formulation. (Pan, Ballot, & Fontane, 2013) propose mathematical models to study the environmental impact of pooling of supply chains. They use data from two French retail companies and calculate the emission using their developed optimization models for two rail and road transportation modes. The authors conclude that supply network pooling is an efficient approach in reducing CO2 emissions.
Table 4 characterizes each study by the problem of interest, as well as by how the authors model holding and transportation costs and emissions. Also, the studies are characterized by if the studies consider segmentation of the holding area or transportation unit capacity. As illustrated by Table 4, none of the studies consider the segmentation of the holding area. Also, none of the studies consider transportation unit capacity and quantity-dependent transportation cost with emission functions that incorporate the relationship between load weight and the fuel consumption in their inventory models. In addition, in all the works but one (by Bouchery et al., 2012), the holding emission is considered as a constraint, and not an objective function. As shown in Table 4, our contribution is the development of an inventory model for cold items that considers holding and transportation unit capacities for the cost and emissions objective functions. As a result of considering the holding and transportation unit capacities, we develop a non-linear model that requires developing a specific solution approach.

<table>
<thead>
<tr>
<th>Problem of interest</th>
<th>Holding cost</th>
<th>Holding emission</th>
<th>Segmentation of Holding Area</th>
<th>Transportation cost</th>
<th>Transportation emission</th>
<th>Transportation unit capacity</th>
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<td>Lot sizing</td>
<td>Objective Function-Linear</td>
<td>Constraint-Linear</td>
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<td>Objective Function-Linear</td>
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<td>Constraint-Linear</td>
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<td>Not modeled</td>
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<td>(Chen, et al. 2013)</td>
<td>Inventory model (EOQ)</td>
<td>Objective Function-Linear</td>
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<td>Problem of interest</td>
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<td>Bonney, M, Jaber</td>
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<td>Objective Function-Linear</td>
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<tr>
<td>Harris, F. W. (1913)</td>
<td>Inventory model (EOQ)</td>
<td>Objective Function-Linear</td>
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<td>Tsai, C.-Y., &amp; Yeh, S.-W. (2008)</td>
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<td>Ubeda, S., et al. (2011)</td>
<td>Transportation modeling</td>
<td>Not modeled</td>
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<td>Objective Function-Linear</td>
<td>Discrete points</td>
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<tr>
<td>This work</td>
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<td>Objective Function-Non-linear</td>
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<td>Objective Function -Non-linear</td>
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</table>
4.4 Problem Formulation

We consider a single warehouse shipping to a single Distribution Center (DC) that are both part of a cold chain. A single-period model is proposed that considers both holding and transportation unit capacity.

In this model, we assume:

- A single item needs to remain in a temperature-controlled environment.
- The demand for the item is deterministic.
- A set of identical temperature-controlled trucks with a known capacity are used for transporting the inventory from the warehouse to the DC.
- A set of identical temperature-controlled units (which we denote as freezer units) with a known capacity are used for holding the cold items at the DC.
- All the shipments happen at the end of each period.
- During each period, the state of the freezer unit does not change; if a freezer is turned on at the beginning of a period, it remains on for the whole period length.

In this model, we are making the order quantity decision for the DC (the downstream node) and the relevant costs are the holding cost at the DC and the transportation cost from the warehouse to the DC. We assume that all the shipments happen at the end of each period; therefore, the “state” of each freezer unit does not change during a period.

This inventory level decision-making model has two main objective functions: a cost function and an emission function. We note that both functions have two main components: cost and emissions of holding cold inventory, and cost and emissions of transporting cold inventory. In subsequent subsections, we formulate the proposed problem, with $D$ representing the total demand per period, and $Q$ representing the order quantity, which is our decision variable. We
refer to this model as the Cold Items Cost and Emission Minimization (CICEM) model in the remainder of the paper.

4.4.1 Cost function

This section introduces the cost function of the CICEM model. The cost function is the summation of transportation costs and holding costs for cold items; which are derived in Sections 4.1.1 and 4.1.2, respectively.

4.4.1.1 Transportation cost

First, we provide the list of parameters used in the formulation of the transportation cost function.

Parameters

- $F_t$: Fixed cost of using a truck unit for transportation ($/truck unit)
- $T_c$: Truck unit capacity (number of items)

The total transportation cost is

$$\text{Total Transportation Cost } TC(Q) = \frac{D}{Q} \times \left\lceil \frac{Q}{T_c} \right\rceil \times (F_t) \quad (1)$$

Where $\left\lceil \frac{Q}{T_c} \right\rceil$ represents the number of trucks used per delivery, and the term $\frac{D}{Q}$ is the number of deliveries required to satisfy the period demand for order quantity of $Q$. Assuming a single warehouse and a single DC implies that the transportation cost per delivery $(F_t)$ is a constant value.
Figure 5 shows the shape of the transportation cost function in equation (1) and illustrates graphically that the total transportation costs decrease as the order quantity increases, except at a jump discontinuity point. This occurs when $Q$ exceeds the truck unit capacity and a fixed cost is incurred for an additional truck.

### 4.4.1.2 Holding cost model

Model parameters

$F_h$: The cost of turning on one freezer unit ($/freezer unit/period$)

$b$: the capacity of each freezer unit (items/ unit)

$i$: interest rate per period

$c$: Cost of each item ($/item$)

The total holding cost is:

$$Total \; Holding \; Cost \; HC(Q) = \left[ \frac{Q}{b} \right] \times F_h + iCQ$$  \hspace{1cm} (2)
We assume the state of a freezer does not change during a period and all the shipments happen at the end of each period; therefore, the opportunity cost of capital for the average inventory is represented as $iCQ$ in our model.

Figure 6 is a graphical representation of the shape of the holding cost function and illustrates that the holding cost function has a constant non-decreasing rate within each interval enclosed between two consecutive jump discontinuities. When $Q$ exceeds the capacity of a freezer unit, we observe a jump discontinuity indicating the need to turn on an additional freezer unit, incurring the fixed cost $F_h$.

4.4.1.3 Total cost function

The total cost per period consists of the holding cost and the transportation cost as follows:

$$Total\ Cost\ C(Q) = \left( \left\lfloor \frac{Q}{b} \right\rfloor \times F_h + iCQ \right) + \left( \frac{D}{Q} \times \left\lfloor \frac{Q}{T_c} \right\rfloor \times (F_t) \right)$$  \hspace{1cm} (3)
Figure 7: Total cost function $C(Q)$

Figure 7 shows the holding, transportation and total cost functions per period. The total cost function $C(Q)$ has several break points where the function jumps. The break points represent reaching the capacity of the freezer units (see Section 3.1.2), the capacity of the truck (see Section 3.1.1), or both. Within an interval between two jump discontinuities, the function has a smooth behavior. In section 4, we study the function over these intervals and propose a solution method for finding the optimal order quantity.

4.4.2 Emission function

The emission function of the CICEM model has two components: the emissions due to transporting the cold items and the emissions due to holding cold inventory in the DC. We discuss both in the following subsections.
4.4.2.1 Transportation emission function

The decision variable, $Q$, affects the emission of transportation by determining the number of trucks and the number of shipments. As the weight of the cargo increases, the MPG of the truck decreases linearly; and more fuel is needed to transport a heavier truck than a lighter truck across the same distance (Gajendran & Clark, 2003; Hickman, Hassel, Joumard, Samaras, & Sorenson, 1999; Ubeda, Arcelus, & Faulin, 2011). The MPG of a truck loaded with $Q$ items is calculated as in (4):

$$\text{MPG}(Q) = \text{MPG}_0 - \alpha \times Q \times W_i$$ (4)

where $W_i$ is the weight of a unit item, $\alpha$ is the coefficient that represents the effect of weight on the MPG of the truck, and $\text{MPG}_0$ is the base MPG for the empty truck. In (4), as the truck load’s weight increases the MPG decreases. In related studies, $\alpha$ is usually obtained from regression models (Gajendran & Clark, 2003). Due to the linearity of the relationship between the MPG and the weight of the truck, the transportation emission is independent of the load distribution among the trucks (Gajendran & Clark, 2003; Hickman et al., 1999; Ubeda et al., 2011).

The total emission from transportation per period can be calculated as in (5):

$$\text{Total Emission of Transporting Cold items} = \text{TETC}(Q)$$

$$= CEGF \times \left( \frac{M}{\text{MPG}_0 - \alpha \times \frac{D}{Q} \times \left\lfloor \frac{Q}{T_c} \right\rfloor \times W_i} \right) \times \frac{D}{Q} \times \left\lfloor \frac{Q}{T_c} \right\rfloor$$

Which can be simplified to:
\[ TETC(Q) = CEGF \times \left( \frac{M}{MPG_0 - \alpha \times \frac{Q}{T_c} \times W_i} \right) \times \frac{D}{Q} \times \left[ \frac{Q}{T_c} \right] \quad (5) \]

In (5), \( \left[ \frac{Q}{T_c} \right] \) is the number of trucks shipped in each delivery, \( \frac{D}{Q} \times \left[ \frac{Q}{T_c} \right] \) is the total number of shipped trucks per period, \( M \) is the distance between the warehouse and DC (miles), and CEGF (Constant of the Emission of a Gallon of Fuel) is a constant term that represents the emission of burning one gallon of diesel fuel. The denominator of the ratio in the second term considers the truck MPG based on the average load weight of each truck delivery. In Equation (5) the total emission from transportation is related to the average load weight of each truck, the delivery’s distance, and the total number of delivered trucks per period.

Figure 8 shows the general shape of equation (5). Note that it has a similar shape to the transportation cost function of Figure 5.

![Figure 8: Total emission of transportation](image-url)
4.4.2.2 Holding emission function

As discussed in Section 3.1.2, to hold $Q$ items in the warehouse with freezers of unit capacity $b$, we determine the number of freezers to be turned on as: $\left\lfloor \frac{Q}{b} \right\rfloor$. Knowing the number of freezers, we are able to calculate the total energy consumption, and with a coefficient that converts energy to carbon footprint, we can calculate the emission function of holding inventory, as follows:

$$\text{Total emission of holding cold items (TEHC)} = \left\lfloor \frac{Q}{b} \right\rfloor \times TECF \times TCFE$$  \hspace{1cm} (6)

In (6), TECF is the “Total Energy Consumption of a Freezer” and TCFE is “Total Carbon Footprint of 1kWh Energy”\footnote{http://www.epa.gov/cleanenergy/energy-resources/refs.html}. These values may be obtained from the research and reports on this field\footnote{http://www1.eere.energy.gov/buildings/appliance_standards/residential/pdfs/refrig_nopr_tsd_2010-09-23.pdf, Page 241}.
Figure 9: Total emission of holding cold items

Figure 9 shows the total emission of holding cold items and illustrates that the emissions from holding cold inventory is constant over each period enclosed between two jump discontinuities. When $Q$ exceeds the capacity of a freezer, we observe a jump discontinuity indicating the need to turn on an additional freezer, incurring the fixed emission of an extra freezer.

### 4.4.2.3 Total emission function

The total emission function can be obtained as the summation of the total emission from holding cold items (TEHC) as represented in (6) and the total emission of transporting cold items (TETC) as in (5):

**Total emission function**

\[
\text{Total emission function} = CEGF \times \left(\frac{M}{MPG_0 - \alpha \times \frac{Q}{T_c} \times W_i}\right) \times \frac{D}{Q} \times \left[\frac{Q}{T_c}\right] + \left[\frac{Q}{b}\right] \times \text{TECF} \times \text{TCFE} \quad (7)
\]
Figure 10: Total emission function

Figure 10 illustrates the graphical representation of the total emission function caused by both holding inventory and transportation. The total emission function has similar properties to the total cost function as it has break points that correspond to multipliers of freezer or truck unit capacity or the number of trucks shipped per delivery.

4.5 Solution Approach

In this section we develop solution algorithms to find the optimum order quantity ($Q^*$) for the cost and emission functions separately. Next, we discuss the solution for the dual-objective problem that includes both the cost function and emission function.
4.5.1 Finding $Q^*$ that minimizes the total cost function

In this section we analyze and propose a solution algorithm for the emission or cost function. We present two cases for the cost function: the special case for which $iC = 0$, and the general case when $iC > 0$.

4.5.1.1 Cost function when: $iC = 0$

We propose a solution method for the cost function when $iC=0$. This special case for the cost function is valid when the opportunity cost of capital tied up in inventory of an item is negligible compared to the cost of controlling the inventory’s temperature. This is applicable for frozen food items that generally have low unit price. For these items, the cost of keeping the temperature at freezing (or below freezing) levels is typically higher than the opportunity cost of capital.

The cost function in equation (3) is comprised of two separable functions: transportation and holding cost functions. Due to the presence of the ceiling function that ensures an integer number of trucks are used and the resulting discontinuity, the transportation cost function is not differentiable on the whole domain, but only on the intervals. Therefore, we break the function into multiple intervals. We make the function differentiable on each interval by substituting the ceiling function with associated integer value, which is a constant number ($s$) over each interval.

Specifically, the discontinuities occur when $\frac{Q}{T_c}$ is an integer, and the term $\left\lfloor \frac{Q}{T_c} \right\rfloor$ has a constant value on any interval of the form: $(s \times T_c, (s + 1) \times T_c]$. The derivative of the transportation cost over each interval is:

$$\frac{d(\text{Transportation Cost})}{d(Q)} = \frac{d}{d(Q)} \left( \frac{D}{Q} \times \left\lfloor \frac{Q}{T_c} \right\rfloor \times (F_t) \right) = -\frac{D \times s \times F_t}{Q^2} \quad (8)$$
With similar reasoning for the holding cost function, \([Q_b]\) is substituted by its value over each period \((t)\), which transforms the holding cost function over each interval to the form:

\[(t + 1) \times F_h + iCQ \quad (9)\]

and the derivative over each interval is:

\[
\frac{d(\text{Holding Cost})}{d(Q)} = \frac{d((t + 1) \times F_h + iCQ)}{d(Q)} = iC \quad (10)
\]

Since in this special case \(iC = 0\), the derivative becomes zero:

\[
\frac{d(\text{Holding Cost})}{d(Q)} = 0
\]

This result is not surprising because when \(iC = 0\) the holding cost is flat on an interval. Consequently, the first derivative of the total cost function with respect to \(Q\) over each interval is:

\[
\frac{d(\text{Total Cost})}{d(Q)} = \frac{d(\text{Transportation Cost})}{d(Q)} + \frac{d(\text{Holding Cost})}{d(Q)} = -\frac{D \times s \times F_t}{Q^2} \quad (11)
\]

As shown in (11), the derivative of the cost function over each interval is negative; therefore, it is a decreasing function over each interval.

4.5.1.1.1 Solution Procedure

We design our solution procedure based on the structural characteristics of the cost function.

**Proposition 1**: For a continuous function, monotonic on an interval, the minimum value occurs at the beginning of the interval if it is increasing, or at the end of the interval if it is decreasing.

**Proof**: The minimum of a continuous function over an interval is \(x^*\) for which:

\[
\forall x \in \text{Interval}: f(x^*) \leq f(x)
\]
Since the total cost function is continuous and decreasing over each interval, it has a negative slope, 
\[ f(x + \sigma) \geq f(x) \text{ for } \sigma \geq 0. \]

Therefore the last point on the interval is the minimum of the cost function over the specified interval \([a, b]\).

For the case of an increasing function, \(x^*\) is the minimum:

\[ \forall x \in \text{Interval}: f(x^*) \leq f(x) \]

Since the slope is positive for the increasing function, \(f(x) \leq f(x + \sigma)\) holds for all the points on the interval, the minimum occurs on the beginning point of the interval: \([a, b]\)  \(\therefore\).

The cost function \(C(Q)\) is a decreasing function, and its derivative is negative over each interval. Therefore, based on Proposition 1, the minimum value of the function on each interval occurs at the end of that interval. The end point on each interval means the largest integer value on the interval.

Based on the discussion above, we need to consider only the endpoint (largest integer point) of each interval. Therefore, a solution procedure to find the minimum value is to calculate the cost function value for all of the end points and choose the endpoint with the smallest function value.

We develop algorithm (1) to find the minimum cost point.
Considering \( n = S + T \), the order of this algorithm is \( O |n| \). It has a one-time sort, no inter loops, and the outer loop is visited at most \( (S+T) \) times (i.e., the total number of break points).

### 4.5.1.2 Cost function in the case \((iC > 0)\)

In the general case we consider the opportunity holding costs and the holding costs for temperature control. This case is applicable when the item held has a high unit cost (i.e., temperature controlled pharmaceuticals).

To find the minimum value of the total cost, we apply a modified version of what we proposed for the special case when \( iC = 0 \). For the general case, the derivative of the transportation cost function is:

\[
\frac{d}{dx} [C(x)] = \begin{cases} 
T & \text{if } x < b \\
T & \text{if } x > b \\
0 & \text{otherwise}
\end{cases}
\]

This derivative is used in the algorithm to find the minimum value of the total cost.
function remains the same; however, the derivative of the holding cost function has an extra term, which is $iC \left( \frac{d\left(iCQ\right)}{d\left(Q\right)} \right)$. Therefore, we have:

$$\frac{d(\text{Total Cost})}{d(Q)} = - \frac{D \times s \times F_t}{Q^2} + iC $$ (12)

Here, we have:

$$\begin{cases} 
- \frac{D \times s \times F_t}{Q^2} + iC \leq 0; & \text{if } Q \leq \sqrt{\frac{D \times s \times F_t}{iC}} \\
- \frac{D \times s \times F_t}{Q^2} + iC \geq 0; & \text{if } Q \geq \sqrt{\frac{D \times s \times F_t}{iC}}
\end{cases} $$ (13)

Therefore, for the values of $Q$ less than or equal to $\sqrt{\frac{D \times s \times F_t}{iC}}$, the $C(Q)$ has a negative value for its slope, and is decreasing, but for the values of $Q$ greater than $\sqrt{\frac{D \times s \times F_t}{iC}}$, the $C(Q)$ has a positive slope and is increasing. Based on proposition 1, the intervals with $Q \leq \sqrt{\frac{D \times s \times F_t}{iC}}$ has the minimum point occurs at their end points (i.e., right most), while for the intervals with $Q \geq \sqrt{\frac{D \times s \times F_t}{iC}}$, their minimum point occurs at the beginning (i.e., left most, the smallest integer value on the interval) point of the interval. Algorithm 2 is developed to find the minimum of the $C(Q)$ function for the general case.
Algorithm (2): Finding the minimum value for the general case of $iC > 0$
For the general case, the developed algorithm for the special case is modified to find the minimum cost point consisting of the following steps.

Step 1: find all the break points for the holding cost function (S points)
Step 2: find all the break points for the transportation cost function (T points)
Step 3: sort all the break points from step 1 and 2 in the order of smallest to largest and save those in Array A
Step 4: setup initial values:
  Global minimum = infinity;
  Previous Total Cost Value = 0;
  Current Total Cost Value = 0;
  K = 0;
Step 5:
while K <= S+T
  K = K+1;
  if $Q < \frac{D \times S \times F_t}{iC}$
    (Previous Total Cost Value) = (Current Total Cost Value);
    Current Point: A[K];
    Calculate the Total cost of current end (most right) point, using equation (3)
    Current Total Cost Value = Total cost of current point
    if (Total cost of current point) $\leq$ global minimum
      Global minimum = (Total cost of current point)
      Optimum point = A[k]
    end
  else
    (Previous Total Cost Value) = (Current Total Cost Value);
    Current Point: A[K];
    Calculate the Total cost of current beginning (most left) point, using equation (3)
    Current Total Cost Value = Total cost of current point
    if (Total cost of current point) $\leq$ global minimum
      Global minimum = (Total cost of current point)
      Optimum point = A[k]
    end
  end
end
4.5.2 Finding $Q^*$ that minimizes the total emission function

We propose a solution method for the emission function. The emission function in equation (7) is comprised of two separable functions: transportation and holding emission functions.

A similar approach for the transportation and holding emission functions is taken as in 4.5.1. by substituting the integer value of the ceiling functions over each interval.

The derivative of the transportation emission over each interval follows from the formula below:

$$\frac{d(T_{\text{Transportation Emission}})}{d(Q)} = \frac{d}{d(Q)} \left( CE GF \times \frac{M}{(MPG_0 - \alpha \times \frac{Q}{T_c} \times W_i)} \times \frac{D}{Q} \times \left\lfloor \frac{Q}{T_c} \right\rfloor \right)$$

$$= CE GF \times D \times s \times M \left( \frac{MPG_0 - 2 \times \alpha \times \frac{Q}{s} \times W_i}{(MPG_0 - \alpha \times \frac{Q}{s} \times W_i)^2} \right) \quad (14)$$

With similar reasoning for the holding cost function, $\left\lfloor \frac{Q}{b} \right\rfloor$ is substituted by its value over each period, which transforms the holding cost function (6) over each interval to the form:

$$H_{\text{Holding Emission}} = t \times TECF \times TCFE$$

And the derivative over each interval is:

$$\frac{d(H_{\text{Holding Emission}})}{d(Q)} = \frac{d(t \times TECF \times TCFE)}{d(Q)} = 0 \quad (15)$$

Consequently, the first derivative of the total emission function with respect to $Q$ over each interval is:
\[
\frac{d(Total\ Emission)}{d(Q)} = \frac{d(Transportation\ Emission)}{d(Q)} + \frac{d(Holding\ Emission)}{d(Q)}
\]

\[
= CEGF \times D \times s \times M \left( \frac{-\left( MPG_0 - 2 \times \alpha \times \frac{Q}{s} \times W_i \right)}{\left( MPG_0 - \alpha \times \frac{Q}{s} \times W_i \right)^2} \right) \quad (16)
\]

### 4.5.2.1 Solution procedure

The solution procedure for the emission function is similar to the cost function in the general case. Here, we borrow the main idea from 4.5.2 and proposition 1. The emission function has a derivative, which is negative for small values of \( Q \), and positive for larger values of \( Q \), after a certain point. Therefore, the general shape of the function can be considered as a convex function with a minimum point.

In (16), we know: \( CEGF \times D \times s \times M \) and \( (MPG_0 - \alpha \times \frac{Q}{s} \times W_i)^2 \) are always positive.

Therefore, we need to check the sign of the term \( (MPG_0 - 2 \times \alpha \times \frac{Q}{s} \times W_i) \) in (16).

In specific, if we solve \( MPG_0 = 2 \times \alpha \times \frac{Q}{s} \times W_i \) for \( Q \), we will have:

\[
Q_0 = \frac{s \times MPG_0}{2 \times \alpha \times W_i} \quad (17)
\]

For values of \( Q \) less than or equal to \( Q_0 \) the emission function is decreasing over each interval, and for values of \( Q \) greater than \( Q_0 \) the emission function is increasing over each interval. Therefore, similar to 4.1.2, the minimum points happen at the end point of an interval, if \( Q \leq Q_0 \), or the beginning point, if \( Q \geq Q_0 \). The solution algorithm proposed in 4.1.2 can be modified to reflect this change. The modified algorithm (Algorithm 3) to find the minimum of the emission function is as follows.
Algorithm (3): Finding the minimum value for the emission function
For the general case, the developed algorithm for the special case is modified to find the minimum emission point consisting of the following steps.
Step 1: find all the break points for the holding emission function (S points)
Step 2: find all the break points for the transportation emission function (T points)
Step 3: sort all the break points from step 1 and 2 in the order of smallest to largest and save those in Array A
Step 4: setup initial values:
   Global minimum = infinity;
   Previous Total Emission Value = 0;
   Current Total Emission Value = 0;
   \( K = 0; Q_0 = \frac{s \times MPG_0}{2 \times \alpha \times W_i} \)
Step 5:
while \( K \leq S + T \)
   \( K = K + 1; \)
   if \( Q \leq Q_0 \)
      (Previous Total Emission Value) = (Current Total Emission Value);
      Current Point: A[K];
      Calculate the Total emission of current end point, using equation (3)
      Current Total Emission Value = Total emission of current point
      if (Total emission of current point) \leq \text{global} \ \text{minimum}
      Global minimum = (Total emission of current point)
      Optimum point = A[k]
   end
   else
      (Previous Total Emission Value) = (Current Total Emission Value);
      Current Point: A[K];
      Calculate the Total emission of current beginning point, using equation (3)
      Current Total Emission Value = Total emission of current point
      if (Total emission of current point) \leq \text{global} \ \text{minimum}
      Global minimum = (Total emission of current point)
      Optimum point = A[k]
   end
end
4.5.3 Finding the dominant (frontier) set of solutions for the CICEM model

In order to find a unique order quantity, we need to develop a set of frontier solutions to partially satisfy both economic and emission functions. In this section, we provide the procedure to generate the set of frontier solutions.

Let \( Q_1^* \) be the optimal order quantity for the cost function; let \( Q_2^* \) be the optimal order quantity for the emission function. In proposition 2 we show that the frontier set occurs only on the interval \([Q_1^*, Q_2^*]\).

**Proposition 2:** Without loss of generality, we can assume that \( Q_1^* < Q_2^* \).

The frontier solution set occurs only on the interval of \([Q_1^*, Q_2^*]\).

**Proof:** Let us consider any point out of this interval. It can be on the left side of the interval, such as: \( Q_3 < Q_1^* \) or on the right side of the interval: \( Q_4 > Q_2^* \).

We start with \( Q_3 \). For \( Q_3 \), we know that Cost \((Q_3) > Cost (Q_1^*)\), since \( Q_1^* \) is the optimal point and Emission \((Q_3) > Emission (Q_1^*)\), since \( Q_1^* \) is closer to \( Q_2^* \), the optimal value for the emission function. Since the second derivative of the emission function is positive, it has one minimum and it increases as the points are further from its minimum. Therefore, \( Q_1^* \) outperforms \( Q_3 \).

Similar reasoning can be stated for \( Q_4 \) to show that the interval of \([Q_1^*, Q_2^*]\) cannot be outperformed by any other points.

### 4.6 Numerical Experiments

In this section we conduct numerical experiments to study the CICEM model and solution approaches developed in Sections 3 and 4 under different parameters values. We conduct the experiments using ratios of two parameters, rather than absolute values. The goal of our numerical experiments is two-fold: First, we compare the solution from the CICEM to the solution from the EOQ model (for the cost objective) and the SOQ model (for the emissions
objective) to provide insights into when it is appropriate to use the CICEM model and when it is reasonable to approximate the solution with the EOQ or SOQ model. Second, we analyze the impact of considering a financial versus an environmental objective to set the order quantity for environments where temperature-controlled items need to be stored at a certain, non-ambient temperature and to do so modular temperature-control units are used.

We group the experiments into three categories. For the first two categories, we test and compare the CICEM model against the EOQ model for the financial objective. First, we consider the cost function when $iC=0$ and second we consider the cost function when $iC>0$. In the third category, we consider the emission function based on the CICEM model and the SOQ model. Finally, we compare the emission function against the cost function and study the tradeoff between the two functions.

4.6.1 Adjusting the parameters for the EOQ model

To use the EOQ model to set the order quantity in an environment with holding and transportation unit capacities, we adjust the parameters for the EOQ model according to Table 5.

As an example, assume the holding cost to turn on a freezer for our model is $1000 and the freezer has a unit capacity of 100 items. To create a comparable holding cost for the EOQ model, we divide $1000 by 100 to get the holding cost per unit, which is: $1000/100=$10. Similarly, the transportation cost is also adjusted for the EOQ model as given in Table 5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CICEM</th>
<th>EOQ equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit holding cost</td>
<td>$\left\lfloor \frac{Q}{b} \right\rfloor \times F_h + ic$</td>
<td>$\frac{F_h}{b} + ic$</td>
</tr>
<tr>
<td>Unit transportation cost</td>
<td>$\frac{D}{Q} \times \left\lfloor \frac{Q}{T_c} \right\rfloor \times F_t$</td>
<td>$\frac{F_t}{T_c} \times D$</td>
</tr>
</tbody>
</table>

Table 5: Parameter adjustment from CICEM values to EOQ values

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4.6.2 The case for the cost function when iC = 0

In this section, we conduct our numerical experiments using the solution approach developed in 4.1.1 and consider the objective function of minimizing the total cost function for the case when iC=0.

4.6.2.1 Set #1: Effect of the relative change of holding cost to transportation cost

For the first set of experiments we develop a set of scenarios by changing the value of the ratio of holding cost to transportation cost, which we denote with λ. This experiment allows us to study the primary trade-off explored by the traditional EOQ model, specifically balancing holding and transportation cost.

\[
\lambda = \frac{\text{holding cost of a freezer}}{\text{transportation cost of a single shipment}} = \frac{F_h}{F_t}
\]  

(18)

In the Table 6, we present seven scenarios with different values for this ratio, with the demand fixed at 2500. The results show that for different values of λ the solution for the EOQ model and the CICEM model result in different optimal order quantities. This occurs because the EOQ assumes a linear increase in the holding cost function, which does not consider the unit capacities associated with holding and transportation in the cold chain. Based on this experiment, for different values of λ, the CICEM outperforms the EOQ model by 49-58%. Therefore, using the EOQ instead of the CICEM can result in a considerably higher cost. For example, using the EOQ model (rather than the CICEM) for λ =0.5, increases the cost by 58%, for λ=1.5 the increase is 57 % and for λ= 5, the increase is 58%.
4.6.2.2 Set #2: Effect of the total demand value

In this section, we study the effect that demand volume has on the solution of the CICEM method and compare the solutions produced with the CICEM model to the solutions from the EOQ model. To do so we define the following ratio:

$$\delta = \frac{\text{Demand}}{\text{Holding capacity of a freezer}} = \frac{D}{b} \quad (19)$$

We consider the range of (10, 250) for $\delta$, which is equivalent when $D=2500$ to having the unit holding capacity as low as 10 units and as high as 250 units. In Table 7 we present the percent increase in the cost objective function if the EOQ model is used instead of the CICEM model for each value of $\delta$. For this experiment, the CICEM outperforms the EOQ model with a range between 37% and 71%. This difference is due to the different structure of the two modeling approaches: the CICEM considers the case where batches of cold items are kept together, and a single holding cost is paid for the whole batch, while in the EOQ model, each item has its own holding cost. The structure of the CICEM model is similar to the general form of a step function, while EOQ presents a continuous model. Based on our experiments, it is recommended to use CICEM model rather than EOQ model in an environment with capacitated temperature-controlled units for holding and transportation.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2.5</th>
<th>3.5</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q* from EOQ</td>
<td>1000</td>
<td>707</td>
<td>577</td>
<td>447</td>
<td>377</td>
<td>316</td>
<td>223</td>
</tr>
<tr>
<td>% increase in Cost Objective from using EOQ versus CICEM</td>
<td>58%</td>
<td>57%</td>
<td>57%</td>
<td>49%</td>
<td>56%</td>
<td>58%</td>
<td>49%</td>
</tr>
<tr>
<td>Q* from CICEM</td>
<td>300</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 7: Percent increase in cost objective from using the EOQ versus the CICEM model for different demand ratios

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \delta = 10 )</th>
<th>( \delta = 25 )</th>
<th>( \delta = 50 )</th>
<th>( \delta = 100 )</th>
<th>( \delta = 200 )</th>
<th>( \delta = 250 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>52%</td>
<td>58%</td>
<td>71%</td>
<td>53%</td>
<td>47%</td>
<td>52%</td>
</tr>
<tr>
<td>1</td>
<td>60%</td>
<td>57%</td>
<td>62%</td>
<td>60%</td>
<td>49%</td>
<td>42%</td>
</tr>
<tr>
<td>1.5</td>
<td>62%</td>
<td>57%</td>
<td>62%</td>
<td>37%</td>
<td>53%</td>
<td>43%</td>
</tr>
<tr>
<td>2.5</td>
<td>47%</td>
<td>49%</td>
<td>58%</td>
<td>49%</td>
<td>42%</td>
<td>59%</td>
</tr>
<tr>
<td>3.5</td>
<td>44%</td>
<td>56%</td>
<td>39%</td>
<td>47%</td>
<td>52%</td>
<td>54%</td>
</tr>
<tr>
<td>5</td>
<td>48%</td>
<td>58%</td>
<td>49%</td>
<td>58%</td>
<td>63%</td>
<td>49%</td>
</tr>
<tr>
<td>10</td>
<td>64%</td>
<td>49%</td>
<td>58%</td>
<td>63%</td>
<td>58%</td>
<td>47%</td>
</tr>
</tbody>
</table>

4.6.3 The case for cost function when \( iC > 0 \)

In this section we consider the general case of the holding costs of cold items, which includes the capital required for holding inventory, as well as the cost of refrigeration and temperature control, \( iC > 0 \). We introduce \( \gamma \), which defines the relationship between the item cost and the transportation cost:

\[
\gamma = \frac{\text{unit price for item (\$)}}{\text{fixed cost of transportation (\$)}} = \frac{C}{F_T} \tag{20}
\]

With this ratio defined, we are able to study a broad range of cold items in our analysis. Products such as ice cream or frozen food have a relatively low item costs. In addition, because they are stored and transported frozen (about -22 F or less), the refrigeration costs for transportation and holding are high. In comparison, pharmaceutical items are more expensive, and because most are required to be stored and transported in (60-70 F) their refrigeration costs for transportation and storage are not as expensive as for frozen food. Such differences in unit prices are reflected in the nominator of the \( \gamma \) ratio, while the transportation refrigeration costs are reflected in the denominator. Several values for this ratio are considered, and the results are summarized in Table 8 for the optimal order quantity and Table 9 for the percent increase in the cost objective.
function of using the EOQ model versus the CICEM model. We set the $\delta$ value to be 25 for this set of experiments.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.05</th>
<th>0.25</th>
<th>0.5</th>
<th>1.25</th>
<th>2.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>CICEM</td>
<td>EOQ</td>
<td>CICEM</td>
<td>EOQ</td>
<td>CICEM</td>
<td>EOQ</td>
</tr>
<tr>
<td>0.5</td>
<td>500</td>
<td>559</td>
<td>300</td>
<td>354</td>
<td>300</td>
<td>267</td>
</tr>
<tr>
<td>1</td>
<td>300</td>
<td>439</td>
<td>300</td>
<td>316</td>
<td>300</td>
<td>250</td>
</tr>
<tr>
<td>1.5</td>
<td>300</td>
<td>373</td>
<td>300</td>
<td>289</td>
<td>200</td>
<td>236</td>
</tr>
<tr>
<td>2.5</td>
<td>300</td>
<td>299</td>
<td>200</td>
<td>250</td>
<td>200</td>
<td>213</td>
</tr>
<tr>
<td>3.5</td>
<td>200</td>
<td>257</td>
<td>200</td>
<td>224</td>
<td>200</td>
<td>196</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>156</td>
<td>100</td>
<td>147</td>
<td>100</td>
<td>139</td>
</tr>
</tbody>
</table>

Table 8 reports the optimum order quantity for each combination of $(\lambda, \gamma)$ for the CICEM and EOQ models, which enables us to investigate the effect of each parameter, as well as the effect of changing both parameters simultaneously, on the optimum order quantity. As an example, for the case of $\lambda = 5$ and $\gamma = 0.05$, the CICEM model sets $Q^* = 200$, while the EOQ model sets $Q^* = 217$.

Table 9 has related information on the percent increase in the cost objective function of using the EOQ model versus the CICEM model to set $Q^*$. As is shown in Table 9, for larger values of $\gamma$, the difference between the EOQ and the CICEM reduces, in most of the cases. As an example, for the case of $\lambda = 3.5$ and $\gamma = 0.05$, if the EOQ model is used to find the optimum order quantity, a 13% increase in the cost function compared to the case that CICEM is used to find the optimum order quantity occurs. However, when $\lambda = 3.5$ and $\gamma = 5$, the percentage increase is 0%.

This behavior may be explained as the relative effect of the unit price. When the unit price is low, (small values for $\gamma$), transportation and holding costs dominate the cost function, but as the
unit price increases (for expensive items, with higher values of γ), the unit price and the term “iC” plays a more significant role in determining the order quantity and as a result, the CICEM model can be approximated using the EOQ model.

Table 9: Percent increase in cost objective function values of using the EOQ model versus the CICEM model for different values of λ and γ

<table>
<thead>
<tr>
<th>λ</th>
<th>γ</th>
<th>0.05</th>
<th>0.25</th>
<th>0.5</th>
<th>1.25</th>
<th>2.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1%</td>
<td>5%</td>
<td>5%</td>
<td>1%</td>
<td>1%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2%</td>
<td>3%</td>
<td>0%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>8%</td>
<td>3%</td>
<td>3%</td>
<td>1%</td>
<td>2%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>5%</td>
<td>1%</td>
<td>7%</td>
<td>4%</td>
<td>3%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>13%</td>
<td>8%</td>
<td>3%</td>
<td>5%</td>
<td>6%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>14%</td>
<td>12%</td>
<td>11%</td>
<td>5%</td>
<td>7%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>16%</td>
<td>14%</td>
<td>11%</td>
<td>4%</td>
<td>9%</td>
<td>1%</td>
<td></td>
</tr>
</tbody>
</table>

4.6.4 Emission function versus emission function

In this section, we compare the emission objective function values resulting from using the CICEM model against the emission function values using the model developed by Bouchery et al. (2012), which is the environmental version of the EOQ model or the sustainable order quantity (SOQ) model, as named by the authors. In order to do so, we adjust the variables from the CICEM for the SOQ model. This variable adjustment is summarized in Table 10.

Table 10: Parameter adjustment from CICEM values to SOQ values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CICEM</th>
<th>SOQ equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Holding emission</td>
<td>|Q| \times TECF \times TCFE</td>
<td>|TECF \times TCFE| \times \tfrac{b}{b}</td>
</tr>
<tr>
<td>Unit Transportation emission</td>
<td>(\left(\frac{M}{MPG_0 - \alpha \times \frac{Q}{T_c} \times W_i}\right) \times \frac{D}{Q} \times \frac{Q}{T_c})</td>
<td>(\left(\frac{M}{MPG_0 - \alpha \times T_c \times W_i}\right) \times \frac{D}{T_c})</td>
</tr>
</tbody>
</table>
The parameters that are used to run the experiments are given in Table 11. Equation 7 is used to calculate the total emission function for the CICEM model. For the SOQ model, the optimal order quantity and the total emission are calculated as derived by (Bouchery et al., 2012):

\[ Q_e^* = \sqrt{\frac{2 \times O_e \times D}{h_e}} \]  
(21)

And

\[ E (SOQ) = \frac{O_e \times D}{Q_e^*} + \frac{Q_e^* \times h_e}{2} \]  
(22)

Table 11: Parameters used for dual objective trade-off set of experiments

<table>
<thead>
<tr>
<th>Demand</th>
<th>2500</th>
<th>Units/period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation cost</td>
<td>200</td>
<td>USD</td>
</tr>
<tr>
<td>Transportation capacity</td>
<td>500</td>
<td>Units</td>
</tr>
<tr>
<td>Empty truck weight</td>
<td>4500</td>
<td>Pounds</td>
</tr>
<tr>
<td>Weight of each item</td>
<td>5</td>
<td>Pounds</td>
</tr>
<tr>
<td>Distance</td>
<td>100</td>
<td>Miles</td>
</tr>
<tr>
<td>CO2 of 1 gallon diesel fuel</td>
<td>22.38</td>
<td>Pound</td>
</tr>
<tr>
<td>Holding Capacity</td>
<td>100</td>
<td>Units</td>
</tr>
<tr>
<td>Total Inventory Capacity</td>
<td>1000</td>
<td>Units</td>
</tr>
<tr>
<td>Emission of a freezer per period</td>
<td>1000</td>
<td>Pounds (of CO2)</td>
</tr>
<tr>
<td>i (interest rate per period)</td>
<td>12%</td>
<td>USD/period</td>
</tr>
<tr>
<td>C (item price)</td>
<td>250</td>
<td>USD</td>
</tr>
</tbody>
</table>

We calculate the optimum order quantity based on the CICEM model considering the emission function only (denoted as \( Q_{Emission}^* \)), and the associated total emission function value. Next, we calculate the total emission function based on the SOQ model, as in Equation 22. As discussed in
5.2 and 5.3, we define a ratio that ties the holding emission and transportation emission. This would help on being able to analyze a wider range of values, and make a more general conclusion. For this purpose, “θ” is defined as follows:

\[
\theta = \frac{\text{emission of one freezer per period}}{\text{emission from burning 1 gallon of fuel}} = \frac{\text{TECF} \times \text{TCFE}}{\text{CEGF}} \quad (23)
\]

Based on the Table 11, the current value for θ is:

\[
\theta = \frac{1000}{22.38} = 44.68
\]

Table 12 summarizes the experiments’ results.

| Table 12: Percent increase in emission objective from using the SOQ versus the CICEM model for different demand and θ ratios |
|-----------------|---|---|---|
| θ               | δ = 5 | δ = 25 | δ = 50 |
| 22.34           | 15%  | 78%  | 126%  |
| 44.68           | 35%  | 95%  | 141%  |
| 67.02           | 43%  | 99%  | 141%  |
| 111.70          | 48%  | 97%  | 133%  |
| 446.8           | 49%  | 88%  | 98%   |

According to the Table 12, for larger values of δ, the CICEM model performs better, for the same value of θ, than the SOQ model. Therefore, we report only the three first set of experiments with smaller values for the demand ration (δ), just to show the trend and how well the CICEM respond to the structure of the emission for cold items.

4.6.5 Cost versus emission function: The trade-off

In this section, we analyze the tradeoff between the cost function and emission function to help the decision maker understand the impact of considering each objective independently (either
solely costs or solely emissions). For this purpose, a set of experiments are run with parameters as shown in Table 11 and the results are reported in Table 13.

Equation 3 and Equation 7 are used to calculate the total cost and emission functions, respectively. For each value of λ, we calculate the optimum order quantity based on the CICEM model considering the cost function only (denoted as $Q^*_{\text{Cost}}$), and the associated total cost function value. Next, we calculate the total emission function based on if we ordered $Q^*_{\text{Cost}}$. We then calculate the optimum order quantity based on the emission function only (denoted as $Q^*_{\text{Emission}}$). Finally, we determine the cost function using $Q^*_{\text{Emission}}$.

<table>
<thead>
<tr>
<th>λ</th>
<th>$Q^*_{\text{Cost}}$</th>
<th>$Q^*_{\text{Emission}}$</th>
<th>Using $Q^*_{\text{Emission}}$ for both functions</th>
<th>Using $Q^*_{\text{Cost}}$ for both functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost</td>
<td>Emission</td>
<td>% deviation from optimal</td>
<td>% deviation from optimal</td>
</tr>
<tr>
<td>0.5</td>
<td>300</td>
<td>200</td>
<td>4%</td>
<td>---</td>
</tr>
<tr>
<td>1.0</td>
<td>300</td>
<td>200</td>
<td>1%</td>
<td>---</td>
</tr>
<tr>
<td>1.5</td>
<td>200</td>
<td>200</td>
<td>0%</td>
<td>---</td>
</tr>
<tr>
<td>2.5</td>
<td>200</td>
<td>200</td>
<td>0%</td>
<td>---</td>
</tr>
<tr>
<td>3.5</td>
<td>100</td>
<td>200</td>
<td>3%</td>
<td>---</td>
</tr>
<tr>
<td>5.0</td>
<td>100</td>
<td>200</td>
<td>6%</td>
<td>---</td>
</tr>
<tr>
<td>10.0</td>
<td>100</td>
<td>200</td>
<td>22%</td>
<td>---</td>
</tr>
</tbody>
</table>
We use Table 13 to illustrate how our models can aid decision makers with a better understanding of the tradeoffs between the two objective functions. As an example with $\lambda=0.5$, $Q_{Emission}^*$ is equal to 200, while the $Q_{Cost}^*$ is equal to 300. The emission function value using $Q_{Cost}^* = 300$ results in a 42% increase in the emission function. On the other hand, using $Q_{Emission}^*$ to calculate the cost function results in only a 4% increase in the cost function value. This simple analysis illustrates that if the decision maker considers only the emission function to be optimized, it causes the cost function to increase by 4%, while optimizing the cost function only and ignoring the emission function ends in an increase of 42% in the emission function.

Due to the structure of the two functions within each interval, the emission function is more sensitive to deviation from optimality than the cost function when $iC > 0$. As shown in Figure 7, the cost function is the summation of the increasing holding cost (due to the presence of $iC$) and the decreasing transportation cost. While, as shown in Figure 10: Total Emission function, the emission function is the summation of the constant holding emission and the decreasing transportation emission. Therefore, within an interval, the cost function (which sums an increasing and decreasing function) is less sensitive to order quantity changes than the emission function (which sums a constant and decreasing function).

4.7 Conclusion

In this work we have introduced a new inventory model entitled the CICEM (Cold Items Cost and Emission Model) to determine the optimal order quantity in an environment with capacitated refrigerated units for holding and transportation. The model considers the holding cost at the distribution center and the transportation costs from the warehouse to the distribution center of cold item inventory. Thus, the CICEM model is a variation of the EOQ model with holding and transportation unit capacities that considers objectives of minimizing both costs and emissions.
The transportation cost, holding cost and total cost are modeled in 4.4.1.1, 4.4.1.2 and 4.4.1.3, while the transportation emission, holding emission, and the total emission functions are modeled in 4.4.2.1, 4.4.2.2 and 4.4.2.3. For the CICEM model, we consider the emission function and two cases for the holding cost function: 1) not considering the interest rate of the investment capital as a part of the holding cost ($iC=0$), and 2) considering the investment opportunity of the items ($iC>0$). To model the holding and transportation unit capacity, all of the mentioned functions are non-linear and non-continuous. We develop exact algorithms to find the optimum value for the cost function when $iC=0$ and $iC>0$, as well as for the emission function. The solution algorithm for the second cost case has a similar structure to the solution algorithm of the emission function (as both algorithms search among the end points up to a point and then search the beginning points for the intervals after).

A set of numerical experiments were run comparing the cost objective of the CICEM model to the EOQ model and the emission objective of the CICEM to the SOQ model. The results confirmed the effectiveness of the CICEM model for different parameter settings, and provided the following managerial insights into the cold item inventory environment that has segmented holding and transportation units.

- For the cost function when $iC=0$, the CICEM outperforms the EOQ model for different values of $\lambda$, which is the ratio of holding cost to transportation cost and $\delta$, which is the demand to unit capacity ratio.
- For the case of $iC>0$, we run experiments to analyze the effect of item cost ($C$) on the optimum order quantity and the performance of the CICEM and EOQ models. Our results (Table 8 & Table 9) show that for small values of $C$, the two models produce largely different cost objectives. But as the item price increase, the differences
between the two models’ cost functions become smaller and for large values of item price, the CICEM model can be approximated using the EOQ model.

- For the emission function, the CICEM outperforms the SOQ model (sustainable version of the EOQ), for different values of $\theta$, which is the ration of holding emission to the transportation emission and $\delta$, which is the demand to unit capacity ratio. Our results (Table 12) show that for larger values of $\delta$, the CICEM presents a better functionality.

- Finally, to explore the tradeoff between the cost and emission function of the cold chain inventory problem, the optimum value for each function of the CICEM model is calculated. We then conduct a trade-off analysis to determine the impact that only considering the cost function has on the environmental function (and vice versa). Due to the structure of the two functions within each interval, the emission function is more sensitive to deviation from optimality than the cost function when $iC>0$. This is due to the fact that within each interval the transportation cost and emission functions are decreasing, yet the holding emission is a constant function and the holding cost is an increasing function (due to the presence of $iC$). The results illustrate that using the emission function to set the order quantity results in smaller deviations in the cost function than using the cost function and calculating the emission function based on that. As an example, according to Table 13, if the optimum order quantity of the emission function is used for the cost function (and $\lambda=0.5$), the cost function would increase only by 4% than its optimum value. However, if the optimum order quantity is determined by the cost function, the emission function (using the order quantity determined via the cost function) would increase 42% over the optimal order quantity.
found using the emission function. Due to the structure of the two functions within each interval, the emission function is more sensitive to deviation from optimality than the cost function.

### 4.8 Future Research

We identify a list of possible future research directions that would be interesting in the field of cold chain supply chain management:

- The CICEM model is a simplified model that could be extended into a more general model for a multi-product, multi-transporter size, multi-transportation mode, multi-capacities, and a multi-period model. Also, the CICEM model is developed considering only a single warehouse and a single distribution center as the network. This supply chain network could be expanded to have additional nodes of different types such as warehouses, distribution centers, retailers, and manufacturers. More general models would require additional modeling and algorithmic development. In addition, several other problems might be jointly considered with the inventory model, such as the Vehicle Routing Problem, the Vendor Managed Inventory Problem, or the Inventory-Routing Problem.

- Finally, it is often difficult to provide an accurate estimation for the emission function parameters, e.g., the emission from using a passenger vehicle is a function of many known and unknown variables. The vehicle’s emission may be caused by a variety of factors including driving behavior, vehicle condition, outside temperature, road surface, and vehicle weight. Thus, an interesting future research direction is to explore applying additional methodologies that model such uncertainty.
4.9 References


5 MULTI-PRODUCT MODELS

It is shown in the literature that supply chain pooling, or sharing supply chain network entities can reduce the costs (Ballot & Fontane, 2010), as well as the generated emissions (Pan, Ballot, Fontane, & Hakimi, 2014). For cold items, not every item can be shipped or held together, due to several issues, the most important one is having different holding temperature requirements. Yet, there are still families of products that can share the holding or transportation capacity, such as dairy products (milk, chocolate milk, etc.). Capacity sharing of capacitated refrigerated units for holding and transportation may reduce the costs and the emissions associated with fulfilling the requested inventory of cold items.

In chapter 4, we assumed that inventory decisions for each product are made independently, and therefore we considered only one type of product. In this chapter, we relax the assumption of considering only one type of product, and develop a multi-product CICEM model. In the multi-product version of the CICEM model, we determine the order quantity of multiple cold items that considers cost and emission as the objective functions.

5.1 Multi-Product Model Formulation

Problem definition: Consider a network that consists of a single provider (which can be a warehouse or manufacturer) and a single Distribution Center (DC). There are “n” types of products that are ordered by the DC from the warehouse. The objective is to find the optimal order quantity, as well as optimal frequency of ordering for each product family. A product family is a group of products that are ordered, transported, and kept together. A solution to the multi-product inventory problem can have families of products that share transportation and holding capacity, and some single products that do not share the transportation or holding capacity.
In order for a family of products to share unit capacity, the following must occur.

1. Products within a family should share holding capacity, but products from two different families should not share holding capacities.

2. Products within a family should share the transportation capacity. To share the transportation capacity, the multiple products are to be transported together in the same shipment. As a result, the products in a family are required to be ordered together and the frequency of the orders for different products will be the same.

3. The products within a family should be compatible with each other. This constraint is required to be satisfied, as the temperature and certain condition of holding or transportation for different products may vary.

In our modeling, we assume that different products take the same space, and also have the same weight.

We hereby summarize our notation, as follows:

**Sets and indices**

\( J: j=1,2,..., \ n \) set of product types

\( I: i=1,2,..., \ |I| \) set of families
Parameters

intr: interest rate (we use this to avoid confusion with index i)

$D_j$: Demand for product type j

$B_{jk}$: Binary parameter, which is 1: if product type j and product type k can be grouped into one family, and 0: otherwise

b: the capacity of each freezer unit (products/unit)

$T_c$: Truck unit capacity (number of products/unit)

$F_t$: Fixed transportation cost of each transportation unit capacity

$F_h$: Fixed holding cost (of each freezer unit)

$\alpha$: The coefficient that represents the effect of weight on the MPG of the truck

W: is the weight of a unit product

$MPG_0$: The base MPG for the empty truck

CEGF: (Constant of the Emission of a Gallon of Fuel) a constant term that represents the emission of burning one gallon of diesel fuel

TECF: Total Energy Consumption of a Freezer

TCFE: Total Carbon Footprint of 1kWh Energy

Decision variables

$R_i$: Frequency of ordering for the $i^{th}$ family of products

$Q_j$: Order quantity of product type j

$x_{ij}$: Binary decision variable, which is 1: if product type j belongs to the family of products $i$, 0: otherwise

5.1.1 Model for cost function

In this section, we propose a mathematical formulation for the multi-product problem. The objective function in this section is the inventory cost function. The problem formulation is as follows:
\[ P(1) - \text{Cost} \]

\[ \begin{align*}
\text{Min: } f(Q_j, R_i, x_{ij}) &= \\
&= \sum_{i \in I} \left( \left[ \frac{\sum_{j \in J} x_{ij} Q_j}{b} \right] \times F_h \right) + \sum_{j \in J} \text{intr. } C_j Q_j + \sum_{i \in I} \left( \frac{\sum_{j \in J} x_{ij} D_j}{\sum_{j \in J} x_{ij} Q_j} \times \left[ \frac{\sum_{j \in J} x_{ij} Q_j}{T_c} \right] \times (F_t) \right) \end{align*} \]  

Subject to:

\[ \forall i \in I \quad \sum_{j \in J} x_{ij} D_j = R_i \text{ (the frequency of the order)} \quad (2) \]

\[ \forall i \in I, \forall j, k \in J, \quad j \neq k \quad B_{jk} \geq x_{ij} x_{ik} \quad (3) \]

\[ \forall j: 1,2, \ldots, n \quad \sum_{i \in I} x_{ij} = 1 \quad (4) \]

\[ \forall i \in I, \forall j \in J, \quad x_{ij} = \{0,1\} \quad (5) \]

\[ \forall j \in J \quad Q_j > 0 \quad (6) \]

\[ \forall j \in J \quad R_j > 0 \quad (7) \]

In the above formulation, the first constraint (2) ensures that all of the products in the same family are ordered together, by enforcing their order frequency to be equal. The second constraint (3) ensures that any two products that are grouped in the same family are compatible with each other. This constraint is presented due to the different requirements on temperature or other conditions for holding or transporting different types of products. If two products are not compatible \((B_{jk} = 0)\), at least one of the two variables of \(x_{ij}\) or \(x_{ik}\) should be zero, which means they may not be grouped in the same family. The third constraint (4) ensures that a product must be assigned to exactly one family. If at optimality a product type is to be shipped and stored individually, it will be a family of only one type of product. In the objective function, the first term represents the operational holding cost of the products per period. The second term in the objective function represents the cost of capital investment for the products in the inventory. The
third term of the objective function represents the transportation cost of the products per period.

In this model, we consider the order quantities $Q_j$ to be continuous variables. The same assumption can be found in (Guerrero, Yeung, & Guéret, 2013; Haksever & Moussourakis, 2008; B. Zhang, 2012), among others.

5.1.2 Model for emission function

In this section, we formulate the inventory problem model for multiple product types with the emission objective function. The model formulation is as follows:

$P(1)$ - Emission

\[
\begin{align*}
\text{Min: } f(Q_j, R_i, x_{ij}) &= \\
&= CEGF \times \\
&= \frac{M}{\left(\text{MPG}_0 - \alpha \times \left[\frac{\sum_{j \in J} x_{ij} Q_j}{\sum_{j \in J} x_{ij}}\right] \times W\right)} \times \left[\frac{\sum_{j \in J} x_{ij} Q_j}{T_c}\right] \\
&\quad + \left[\frac{\sum_{j \in J} x_{ij} Q_j}{b}\right] \times TECF \times TCFE
\end{align*}
\]

Subject to:

\begin{align*}
\forall i \in I & \quad \frac{\sum_{j \in J} x_{ij} D_j}{\sum_{j \in J} x_{ij} Q_j} = R_i \text{ (the frequency of the order)} \\
\forall i \in I, \forall j, k \in J, & \quad j \neq k \quad B_{jk} \geq x_{ij} x_{ik} \\
\forall j: 1,2, ..., n & \quad \sum_{i \in I} x_{ij} = 1 \\
\forall i \in I, \forall j \in J, & \quad x_{ij} = \{0,1\} \\
\forall j \in J & \quad Q_j > 0 \\
\forall i \in J & \quad R_i > 0
\end{align*}

In the above formulation, the constraints are similar to the constraints of $P(1)$-Cost in 5.1.1. In addition, we consider the order quantities $Q_j$ to be continuous variables, as in 5.1.1. In the objective function, the first term represents the emission of each shipment, multiplied by the total
number of shipments per period. The second term in the objective function represents the emission from holding a family of products in a cold holding area by considering the number of freezers units needed, times the emission associated with each freezer unit.

In what follows, we develop a solution approach for the multi-product models with cost and emission objective functions.

5.2 Solution Approach

In this section, we propose a solution algorithm to solve P(1)-Cost and also P(1)-Emission. The overview of the solution algorithm is depicted as a flowchart in Figure 11.

![Figure 11: Solution algorithm flow chart](image-url)
The solution algorithm in high level is summarized as follows.

**Step 1:** Form the feasible partitions of the set of products by grouping the products into different families using procedures in 5.2.1.1 or 5.2.1.2, and finding the associated combinations of families for each partition.

**Step 2:** For each feasible partition, solve the inventory management problem for each family of products in the partition, using the algorithms proposed in 5.2.2.1 (if intr.C = 0, or is a relatively small value), 5.2.2.2 (for general case of intr.C >0), or 5.2.3 for the emission function.

**Step 3:** Find the total emission (or cost) for each partition by adding the emission (or cost) of each family in the partition, using the objective function of the problems in 5.2.2.1 (if intr.C = 0, or is a relatively small value), 5.2.2.2 (for general case of intr.C >0), or 5.2.3 for the emission function.

**Step 4:** Compare the total emission (or cost) of all the feasible partitions. Select the product-family partition with minimum emission (or cost).

Our solution approach to solve both problems of P(1)-Cost and P(1)-Emission consist of two main steps: (1) grouping (partitioning) the products into different families, and (2) solving the inventory problem for each family, to minimize the emission function (for Problem P(1)-Emission) or the cost function (for Problem P(1)-Cost). In the first step, we decide upon the binary variables only, as we group the products into families. Our solution approach is sequential, because first we form all the partitions, and then in the second step, we solve the inventory problem for each partition.

In 5.2.1 we present two different procedures for grouping the products into families, which can be applied for both P(1)-Emission and P(1)-Cost problems, and solution algorithms for different cases for the inventory problem within each family.
5.2.1 Procedures for grouping the products

For grouping the products into families, we propose an exact algorithm (based on enumeration), as well as a heuristic algorithm. In section 5.3 we run experiments using both algorithms to study their performance and solution quality.

5.2.1.1 An exact, enumeration-based algorithm for grouping the products

The exact algorithm finds the best partition of product families by checking all the feasible combinations of forming families on the set of products. From the set theory, we form and check all the possible partitions of the set of the entire products. Please note that we exclude the infeasible partitions. A partition is infeasible, if it has grouped incompatible product types ($B_{jk} = 0$) in a family ($x_{ij} = x_{ik} = 1$).

The exact procedure of grouping products is stated as follows:

<table>
<thead>
<tr>
<th>Step 1: make a set of all possible partitions over the set of product types</th>
<th>Step 2: for every partition in the set generated in step 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>if we have: ($x_{ij} = x_{ik} = 1$) &amp;&amp; ($B_{jk} = 0$)</td>
</tr>
<tr>
<td></td>
<td>Remove the partition from the set of feasible partitions</td>
</tr>
<tr>
<td>end</td>
<td></td>
</tr>
</tbody>
</table>

The procedure of finding all the feasible partitions requires computational effort, and for large instances of the product set is impractical as the run time grows exponentially. To reduce the search space of the algorithm, we propose an improvement procedure.

An improvement procedure for the exact algorithm

The number of partitions of a set follows the series of numbers called “Bell Numbers” (Aigner, 1999; “Bell number,” 2014). The exponential generating function of the Bell numbers is (Cohn, Even, Menger, & Hooper, 1962; Rota, 1964)
\[ B(x) = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n = e^{e^x-1} \]

The series of Bell number has a faster growth rate than the number of non-empty subsets of a set \((2^n - 1)\), and for large values of \(n\) \((n > 5)\), we have:

\[ (2^n - 1) \ll \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n = e^{e^x-1} \]

Table 14 demonstrates the first ten numbers in the series of Bell numbers, and also the number of non-empty sets.

<table>
<thead>
<tr>
<th>Number of products in a set</th>
<th>Number of partitions {Bell Number series}</th>
<th>Number of non-empty subsets</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>52</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>203</td>
<td>63</td>
</tr>
<tr>
<td>7</td>
<td>877</td>
<td>127</td>
</tr>
<tr>
<td>8</td>
<td>4140</td>
<td>255</td>
</tr>
<tr>
<td>9</td>
<td>21147</td>
<td>511</td>
</tr>
<tr>
<td>10</td>
<td>115,975</td>
<td>1023</td>
</tr>
</tbody>
</table>

Therefore, if the number of products in a set is greater than 5 we can save on the computational time if we reduce our search through the subsets of a set, instead of the partitions of a set. Referring to the final procedure of the solution, we need to find the cost of each family in a partition and then add the costs of all the families to obtain the total cost associated with each partition. A family of products may appear in more than one partition. The minimum inventory cost of a family is the same in each partition, as it consists of the same products, as shown in 5.2.2. We consider all the non-empty subsets of the products set (which are \(2^n - 1\) for an \(n\)-element set).
set). We can then use these values to calculate the total cost for each partition by adding the calculated cost of each family in a given partition. We provide a numerical example in 5.3.

5.2.1.2 A heuristic approach for grouping the products

As discussed in 5.2.1.1, the exact algorithm for partitioning products into families identifies all the possible partitions (or subsets) over the set of product types. For small instances, the exact algorithm can be applied, but for large instances, it is not practical to check all the feasible partitions.

Heuristic procedure: we can group the product types into equal-size (or almost equally sized) families as shown in the following procedure:

**Step 1:** sort all the products in descending order by their demand values, and save into an ordered array of products

**Step 2:** \( f = 1 \)  \{initializing the family number\}

\( s \) = number of products in each family (given value)

while the set of available products \( <> \) empty  do

\( j = 0; \)

\( k = 1; \)

while \( j < s \)

if (it is feasible to have the \( k \)th product in the ordered array of products in the current family \( f \))

- Put the \( k \)th product type into the family number \( "f" \)
- Remove the \( k \)th product from the array of available products & update the set of available products

\( j = j+1 \)

else

\( k = k+1 \)

end

if (there are no more compatible product)

- \( j=s \)

end

sort all the available products in descending order by their demand values, and save into an ordered array of available products

\( f = f + 1 \)

end
In this algorithm, we put the products from the ordered array into the next compatible family, and will do so until a family contains “s” product types, or there are no more products to be added to the family that is compatible with all the current products in the family. As we add each product type, we check the feasibility of having the new product type with the products already put in the family. If they are compatible, we include the new product type and move on to the next product in the ordered array, and if not, we skip this product and check the next product in the ordered array. The skipped products remain in the ordered array and are available to be checked for feasibility for the next family.

We develop the heuristic algorithm in such a way to have products with similar demand frequency grouped together into a product family. We anticipate that grouping products with similar demand frequencies together because similar demanded products will have similar order quantities for a fixed order and holding costs.

Please note that the described algorithm is a simple heuristics to be used in practice. One can use more advanced analytical methods, such as clustering algorithms. (see for example Milligan (1980) for a set of clustering algorithms). ABC analysis might also be conducted to group the product types. It is also possible that a decision maker has already formed the family of products. This may happen due to different reasons: (1) the given family of products is the only set of products delivered from the warehouse (or manufacturer); (2) there are some preferences to have a family of products shipped and held together.
5.2.2 Procedures for solving the inventory problem for a family of products to minimize the cost function

After finding the set of product families that will share capacities, it is of interest to find the optimal order quantity for each of the product types within one family of products.

For this step, the problem is defined as follows: Given the set of product families, find the optimal order quantity for each product type within a family. By determining the families of products, the problem P(1)-Cost is reduced to P(2)-Cost.

\[ P(2)-\text{Cost} \]

Define a set of products that belong to the \( i \)th family as \( J_i \):

\[
\text{Min: } f(Q_1, Q_2, ..., Q_n, R_i) = \left\lfloor \frac{\sum_{j \in J_i} Q_j}{b} \right\rfloor \times F_h + \sum_{j \in J_i} \text{intr.} C_j Q_j + \left( \frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j} \times \left\lfloor \frac{\sum_{j \in J_i} Q_j}{T_c} \right\rfloor \times (F_t) \right)
\]

Subject to:

\[
\forall j: j \in J_i \quad \frac{D_j}{Q_j} = R_i \text{ (the frequency of the order)}
\]

\[
\forall j: j \in J_i \quad Q_j > 0
\]

\[ R_i > 0 \]

In what follows, we discuss solution approaches to a special case of the problem P(2)-Cost, followed by a solution approach for the general case. Here, we investigate the properties of P(2)-Cost. We consider the special case as: (\( \text{intr.} C = 0 \)), and the general case (\( \text{intr.} C > 0 \))

5.2.2.1 Considering the case of: \( \text{intr.} C_j = 0 \) \( \forall j \in J \)

For the case of \( \text{intr.} C_j = 0 \), the objective function of P(2) becomes:

\[
\left\lfloor \frac{\sum_{j \in J_i} Q_j}{b} \right\rfloor \times F_h + \left( \frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j} \times \left\lfloor \frac{\sum_{j \in J_i} Q_j}{T_c} \right\rfloor \times (F_t) \right)
\]

For this problem, by considering \( \sum_{j \in J_i} Q_j = Q' \) and \( \sum_{j \in J_i} D_j = D' \), the problem becomes:
P(3)-Cost

\[ \text{Minimize: } f(Q', R_i) = \left[ \frac{Q'}{b} \right] \times F_h + \left( \frac{D'}{Q'} \times \left[ \frac{Q'}{T_c} \right] \times F_t \right) \]

Subject to:

\[ \forall j: j \in J_i \quad \frac{D_j}{Q_j} = R_i \text{ (how many times per period do we need to order)} \quad (1) \]

\[ \sum_{j \in J_i} Q_j = Q' \quad (2) \]

\[ \sum_{j \in J_i} D_j = D' \quad (3) \]

\[ \forall j: j \in J_i \quad Q_j > 0 \]

\[ R_i > 0 \]

In P(3)-Cost, \( D' \) is a constant and known parameter as it is the summation of all the products’ demands, and each product’s demand is assumed to be a constant and known value. The unconstrained P(3)-Cost can be solved by the same solution approach that is developed for the CICEM model in the case of \( \text{intr.C} = 0 \), in Chapter 4. Given the value of \( Q' \) we can obtain the value for \( R_i \) by \( \frac{D'}{Q'} = R_i \). In the next step, the order quantity for each product may be calculated, proportional to its total demand (\( D_j \)), as follows:

\[ \forall j: j \in J_i \quad \frac{D_j}{Q_j} = R_i \text{ (how many times per period do we need to order)} \]

This gives:

\[ \forall j: j \in J_i \quad D_j = R_i \times Q_j \quad (20) \]

So we can state:

\[ \sum_{j \in J_i} D_j = \sum_{j \in J_i} R_i \times Q_j \quad (21) \]

And under the optimality of the CICEM model, we have:
\[ \sum_{j \in J_l} D_j = \sum_{j \in J_l} R_l^* \times Q_j^* \quad (22) \]

And since neither \( R_l \) nor \( R_l^* \) are changed through changing the index \( j \), we have:

\[ \sum_{j \in J_l} D_j = R_l^* \times \sum_{j \in J_l} Q_j^* \quad (23) \]

By (23) and the constraint (1) of P(3)-cost, we have:

\[ \frac{\sum_{j \in J_l} D_j}{\sum_{j \in J_l} Q_j^*} = R_l^* = \frac{D_j}{Q_j^*} \]

Then, if we substitute \( \sum_{j \in J_l} Q_j = Q' \), and we solve for \( Q_j^* \) in the above equation for a given \( D_j \) and \( Q' \), we can determine the order quantity for different products, \( Q_j^* \):

\[ Q_j^* = \frac{D_j}{\sum_{j \in J_l} D_j} \times \sum_{j \in J_l} Q_j^* = \frac{D_j}{D'} \times Q^* \quad (24) \]

To summarize, the solution procedure for solving P(2)-Cost can be stated as:

**Step 1:** Aggregate the demand for all products in a family, call it \( D' \).

**Step 2:** Substitute the \( D' \) value from step 1 into the cost function of P(3)-Cost and solve using the solution approach presented in 4.3 for the CICEM model, to find the \( Q^* \)

**Step 3:** Use the Eq. 24 to find the order quantity for each product

**5.2.2.2 Considering the general case in which: intr. \( C_j > 0 \)**

In this case, the objective function of P(1)-Cost can be re-written as:

\[ f(R_l, Q_j) = \left[ \frac{\sum_{j \in J_l} Q_j}{b} \right] \times F_h + \sum_{j \in J_l} \text{intr.} \ C_j \frac{D_j}{R_l} + \left( \frac{\sum_{j \in J_l} D_j}{\sum_{j \in J_l} Q_j} \right) \times \left( \frac{\sum_{j \in J_l} Q_j}{T_c} \right) \times (F_t) \]

Since \( R_l \) does not change with the change of \( j \), it can be taken out from the summation:
\[ f(R_i, Q_j) = \left[ \frac{\sum_{j \in J_i} Q_j}{b} \right] \times F_h + \frac{1}{R_i} \sum_{j \in J_i} \text{intr.} C_j D_j + \left( \frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j} \times \left[ \frac{\sum_{j \in J_i} Q_j}{T_c} \right] \times (F_t) \right) \]  \tag{26}

Substituting Eq. 22 gives:

\[ f(R_i, Q_j) = \left[ \frac{\sum_{j \in J_i} Q_j}{b} \right] \times F_h + \frac{\sum_{j \in J_i} Q_j}{\sum_{j \in J_i} D_j} \sum_{j \in J_i} \text{intr.} C_j D_j + \left( \frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j} \times \left[ \frac{\sum_{j \in J_i} Q_j}{T_c} \right] \times (F_t) \right) \]  \tag{27}

By substituting \( \frac{\sum_{j \in J_i} C_j D_j}{\sum_{j \in J_i} D_j} = C' \)

\[ f(R_i, Q_j) = \left[ \frac{\sum_{j \in J_i} Q_j}{b} \right] \times F_h + \text{intr.} C' \sum_{j \in J_i} Q_j + \left( \frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j} \times \left[ \frac{\sum_{j \in J_i} Q_j}{T_c} \right] \times (F_t) \right) \]  \tag{28}

We now can use the same approach as we used for the special case of (intr.C =0) in 1.1: by considering \( \sum_{j \in J_i} Q_j = Q' \) and \( \sum_{j \in J_i} D_j = D' \), the problem is:

P(4)-Cost

\[ \text{Minimize: } f(Q', R_i) = \left[ \frac{Q'}{b} \right] \times F_h + \text{intr.} C' Q' + \left( \frac{D'}{Q'} \times \left[ \frac{Q'}{T_c} \right] \times (F_t) \right) \]

Subject to:

\( \forall j: j \in J_i \quad \frac{D_j}{Q_j} = R_i \) (how many times per period do we need to order)

\( \sum_{j \in J_i} Q_j = Q' \)

\( \sum_{j \in J_i} D_j = D' \)

\( \frac{\sum_{j \in J_i} C_j D_j}{\sum_{j \in J_i} D_j} = C' \)

\( \forall j: j \in J_i \quad Q_j > 0 \)

\( R_i > 0 \)
The term $C'$ can be interpreted as the weighted average price for all products. This problem P(4)-Cost can be solved with the solution approach proposed in Chapter 4, for the general case of (intr.C>0) for the CICEM model.

5.2.3 Procedure for solving the inventory problem for family of products to minimize the emission function

In this section, we propose a solution approach for the inventory model of a given family of products to minimize the emission function. In this step, the families of products are formed. The objective is to find the order quantity of each product that minimizes the emissions of the given family of products. For this case, let a set of products that belong to the $i^{th}$ family be denoted as $J_i$, and the objective function of P(1)-Emission is:

$$\text{Min: } f(Q_j, R_i) =$$

$$CEGF \times \frac{M}{\left(\frac{MPG_0 - \alpha \times \frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j} \times W}{\sum_{j \in J_i} Q_j} \right)} \times \frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j} \times \left[\frac{\sum_{j \in J_i} Q_j}{T_c}\right]$$

$$+ \left[\frac{\sum_{j \in J_i} Q_j}{b}\right] \times TECF \times TCFE$$

By considering $\sum_{j \in J_i} Q_j = Q'$ and $\sum_{j \in J_i} D_j = D'$, the problem is:

P(3)- Emission: Minimize: $f(Q', R_i) =$

$$CEGF \times \frac{M}{\left(\frac{MPG_0 - \alpha \times \frac{Q'}{T_c} \times W}{Q'}\right)} \times \frac{D'}{Q'} \times \left[\frac{Q'}{T_c}\right] + \left[\frac{Q'}{b}\right] \times TECF \times TCFE$$

Subject to:

$$\forall j: j \in J_i \quad \frac{D_j}{Q_j} = R_i > 0 \text{ (how many times per period do we need to order)}$$
\[
\sum_{j \in J_i} Q_j = Q'
\]
\[
\sum_{j \in J_i} D_j = D'
\]
\[\forall j: j \in J_i \quad Q_j > 0, \quad R_i > 0\]

In P(3)-Emission, \( D' \) is a constant and known parameter, as it is the summation of all the products’ demands. The unconstrained P(3)-Emission can be solved by the same solution approach that is developed for the CICEM model in the emission function, in Chapter 4. Given the value of \( Q' \) we can obtain the value for \( R_i \) by \( \frac{D'}{Q'} = R_i \). At the next step, the order quantity for each product is calculated, using the Eq. (23), as discussed in 5.2.2.1.

To summarize, the solution procedure for solving P(3)-Emission can be stated as:

**Step 1:** Aggregate the demand for all products in a family, call it: \( D' \)

**Step 2:** Substitute the \( D' \) value from step 1 into the emission function of P(3)-Emission and solve using the solution approach presented in 4.4.2 for the CICEM model, to find the \( Q^* \)

**Step 3:** Use the Eq. (23) to find the order quantity for each product

### 5.3 Numerical Example

In this section, we propose a set of numerical experiments. The experiments are designed to address the following questions:

1) How to apply the exact and its improvement procedure, and the heuristic grouping procedures?

2) What is the effect of demand variability on the cost function value?

3) What is the effect of compatibility of different products on the cost function value?
4) What is the effect of holding and transportation unit capacity on each objective function?

5) What is the effect of the change in transportation and holding costs on the cost function?

We first, demonstrate how to use the proposed procedures for grouping the products, followed by the set of experiments to get insights from the model and numerical results.

5.3.1 Illustrating the use of the exact algorithm and its improvement procedure

The objective of this experiment is to demonstrate how to use the exact grouping algorithm. Consider a case with 4 different types of products with different demand values. The demand for each product type is given as in Table 15, and the input parameters for the experiment are in Table 16. We assume that all the four product types are compatible to be grouped in one family.

<table>
<thead>
<tr>
<th>Product type</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2500</td>
</tr>
<tr>
<td>2</td>
<td>4300</td>
</tr>
<tr>
<td>3</td>
<td>3750</td>
</tr>
<tr>
<td>4</td>
<td>6500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 15: Data for multi-product case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product type</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 16: Data for multi-product case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tc</td>
</tr>
<tr>
<td>750</td>
</tr>
</tbody>
</table>

Recalling from the improvement procedure for the exact algorithm on grouping product types in 5.2.1.1, we consider all the non-empty subsets of the set of product types, and the associated cost of inventory to each subset (which can be considered as a family of products, using the objective function of the P(3)-Cost, as described in 5.2.2.1.) The results are summarized in Table 17.
Table 17: The total cost for “subsets” of the demand set

<table>
<thead>
<tr>
<th>Subset</th>
<th>Total Demand of Subset</th>
<th>Total Cost of Subset</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>2500</td>
<td>189.28</td>
</tr>
<tr>
<td>$D_2$</td>
<td>4300</td>
<td>271.57</td>
</tr>
<tr>
<td>$D_3$</td>
<td>3750</td>
<td>246.42</td>
</tr>
<tr>
<td>$D_4$</td>
<td>6500</td>
<td>367.33</td>
</tr>
<tr>
<td>($D_1, D_2$)</td>
<td>6800</td>
<td>380.13</td>
</tr>
<tr>
<td>($D_1, D_3$)</td>
<td>6250</td>
<td>356.66</td>
</tr>
<tr>
<td>($D_1, D_4$)</td>
<td>9000</td>
<td>474.00</td>
</tr>
<tr>
<td>($D_2, D_3$)</td>
<td>8050</td>
<td>433.46</td>
</tr>
<tr>
<td>($D_2, D_4$)</td>
<td>10800</td>
<td>550.80</td>
</tr>
<tr>
<td>($D_3, D_4$)</td>
<td>10250</td>
<td>527.33</td>
</tr>
<tr>
<td>($D_1, D_2, D_3$)</td>
<td>10550</td>
<td>540.13</td>
</tr>
<tr>
<td>($D_1, D_2, D_4$)</td>
<td>13300</td>
<td>657.46</td>
</tr>
<tr>
<td>($D_1, D_3, D_4$)</td>
<td>12750</td>
<td>634.00</td>
</tr>
<tr>
<td>($D_2, D_3, D_4$)</td>
<td>14550</td>
<td>710.80</td>
</tr>
<tr>
<td>($D_1, D_2, D_3, D_4$)</td>
<td>17050</td>
<td>817.46</td>
</tr>
</tbody>
</table>

Now, we can create all the possible partitions of the set of products, and calculate the total cost for each partition, using the values given in Table 17. The results are reported in Table 18.
As shown in Table 18, for the given set of parameters, the minimum cost among all the possible partitions occurs when all the products are forming a single family, and the maximum cost is when they are all individual families, with no sharing.
5.3.2 Using the heuristic algorithm for grouping the product types

In this section, we use the proposed heuristic algorithm in 5.2.1.2 to group the product types into product families. We compare the results obtained from the heuristic grouping method against the ones from the exact algorithm to check the solution quality of the proposed algorithms. We assume that the recommended number of product types in each family is two ($s=2$). The same sets of parameters are used as in Table 15 and Table 16. Consider the compatibility matrix $B$ as:

$$B = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 1
\end{bmatrix}$$

By performing the heuristic algorithm for grouping, we have:

**Step 1:** sort the products according to their demand values:

Array (D) = \{1200(6), 3400(1), 5500(4), 7750(3), 9000(5), 15000(2)\}

**Step 2:** group every two products into a family, until all the products are assigned, by checking the compatibility criteria for each family:

$F_1 = \{1200, 3400\}, F_2 = \{5500, 7750\}, F_3 = \{9000\}, F_4 = \{15000\}$

Since product type 2 and 5 may not be grouped in one family, they form two families of individual items.

**Step 3:** solve the inventory problem for each family.

We define the ratio of holding cost to the transportation cost, as:

$$\lambda = \frac{\text{holding cost of a freezer}}{\text{transportation cost of a single shipment}} = \frac{F_h}{F_t}$$

The results for the case of $\lambda=0.64$ are reported in Table 19.
Table 19: Total cost of each family using heuristic algorithm for grouping

<table>
<thead>
<tr>
<th>Family</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1 = {1200,3450}</td>
<td>1551</td>
</tr>
<tr>
<td>F2 = {5500, 7750}</td>
<td>3426</td>
</tr>
<tr>
<td>F3 = {9000}</td>
<td>2520</td>
</tr>
<tr>
<td>F4 = {15000}</td>
<td>3800</td>
</tr>
</tbody>
</table>

The total cost of this partition is calculated by adding the cost of each family within the partition: 1551+3426+2520+3800 = 11297. Comparing this number with the optimal value from Table 20, we calculate the deviation from the optimal value (which is 10728) as:

\[
\frac{11297 - 10728}{10728} \approx 5.3\%
\]

We apply the same procedure for different values of \(\lambda\) and report the results in Table 20.

Table 20: Results for different values of \(F_h\) and \(F_t\), using heuristic algorithm for grouping

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>Associated cost of each family of products</th>
<th>Total cost of the partition</th>
<th>Total cost if using the exact algorithm</th>
<th>% deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>({1200, 3400})</td>
<td>({5500, 7750})</td>
<td>({9000})</td>
<td>({15000})</td>
</tr>
<tr>
<td>0.3125</td>
<td>1281</td>
<td>3126</td>
<td>2220</td>
<td>3500</td>
</tr>
<tr>
<td>0.6250</td>
<td>1551</td>
<td>3426</td>
<td>2520</td>
<td>3800</td>
</tr>
<tr>
<td>0.9375</td>
<td>1801</td>
<td>3726</td>
<td>2807</td>
<td>4100</td>
</tr>
<tr>
<td>1.5625</td>
<td>2301</td>
<td>4278</td>
<td>3307</td>
<td>4678</td>
</tr>
<tr>
<td>3.1250</td>
<td>3252</td>
<td>5528</td>
<td>4557</td>
<td>5928</td>
</tr>
</tbody>
</table>

In Table 20, we report the cost values for each family of products when using the objective function of P(3)-Cost, as described in 5.2.2.1. By adding the cost for every family of products in a partition, we obtain the total cost associated with the partition. We then compare this result to the cost values obtained by using the exact grouping algorithm, and calculate the relative
deviation in the value of the cost function. For the given set of parameters, the percent deviation has a range of 3% to 13%. According to Table 20, increasing $\lambda$ increases the % deviation. The percent deviation in cost from the solutions of the exact algorithm for our experiments is less than 12.90% for values of $\lambda \leq 3.125$. As a result of this set of parameters’ values, for small $\lambda \leq 1$ values, the heuristic grouping algorithm might be used instead of the exact grouping algorithm, while expecting at most 10% deviation from optimal cost.

5.3.3 Experimental design setup

In this section and through the rest of 5.3, we design and run experiments on different values of parameters of the problem to obtain insights from the results.

The experiments are conducted on problem instances with $n=3$, 5, and 7 products. For each value of $n$, we change the following parameters.

Unit cost of products: we consider two sets for unit costs as: $C_j = 3 + \eta (n-j)$, with $\eta = 0.2, 0.5$.

Holding cost of a freezer: we consider three values for the holding cost:

- $10\bar{C} = \frac{\sum_{j=1}^{n} C_j}{n}$
- $25\bar{C} = \frac{\sum_{j=1}^{n} C_j}{n}$
- $50\bar{C} = \frac{\sum_{j=1}^{n} C_j}{n}$

Transportation cost of each shipment: we consider three values for the transportation cost:

- $10\bar{C} = \frac{\sum_{j=1}^{n} C_j}{n}$
- $25\bar{C} = \frac{\sum_{j=1}^{n} C_j}{n}$
- $50\bar{C} = \frac{\sum_{j=1}^{n} C_j}{n}$

Holding unit capacity: $1\% \sum_{j=1}^{n} D_j$, $5\% \sum_{j=1}^{n} D_j$, $10\% \sum_{j=1}^{n} D_j$

Transportation unit capacity: $1\% \sum_{j=1}^{n} D_j$, $5\% \sum_{j=1}^{n} D_j$, $10\% \sum_{j=1}^{n} D_j$

In our parameters setup, we consider three cases: (1) equal transportation and holding cost, (2) transportation cost greater than holding cost, and (3) holding cost greater than transportation
cost. As a result, the ratio \( \frac{\text{holding cost}}{\text{transportation cost}} \) is varied from 0.1, to 1 and 10, to represent different environments.

**Demand:** we model demand for each product using a normal distribution, with mean of 3000 and Coefficient of Variance \((cv)\) of 0.1 and 0.3, to represent two levels of demand variability. We also conduct experiments using uniform distribution for demand values, with similar parameters.

**Compatibility index:** we randomly generate compatibility matrices for each product set. The ratio of \( \tau = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}}{n^2} \) which is the “total number of compatible products to the total number of products” is computed and modeled for three levels of (25%-35%), (45%-55%), and (70%-90%).

### 5.3.4 The effect of demand variation

In this section, we study the effect of demand variability on the total cost. Table 21 reports the results from 10 replications for each scenario.

<table>
<thead>
<tr>
<th>( n )</th>
<th>Normal ((cv=0.1))</th>
<th>Normal ((cv=0.3))</th>
<th>Uniform ((cv=0.1))</th>
<th>Uniform ((cv=0.3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Mean 6343</td>
<td>6230</td>
<td>6363</td>
<td>6354</td>
</tr>
<tr>
<td></td>
<td>Stdev 120</td>
<td>373</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>Mean 11576</td>
<td>11855</td>
<td>11639</td>
<td>11650</td>
</tr>
<tr>
<td></td>
<td>Stdev 214</td>
<td>790</td>
<td>13</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>Ave 13423</td>
<td>13651</td>
<td>13556</td>
<td>13553</td>
</tr>
<tr>
<td></td>
<td>Stdev 254</td>
<td>990</td>
<td>18</td>
<td>23</td>
</tr>
</tbody>
</table>

In Table 21, we observe that if the demand has a normal distribution, increase in demand variance causes larger increases in the variability of the total cost values, in comparison to the cases when demand follows a uniform distribution. The results also indicate that the variability
of the total cost value increases as the number of product type’s increase. This observation is consistent for both demand distributions, and also for both levels of variability.

5.3.5 The effect of compatibility index

We define the matrix B as a 2-dimension array of 0-1 parameters, over the set of product types. Each element in B indicates the compatibility of two product types to be grouped in one family or not. In this section, we study the effect of the level of compatibility of different product types on the total cost of a partition. For this purpose, we define a “compatibility index” as:

\[
\text{compatibility index: } \tau = \frac{\text{sum of all the elements of the B matrix} - n}{n^2 - n}
\]

In the nominator, we subtract “n” from the total sum of the elements of B matrix due to the fact that all the elements on the main diagonal of the B matrix are “1” (each product is compatible with itself). The same reasoning is used for the denominator of the \(\tau\) ration. Three levels of (25%-35%), (45%-55%), and (70%-90%) are tested, with 10 replication on each configuration, and the results are summarized in Table 22.

Table 22: The effect of demand variation on total cost, with \(n = 5\)

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>Normal ((cv = 0.1))</th>
<th>Normal ((cv = 0.3))</th>
<th>Uniform ((cv = 0.1))</th>
<th>Uniform ((cv = 0.3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>35%</td>
<td>Mean 11869</td>
<td>Mean 11452</td>
<td>Mean 11845</td>
<td>Mean 11829</td>
</tr>
<tr>
<td></td>
<td>Stdev 234</td>
<td>Stdev 753</td>
<td>Stdev 11</td>
<td>Stdev 17</td>
</tr>
<tr>
<td>48%</td>
<td>Mean 11835</td>
<td>Mean 11646</td>
<td>Mean 11838</td>
<td>Mean 11843</td>
</tr>
<tr>
<td></td>
<td>Stdev 256</td>
<td>Stdev 850</td>
<td>Stdev 10</td>
<td>Stdev 16</td>
</tr>
<tr>
<td>73%</td>
<td>Mean 11674</td>
<td>Mean 11822</td>
<td>Mean 11839</td>
<td>Mean 11841</td>
</tr>
<tr>
<td></td>
<td>Stdev 175</td>
<td>Stdev 792</td>
<td>Stdev 12</td>
<td>Stdev 20</td>
</tr>
</tbody>
</table>

The results from Table 22 indicate that there is a fairly small difference between different levels of products’ compatibility. According to Table 22, the compatibility index value does not greatly affect the total cost value. This might be a surprising result, but further analysis reveals that since
the defined ranges do not include extreme cases (all products compatible $\tau=100\%$, or no compatibility at all $\tau=0\%$). Therefore, in the experimented ranges of compatibility index which does not include the extreme cases, there are always some products that are compatible with each other, and may form a family of products to share the holding and transportation unit capacity. As a result, the values have almost similar mean value, and the difference between different columns is due to the stochasticity of the demand (we generate new set of demand values from the associated probability function for each replication). The results are also consistent with the ones in Table 21 that the demands from uniform distribution cause less variability in the values of total cost, in comparison to the results with normally distributed demands. We believe that this happens because the uniform distribution is truncated (has a maximum and minimum value), while the normal distribution has a wider range of values. According to Table 22, the results from a lower variability ($cv=0.1$) are less scattered than the results from the same distribution with a higher variability ($cv=0.3$), which is intuitive.

5.3.6 The effect of holding cost ($F_h$) and transportation cost ($F_t$) on the cost function value

In this section, we investigate the effect of holding and transportation unit cost on the value of the total cost function. Three levels of holding and transportation costs are tested, for different demand distributions. Table 23 presents the results for the case of having 5 product types ($n=5$), with the compatibility index of 48\%, and $\eta = 0.2$. 

145
Table 23: The effect of $F_h$ and $F_t$ on total cost, with $n = 5$ and $\tau = 48\%$

<table>
<thead>
<tr>
<th>$F_h$</th>
<th>$F_t$</th>
<th>Normal $(cv = 0.1)$</th>
<th>Normal $(cv = 0.3)$</th>
<th>Uniform $(cv = 0.1)$</th>
<th>Uniform $(cv = 0.3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 $\bar{C}$</td>
<td>Mean</td>
<td>3513.70</td>
<td>3510.92</td>
<td>3513.86</td>
<td>3509.77</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>6.06</td>
<td>5.44</td>
<td>6.23</td>
<td>7.07</td>
</tr>
<tr>
<td>25 $\bar{C}$</td>
<td>Mean</td>
<td>8620.91</td>
<td>8623.89</td>
<td>8629.72</td>
<td>8615.80</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>19.77</td>
<td>18.85</td>
<td>23.97</td>
<td>22.96</td>
</tr>
<tr>
<td>50 $\bar{C}$</td>
<td>Mean</td>
<td>17146.21</td>
<td>17161.05</td>
<td>17154.51</td>
<td>17170.42</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>35.80</td>
<td>39.26</td>
<td>30.24</td>
<td>32.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F_h$</th>
<th>$F_t$</th>
<th>Normal $(cv = 0.1)$</th>
<th>Normal $(cv = 0.3)$</th>
<th>Uniform $(cv = 0.1)$</th>
<th>Uniform $(cv = 0.3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 $\bar{C}$</td>
<td>Mean</td>
<td>3668.78</td>
<td>3670.99</td>
<td>3667.77</td>
<td>3649.06</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>4.90</td>
<td>6.09</td>
<td>7.75</td>
<td>9.15</td>
</tr>
<tr>
<td>25 $\bar{C}$</td>
<td>Mean</td>
<td>8773.26</td>
<td>8787.06</td>
<td>8775.43</td>
<td>8775.97</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>19.30</td>
<td>18.64</td>
<td>21.34</td>
<td>24.96</td>
</tr>
<tr>
<td>50 $\bar{C}$</td>
<td>Mean</td>
<td>17301.99</td>
<td>17320.76</td>
<td>17277.87</td>
<td>17310.91</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>23.68</td>
<td>31.41</td>
<td>31.33</td>
<td>35.89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F_h$</th>
<th>$F_t$</th>
<th>Normal $(cv = 0.1)$</th>
<th>Normal $(cv = 0.3)$</th>
<th>Uniform $(cv = 0.1)$</th>
<th>Uniform $(cv = 0.3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 $\bar{C}$</td>
<td>Mean</td>
<td>3923.17</td>
<td>3922.13</td>
<td>3834.48</td>
<td>3920.08</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>8.38</td>
<td>7.33</td>
<td>8.30</td>
<td>8.88</td>
</tr>
<tr>
<td>25 $\bar{C}$</td>
<td>Mean</td>
<td>9042.11</td>
<td>9043.76</td>
<td>9011.08</td>
<td>9010.30</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>13.04</td>
<td>17.56</td>
<td>11.34</td>
<td>12.90</td>
</tr>
<tr>
<td>50 $\bar{C}$</td>
<td>Mean</td>
<td>17551.70</td>
<td>17579.14</td>
<td>17543.01</td>
<td>17567.47</td>
</tr>
<tr>
<td></td>
<td>Stdv</td>
<td>33.74</td>
<td>30.77</td>
<td>29.21</td>
<td>27.51</td>
</tr>
</tbody>
</table>

The results in Table 23 show that for a fixed holding cost value; increasing the transportation cost value results in an increase in the cost function. We also observe that an increase in the cost function value occurs as the holding cost increases, for any fixed value of the transportation cost.

5.3.7 The effect of holding unit capacity ($b$) and transportation unit capacity ($T_c$) on the cost function

We study the effect of holding unit capacity ($b$) and transportation unit capacity ($T_c$) on the cost function for different values of demand. The results of ten replications for each configuration are reported in Table 24.
Table 24: The effect of holding unit capacity (b) and transportation unit capacity (T_c) on cost function value, with n = 5 and τ = 48%

<table>
<thead>
<tr>
<th>T_c</th>
<th>b</th>
<th>Normal (cv = 0.1)</th>
<th>Normal (cv = 0.3)</th>
<th>Uniform (cv = 0.1)</th>
<th>Uniform (cv = 0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Σ_j=1 D_j</td>
<td>Σ_j=1 D_j</td>
<td>Σ_j=1 D_j</td>
<td>Σ_j=1 D_j</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stdv</td>
<td>Stdv</td>
<td>Stdv</td>
<td>Stdv</td>
</tr>
<tr>
<td></td>
<td>1%</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3504.41</td>
<td>3502.35</td>
<td>3498.41</td>
<td>3501.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.17</td>
<td>6.16</td>
<td>13.69</td>
<td>7.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3497.16</td>
<td>3502.48</td>
<td>3497.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.62</td>
<td>6.14</td>
<td>13.27</td>
<td>9.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3500.60</td>
<td>3500.21</td>
<td>3497.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.45</td>
<td>6.44</td>
<td>13.32</td>
<td>9.20</td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1186.68</td>
<td>1182.95</td>
<td>1178.94</td>
<td>1176.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.83</td>
<td>7.47</td>
<td>30.40</td>
<td>36.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>782.02</td>
<td>781.95</td>
<td>779.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>0.25</td>
<td>7.84</td>
<td>8.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>781.97</td>
<td>782.00</td>
<td>779.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.26</td>
<td>0.24</td>
<td>8.98</td>
<td>6.56</td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1138.62</td>
<td>1127.45</td>
<td>1128.14</td>
<td>1127.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.55</td>
<td>25.43</td>
<td>33.83</td>
<td>30.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>544.07</td>
<td>543.38</td>
<td>538.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.09</td>
<td>1.69</td>
<td>15.55</td>
<td>21.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>441.99</td>
<td>442.00</td>
<td>439.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>0.07</td>
<td>7.19</td>
<td>5.05</td>
</tr>
</tbody>
</table>

The results uncover some interesting behavior of the total cost function: for each value of holding capacity (b), as the transportation capacity (T_c) increase, the total cost decreases. On the other hand, for each value of transportation capacity (T_c), the total cost decrease as the holding capacity (b) increase, as far as the holding capacity is smaller or equal to the transportation capacity (b ≤ T_c). This result can be explained as follows: increase in holding (transportation) capacity is equal to decrease in the holding (transportation) costs and a fewer number of holding (transportation) units are needed for units with higher unit capacity levels (since we have: b ≤ T_c). The total cost within each row remains almost constant, since the average demand is the same for different demand distribution. Table 24 also indicates that if the transportation capacity is small,
the total cost shows almost no sensitivity to the change in holding capacity (the differences are caused by demand randomness), but for higher values of transportation capacity, the total costs’ sensitivity to the holding capacity increase. As a result, for example, the effect of having $b=1\% \sum_{j=1}^{n} D_j$ is more significant if we have $T_c = 10\% \sum_{j=1}^{n} D_j$ than if we have: $T_c = 5\% \sum_{j=1}^{n} D_j$. This interesting result can be explained by the structure of the cost function: for the holding cost, the ratio of $\frac{\sum_{j \in J_i} Q_j}{b}$ is multiplied by the constant value of the holding unit cost ($F_h$). On the other hand, for the transportation cost, $\frac{\sum_{j \in J_i} Q_j}{T_c}$ is multiplied by another ratio that has the decision variables in the denominator $\frac{\sum_{j \in J_i} D_j}{\sum_{j \in J_i} Q_j}$.

5.3.8 The effect of holding unit capacity ($b$) and transportation unit capacity ($T_c$) on emission function

In this section, we conduct experiments on the multi-product model with the emission objective function. The emission of a gallon of diesel fuel is considered as 10.18 Kg/Gal\(^{42}\), and the emission of 1 KWh as 0.8 Kg/ KWh\(^{43}\). Table 25 summarizes the values of parameters used in the emission function.

<table>
<thead>
<tr>
<th>Table 25: Parameters used for emission function</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MPG of the empty truck</strong></td>
</tr>
<tr>
<td><strong>Weight of each product</strong></td>
</tr>
<tr>
<td><strong>Distance</strong></td>
</tr>
<tr>
<td><strong>CO₂ of 1 gallon diesel fuel</strong></td>
</tr>
<tr>
<td><strong>Emission of a freezer per period</strong></td>
</tr>
</tbody>
</table>

\(^{42}\)http://www.epa.gov/otaq/climate/documents/420f11041.pdf

\(^{43}\)http://www.eia.gov/tools/faqs/faq.cfm?id=74&t=11
Three levels of holding and transportation unit capacities are experimented, and the results are summarized in Table 26. The emission of each partition is calculated, using the objective function of the P(3)-Emission, as formulated in 5.2.3.

Table 26: The effect of holding unit capacity (b) and transportation unit capacity (Tc) on emission function value, with \( n = 5 \) and \( \tau = 48\% \)

<table>
<thead>
<tr>
<th>Tc</th>
<th>b</th>
<th>Normal ((cv=0.1))</th>
<th>Normal ((cv=0.3))</th>
<th>Uniform ((cv=0.1))</th>
<th>Uniform ((cv=0.3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>Mean</td>
<td>101796.56</td>
<td>101800.36</td>
<td>101776.50</td>
<td>101805.43</td>
</tr>
<tr>
<td></td>
<td>(\Sigma_{j=1}^{n} D_j) Stdv</td>
<td>196.61</td>
<td>198.22</td>
<td>201.25</td>
<td>211.70</td>
</tr>
<tr>
<td>5%</td>
<td>Mean</td>
<td>101822.32</td>
<td>101801.08</td>
<td>101793.37</td>
<td>101792.09</td>
</tr>
<tr>
<td></td>
<td>(\Sigma_{j=1}^{n} D_j) Stdv</td>
<td>179.61</td>
<td>199.72</td>
<td>190.80</td>
<td>213.75</td>
</tr>
<tr>
<td>10%</td>
<td>Mean</td>
<td>101777.18</td>
<td>101796.63</td>
<td>101834.30</td>
<td>101768.77</td>
</tr>
<tr>
<td></td>
<td>(\Sigma_{j=1}^{n} D_j) Stdv</td>
<td>200.31</td>
<td>189.65</td>
<td>208.53</td>
<td>202.57</td>
</tr>
</tbody>
</table>

From Table 26 it is evident that the transportation capacity (Tc) plays a significant role in the total emission value of the best partition. For small values of Tc, we observe a higher level of variability in the final emission function values, for the given values of holding capacity. As Tc increases, the variability of the objective function decreases significantly, but the emission function values increase. We can explain this increase in the emission function value by the effect of the extra load on the transporters, while the number of shipments does not accordingly
decrease. For the next level of T_c(10% \sum_{j=1}^{n} D_j), the emission function values decrease to their initial levels (as were for T_c=1% \sum_{j=1}^{n} D_j), while the variability of the function values reduces one more time, to almost one third of the variability in the previous T_c level. The results are fairly consistent against different demand distributions and demand variability, similar to the cost function in Table 24.

5.4 Analysis, Conclusion, and Future Research

In this chapter, we formulate the CICEM model when there is more than one type of product and the products within family share transportation and holding capacity, but products from different families do not. Therefore, the problem has two types of decision variables: (1) determining if a product is a member of a family or not, and (2) how much to order and how frequently to order for products within each family. We propose a solution procedure according to the decision variable types: (1) a procedure for grouping (partitioning) the products into different families, and (2) a procedure to solve the inventory problem for each family.

For partitioning products into families, we propose an exact algorithm and an improvement for it in 5.2.1.1, and a heuristic algorithm in 5.2.1.2. To solve the inventory problem, we consider two cases for the cost function and develop appropriate solution algorithm for each case: (1) the case of (intr.C = 0) in 5.2.2.1 and (2) the general case of (intr.C >0) in 5.2.2.2. We also provide a solution algorithm for the inventory problem with the emission objective function in 5.2.3.

In 5.3, we present a set of numerical examples to bring insights from the numerical results. Essentially, the experiments are designed to address the following questions:

1) How to apply the exact and its improvement procedure, and the heuristic grouping procedures?
2) What is the effect of demand variability on the cost function value?

3) What is the effect of compatibility of different products on the cost function value?

4) What is the effect of holding and transportation unit capacity on each objective function?

5) What is the effect of the change in transportation and holding costs on the cost function?

To address Question 1, we designed the experiments in 5.3.1 and 5.3.2. Experiments in 5.3.1 demonstrated how to apply the exact algorithm, and also the improvement procedure for the exact algorithm, to group the products, while in 5.3.2 we show how to implement the heuristic grouping algorithm. The results from 5.3.2 also depict the quality of the results of the proposed heuristic algorithm to group the products (Question 1). The results show that for small values for the ratio of holding to transportation costs ($\lambda \leq 1$), the heuristic grouping algorithm might be used instead of the exact grouping algorithm, while expecting at most 10% deviation from optimal cost.

Question 2 is addressed in 5.3.4, to show the effect of demand variability, and different demand distributions, on the cost function value. The results indicate that the variability of cost function value increases as the number of product types increases. This observation is consistent for both demand distributions (normal and uniform), and also for both levels of demand variability. Table 21 also indicates that for both cases of demand variability ($cv = 0.1, 0.3$), the cost values have higher variability when demand is normally distributed than when demand is uniformly distributed.

In 5.3.5 we answer Question 3, on the effect of products’ compatibility on the final solution. Our findings indicate that there is no significant difference between different levels of products’ compatibility. We also found that the results from a lower variability ($cv = 0.1$) but similar
demand distribution are less scattered than the results from the same distribution, but with a higher variability ($cv=0.3$), which is intuitive.

We address Question 4 in 5.3.7 on the effect of holding and transportation capacity on the total cost and 5.3.8 on the total emission. The results show that the cost function is more sensitive to the transportation unit capacity, than the holding unit capacity. In addition, the total cost decreases as the transportation unit capacity increase. This is an intuitive result, as having transporters with larger capacity reduces the number of shipments, and results in cost reduction for the system. The emission function also shows higher sensitivity to the transportation unit capacity, than the holding unit capacity. In addition, the variability of the results is higher for smaller values of transportation unit capacity, and decrease with increases in $T_c$.

Question 5 is answered in 5.3.6, where we studied the effect of holding and transportation cost on the solution of the inventory problem with the cost function as its objective function. The results show that the total cost function increases when the holding or transportation cost values increase.

A result worth mentioning is that the model never chooses to have more than 1 full transporter. By checking the model, we determine that by increasing the number of transporters from 1 to n, there is no improvement in cost or emission. The reason is if the number of transporters gets “n times” more, it reduces the shipment frequency to $\frac{1}{n}$, which makes the total transportation cost (emission) constant. On the other hand, in order to receive the order from “n” fully loaded trucks, we might need to turn more holding units on, which increases the cost (emission) of holding. As a result, it will never happen to use two (or more) transporters for a shipment.

*Future researches could be conducted in several aspects:*
The case may be studied in which, products are shipped from different warehouses, into a single DC. As a result, the families may not share the transportation capacity, but the holding capacity may be shared among different families.

5.5 References


6 CONCLUSION AND FUTURE RESEARCH

This chapter includes a brief narrative of the overall research approach, final conclusions and future research directions.

6.1 Contributions

Handling, holding and transportation of temperature-sensitive products along a supply chain is known as the cold supply chain (or cold chain). The cold chain represents an important and substantial part of the supply chain. According to a survey by USDA in 2012, there is a total gross refrigerated storage capacity of 3.96 billion cubic feet in the United States out of which, 3.22 billion cubic feet is the usable refrigerated storage capacity.44

The cold chain is responsible for approximately 1%-3.5% of the emission in the world (James & James, 2010) 45. According to a report by the Center for Sustainable Systems, there are some greenhouse gases in the atmosphere only because of industrial activities. HFC, the main refrigeration coolant currently used, is one of these gases, which has a large GWP46 number (14800). Leakage of the refrigerators is another important contributor to emission, which can also results in significant financial costs for firms.

The primary goal of this dissertation is to study supply chain management issues in cold chains, identify the gaps in the analytical modeling literature for supply chain decision-making in cold chains, and develop models for inventory management of cold products that consider both costs and emissions.

44 http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1034
46 Global Warming Potential
A thorough literature review is conducted on the supply chain network design articles. The three levels of decisions in the supply chains are studied, with the focus on the models that consider at least two levels of decisions together. These analyses also include the researches with emission consideration in their models.

Three main contributions of this research are as followed:

6.1.1 Contribution 1
The cold chain is studied based on the three levels of decision making in the supply chain: 1) strategic, 2) tactical, and 3) operational. Economic, environmental, and social aspects of the cold chain are described for each decision making level. The differences between the cold chain and the traditional supply chains are discussed afterwards. Based on these differences, the need for developing new models that address the unique nature of the cold chain is emphasized.

To bridge this gap we developed analytical models for the cold chains; specifically, we derived special inventory models for the cold chain.

6.1.2 Contribution 2
We introduced a new inventory model entitled the CICEM (Cold Items Cost and Emission Model) to determine the optimal order quantity in an environment with capacitated refrigerated units for holding and transportation. The model considers the holding cost at the distribution center and the transportation costs from the warehouse to the distribution center of cold item inventory. Thus, the CICEM model is a variation of the EOQ model with holding and transportation unit capacities that considers objectives of minimizing both costs and emissions. The transportation cost and emissions, the holding cost and emissions and the total cost and emission are modeled. For the CICEM model, we consider the emission function and two cases
for the holding cost function: 1) not considering the interest rate of the investment capital as a part of the holding cost (iC=0), and 2) considering the investment opportunity of the items (iC>0). To model the holding and transportation unit capacity, all of the mentioned functions are non-linear and non-continuous. We develop exact algorithms to find the optimum value for the cost function when iC=0 and iC>0, as well as for the emission function. The solution algorithm for the second cost case has a similar structure to the solution algorithm of the emission function (as both algorithms search among the end points up to a point and then search the beginning points for the intervals after).

The research questions to be addressed are:

*Considering the holding unit capacity and transportation unit capacity, what is the order quantity that minimizes the cost function?*

*Considering the holding unit capacity and transportation unit capacity, what is the order quantity that minimizes the emission function?*

*Considering the holding unit capacity and transportation unit capacity, what is the trade-off between the cost and emission functions for the inventory problem of the cold items?*

A set of numerical experiments were run comparing the cost objective of the CICEM model to the EOQ model and the emission objective of the CICEM to the SOQ model. The results confirmed the effectiveness of the CICEM model for different parameter settings.

Moreover, to explore the tradeoff between the cost and emission function of the cold chain inventory problem that has segmented holding and transportation units, the optimum value for each function of the CICEM model is calculated. We then conduct a trade-off analysis to determine the impact that only considering the cost function has on the environmental function (and vice versa). The results illustrate that using the emission function to set the order quantity
results in smaller deviations in the cost function than using the cost function and calculating the emission function based on that. Due to the structure of the two functions within each interval, the emission function is more sensitive to deviation from optimality than the cost function.

Our managerial insight is that:

“For cold items that use segmented holding and transportation units, using the emission function to set the order quantity results in smaller deviations in the cost function than the deviations resulting from the emissions function that sets the order quantity based on the cost function.”

6.1.3 Contribution 3

The CICEM model is extended for the case of having multi-products. The difference between the case of sharing the transportation and/or holding capacity and no sharing is studied. We consider a group of products that share capacities as a family of products. The products within a family must share transportation and holding capacity, but products from different families may not share any capacity. Therefore, the problem has two types of decision variables: (1) determining if a product is a member of a family or not, and (2) how much to order and how frequently to order for products within each family. We propose a solution procedure according to the decision variable types: (1) a procedure for grouping (partitioning) the products into different families, and (2) a procedure to solve the inventory problem for each family.

For partitioning products into families, we propose an exact algorithm in addition to a heuristic algorithm. To solve the inventory problem, we consider two cases for the cost function and develop appropriate solution algorithms for each case: (1) the case of (intr.C = 0) and (2) the general case of (intr.C >0). In addition, we develop a solution algorithm for the emission
function as well. We run a set of experiments for different values of the model parameters, and also consider two probability distributions for the demand, each with two levels of variability. Our experimental results show that as the number of products increase, the total inventory cost and also the variability of the total costs (for replications of a configuration) increase. It is also concluded from the numerical results that the model is more sensitive to the transportation cost, rather than the holding cost. The results for both cost and emission functions show higher sensitivity of the total emission (or cost) function value to the transportation unit capacity, rather than the holding unit capacity.

6.2 Conclusion

In terms of overall impact, this thesis makes several important contributions.

From a scientific perspective, our most significant contribution is the new formulation approach for modeling the cost and emission functions for the cold items in an inventory management problem. By considering the holding unit capacity and transportation unit capacity in the modeling approach, our modeling approach relaxes a major assumption in existing inventory literature.

From a practical perspective, the emissions generated from the cold chain, as well the financial expenditures are substantial, and our models and algorithms have the potential to improve the decision support systems responsible for managing the cold chain. It is also true that considering only the environmental impacts to find the order quantity results in less deviation from minimum cost, than the deviation in the emission if the order quantity is based on only the cost. Lastly, we hope that this thesis brings greater attention to the cold chain from the analytical and decision science communities, and introduces a new trend for future researches on cold chains.
6.3 Future Research

During our studies, we have identified several interesting topics for future research areas.

- In our CICEM model, we addressed the optimal order quantity with regards to cost and emission. We assumed the demand to be deterministic and known in advance, which may not be the case in some real world applications. It would be an interesting problem to develop an inventory model for cold items that considers stochastic demand, and also find the required safety stock level.

- The allocation problem for cold chain, which as we believe, would be a complicated problem to solve, is another possible research topic. The complexity of the allocation problem for cold items raise from the limited possibility of capacity sharing for cold items unlike regular items, as explained in 3.3, as well as the routing problem considering the last mile and shelf life time of the cold items.

- The CICEM model is a simplified model that could be extended into a more general model for a multi-transporter size, multi-transportation mode, multi-capacities, and a multi-period model. In addition, several other problems might be jointly considered with the inventory model, such as the Vehicle Routing Problem, the Vendor Managed Inventory Problem, or the Inventory-Routing Problem.

- The advantages of considering strategic level decisions with tactical level decisions (such as location and inventory problems) are discussed by other researchers, for traditional supply chains. The same study might be performed for cold items, considering the special cost/emission function structure for the cold items.

- In the CICEM model, we assume infinite capacity for the holding items, as well as the number of transporters. A more realistic model would consider total holding
capacity, and a maximum for the number of transporters. Also, the maximum order fulfillment from supplier on each shipment might be considered.

- The reverse logistics of the cold chain could be a possible research topic. For the forward section of the cold chain, the holding unit capacity and transportation unit capacities should be considered. On the other hand, for the backward section, the items might not need to be returned via refrigerated transporters, or be kept in cold warehouses.

- The CICEM model does not consider the perishability of the cold items. The current literature on the inventory management of the perishable items that consider perishability and shelf life, does not consider the unit capacity. A future research might be done on considering both unit capacity and the shelf life and perishability of the cold items in the inventory management problem.

All of the above mentioned extensions are for the cold chain. In addition, our modeling approaches that consider unit capacity of holding and transportation are applicable in several other industries. For example, supply chains that deal with liquid material can benefit from our inventory models. Liquid hydrogen, different grades and types of liquid fuels, chemicals (such as paint) are examples of such liquid items. Different liquid items should be kept in different holding units, usually called a tank. For transportation of the liquid items, they are shipped through different tanker trucks. The similarity of the supply chain for liquid items to the cold chain is: once a tank (for holding) or a tanker truck (for transportation) is assigned to a product, it has a constant, full cost for the whole tank (or tanker) which does not depend on the volume of the liquid inside.
6.4 References

