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A COMPUTATIONAL MODEL TO ESTIMATE THE THICKNESS OF THE WATERFILM DUE TO RAIN ON THE UPPER SURFACE OF AN AIRFOIL

BY

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THESIS

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ABSTRACT

Based on a two-phase boundary layer approach, a computational model is proposed to estimate the thickness of the waterfilm due to rain on the upper surface of an airfoil. The coupling between the air boundary layer and the water film is established by the conservation of mass and momentum at the interface. By a simple coordinate transformation, the interface is conformed to the finite difference grid system. Trajectory analysis of a raindrop of 1 mm diameter shows that the impingement of drops is high near the leading edge of the airfoil and decreases downstream. The finite difference equations of air/waterfilm are based on a Crank Nicholson scheme. The solution of finite difference equations at the initial station indicates a film thickness of 0.01 mm. Marching downstream along the surface of the airfoil gives raise to stability problems in the finite difference equations.
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# TABLE OF CONTENTS

LIST OF FIGURES ................................................. v
LIST OF TABLES .................................................. vii
NOMENCLATURE .................................................. viii

CHAPTER

I. INTRODUCTION ................................................. 1

II. THEORETICAL CONSIDERATIONS ................................ 5

III. MATHEMATICAL MODEL ........................................ 12
    Equation of Motion of a Raindrop .......................... 15
    Equations of the Airflow ................................... 20
    Equations of the Waterfilm Flow ........................... 22
    Interface Conditions ........................................ 23
    Conservation of Mass ........................................ 25
    Conservation of Momentum ................................... 27
    Boundary Conditions ......................................... 36
    Nondimensionalization ....................................... 36
    Coordinate Transformation .................................. 38

IV. SOLUTION METHODOLOGY ...................................... 44
    Trajectory Analysis of a Raindrop ......................... 45
    Finite Difference Equations ............................... 51

V. RESULTS AND DISCUSSION .................................... 63

VI. OBSERVATIONS AND RECOMMENDATIONS ....................... 72

APPENDICES

Appendix I: Properties ......................................... 74
Appendix II: Construction of a Joukowsky Airfoil and Potential Flow Around the Airfoil ........................................ 75
Appendix III: Curve Fit for CgVsRe ............................... 83
Appendix IV: Numerical Procedures ............................. 84
REFERENCES ....................................................... 107
### LIST OF FIGURES

**FIGURE**

1. Raindrops + Airflow Around An Airfoil ........................................... 6
2. Path of a Raindrop. ............................................................................. 7
3. Formation of Water Film on the Surface of the Airfoil ........................ 8
4. Continuous Water Film and Rivulet Formation on the Surface of a Wing/Airfoil ................................................................. 9
5. Physical Model. .................................................................................. 13
6. Control Volume at Air/Waterfilm Interface ........................................ 14
7. Velocity Components of a Raindrop in the Free Stream ....................... 16
8. Resolution of Drag Components .......................................................... 18
9. Surface Coordinates of the Airfoil ...................................................... 21
10. Magnified Control Volume at Air/Waterfilm Interface .......................... 24
11. Control Volume to Link the Mass Transfer at the Interface to the Free Stream Liquid Water Content .................................................... 30
12. Angle Between the Surface Coordinate \( x \) and the Regular Coordinate \( X \) ................................................................. 33
13. Physical Space Consisting of the Interface ......................................... 40
14. Computational Space Consisting of the Interface After Transformation .... 40
15. Experimental Curve for \( C_d V Re_s \) .................................................... 47
16 Reverse Joukowsky Transformation. .................................. 49
17 Flowchart of Trajectory Analysis. ............................... 52
18 Grid for the Finite-Difference Equations. ........................ 54
19 Grid for Estimating the Initial Profile ..................... 54
20 Flowchart for Solving the Finite Difference Equations ......... 59
21 Trajectory of a Raindrop of Diameter 1 mm ................. 64
22 Impingement Factor vs Distance on the Surface of the Airfoil ......................................................... 67
23 Complex Plane Consisting of Cylinder ...................... 76
24 Joukowsky Airfoil .................................................. 77
25 General Joukowsky Transformation ............................ 79
26 Joukowsky Transformation ........................................ 80
LIST OF TABLES

TABLE

1  Results of Trajectory Analysis of 1 mm Diameter Raindrop ........................................ 65

2  Impingement Factor at Different Locations on the Surface of the Airfoil .................. 66

3  Error Analysis ......................................................... 71
NOMENCLATURE

\( a \)  Radius of the Circular Cylinder
\( \dot{a} \)  Acceleration Vector
\( b \)  Constant Defined by Joukowsky Transformation
\( C \)  Chord of the Airfoil
\( C_d \)  Drag Coefficient
\( d \)  Diameter of the Raindrop
\( \vec{F} \)  Force Vector
\( g \)  Acceleration Vector
\( \vec{I} \)  Unit Vector in X Direction
\( \vec{i} \)  Unit Vector in x Direction
\( i \)  Counter in \( \xi \) Direction
\( j \)  Counter in \( \eta \) Direction
\( m_d \)  Mass of the Raindrop
\( m \)  Iteration Counter
\( \vec{n} \)  Unit Normal Vector
\( N_0 \)  Number Density of Raindrops
\( P \)  Center of the Circular Cylinder
\( p \)  Pressure
\( r_d \)  Radius of the Raindrop
\( Re \)  Reynolds Number
\( t \)  Time
\( u \)  Velocity Component in \( x/\xi \) Direction
Free Stream Velocity

Velocity Component in X-Direction Based on Potential Flow

Velocity Component in Y-Direction Based on Potential Flow

Terminal Velocity of the Raindrop

Velocity Component in y/\eta Direction

Velocity Vector

Relative Velocity Vector with Respect to Control Volume

Relative Velocity Vector of Air with Respect to the Drop

Coordinate Tangential to the Surface of the Airfoil

Coordinate Aligned Parallel to the Chord of the Airfoil

Volume Fraction of Liquid Water in the Free Stream

Coordinate Normal to the Surface of the Airfoil

Coordinate Perpendicular to X

Position of the Raindrop in the Free Stream

Complex Plane (x + iY), Consisting of the Joukowsky Airfoil

Complex Plane Consisting of the Circular Cylinder (x' + iY')

Complex Plane Consisting of the Circular Cylinder (x'' + iY'')

Thickness of the Water Film

Defined by the Transformation y/\delta

Angle Between x and X

Angle Made by the Relative Velocity Vector W_R or the Drag Force F_Drag on the Drop, with Respect to the X-Axis

Defined by the Transformation \xi = x

Complex potential
\( \mu \) Absolute Viscosity
\( \nu \) Kinematic Viscosity
\( \rho \) Density
\( \sigma \) Stress Tensor

**SUBSCRIPTS**

- \( a \) With Respect to Air
- \( w \) With Respect to Water
- \( e \) Velocity of Air at the Surface Based on Potential Flow
- \( o \) Reference Value
- \( d \) With respect to drop
INTRODUCTION

Weather has always been a major factor in aviation safety. Weather-related airplane accidents have been mostly attributed to wind shear and the possibility of rain as a causative factor has been overlooked (1). As a consequence, the National Aeronautics and Space Administration has initiated a research program to assess the effect of rain on airplane performance.

Rain falling on an aircraft will form a thin water film partly due to the shearing action of the airflow. Subsequent impacts of the drops on the waterfilm and the wavy nature of the film will create a rough surface. Air flowing over the waterfilm will induce lift and drag penalties. Based on this hypothesis, Haines and Luers (2) analyzed the effect of heavy rain on the aircraft performance. They equated the film roughness to equivalent sandgrain roughness and predicted drag and lift penalties up to 30%. Their analysis includes treating the airfoil as a flat plate.

Calarese and Hankey (3) have analyzed the effect of rain on airfoils by considering the Navier Stokes equation for air and adding an additional force term to the equations, as a contribution due to the presence of raindrops in the airflow. They have identified scaling parameters such as: the air Reynold's number, the drop Reynold's number and the density ratio of water content of rain to air. The effect of roughness due to the waterfilm on the surface of the airfoil was not addressed.
As the drop strikes the water layer, the water layer is indented and some of the water in the water layer rebounds in the form of a crown. Macklin and Metaxas (4) have studied this problem. They proposed a simple correlation relating the height of the crown formed due to the impact of the drop on a liquid layer to the Weber number of the drop, Froude number of the drop, depth of the liquid film and the crown radius. Hence, the thickness of the waterfilm is an important consideration in estimating the cratering effect of the raindrops.

There have also been some experimental attempts to assess the aerodynamic penalties caused by rain. Hansman and Barsotti (5) have performed water spraying tests on the airfoils coated with gel and wax, respectively. They observed a 75% reduction in maximum L/D ratio for a waxed surface and a 45% reduction in the case of a gel-coated surface. They considered the wettability of the surface to be an important criterion in the study of roughness due to the waterfilm on the surface of the airfoils. Dunham et al. (6) have conducted water spray tests on a NACA 64-210 model airfoil. They, too, observed a reduction in the maximum lift capability of the airfoil and an increase in the drag. They identified the dependency of their results on the velocity of airflow and the liquid water content of the spray. Hastings and Manuel (7) studied the characteristics of the waterfilm on the surface of airfoils with a simulated heavy rain
in a wind tunnel. They report a continuous sheet of water on the forward portion of the airfoil and a rivulet formation towards the end. They suggest film roughness as an important criterion in addressing the aerodynamic penalties.

The present study proposes a theoretical model to estimate the thickness of the waterfilm on the upper surface of the airfoil. Haines and Luers (2) have used a boundary layer type of equation to describe the flow of the waterfilm. By performing mass balance on the waterfilm, they estimated the thickness of the waterfilm. They assumed that air exerts a constant shear on the waterfilm and used this as a boundary condition to solve the equations describing the motion of the waterfilm. Bilinin's approach (8) is also based on single phase flow, i.e., water film flow, alone. These studies do not account for the pressure gradient caused by the curvature of the airfoil.

The air boundary layer can affect the waterfilm and vice versa. Both phases, air and waterfilm, have to be studied simultaneously to gain a clear physical insight of the problem. In addition, a better estimate of the thickness of the waterfilm is possible. The computational model proposed in the present study considers the two-phase nature of the problem. Appropriate equations are derived at the interface to couple the two phases. The effect of pressure gradient
due to the curvature of the airfoil is also included. Further, the momentum exerted by drops on the waterfilm is considered. Later sections in this study deal with the details of the computational model.
THEORETICAL CONSIDERATIONS

The physical problem to be studied is an airflow consisting of raindrops streaming towards airfoil (Fig. 1). Due to the presence of the airfoil, the flow is accelerated. Raindrops are dragged along by the air towards the airfoil. Air, being the lighter of the two, will curve around the airfoil and will form a streamlined pattern. On the other hand, raindrops, being heavier, tend to follow a straight line path and impinge on the surface of the airfoil (Fig. 2).

As the raindrops strike the airfoil, they impart momentum to the airfoil. This effect was considered by Haines and Luers (2). After impingement, the raindrops can break up and bounce back into the main stream in the form of small droplets. Droplets on the surface of the airfoil are flattened by the shearing action of the air. As a result, a thin layer of water is formed on the surface (Fig. 3). A continuous waterfilm formation is possible under conditions discussed below.

The airflow is accelerated by the favorable pressure gradient, caused by the forward portion of the airfoil. This pressure gradient is also transmitted to the waterfilm. Air exerts certain shear on the waterfilm causing it to move. Under the action of external shear, the waterfilm proceeds further downstream (Fig. 3). As the flow enters the adverse pressure gradient region, i.e., the rearward portion of the airfoil, the velocity of the air is reduced. In turn, the shearing action of the airflow is reduced. The waterfilm tends to stagnate, and due to the effect of surface tension, rivulets are formed (Fig. 4).
Fig. 1. Raindrops + Airflow around an Airfoil
Fig. 2. Path of a Raindrop
Fig. 3. Formation of Waterfilm on the Surface of the Airfoil
Fig. 4. Continuous Waterfilm and Rivulet Formation on the Surface of a Wing/Airfoil
Hastings and Manuel (7) have observed a similar pattern in their experimental studies. The rivulet formation is also a function of angle of attack. As the angle of attack increases, the rivulet formation starts earlier and closer to the leading edge. Hansman and Barsotti (5) have reported such behavior in their waterfilm studies. The present theoretical model is applicable to the region where a continuous film is formed.

Goldstein (9) has considered a water spray flow over a circular cylinder and analyzed the problem using a perturbation technique. Airflow was considered to be inviscid. Water drops were assumed to be uniformly distributed in the free stream, possessing the free stream velocity of the air. He neglected the effects of surface tension, wave formation and cratering in his theoretical analysis. This study dealt with the effect of drop impingement on the heat transfer coefficient. The theoretical results obtained by Goldstein are satisfactory compared to the experimental results.

Lu (10) has also analyzed the water spray flow over a circular cylinder, considering the viscous effects of air. He, too, assumed that drops are uniform in size and distribution in the free stream. The effects of surface tension, wave formation and splashing of drops were neglected in his theoretical model. In both cases (9,10) gravity effects were neglected.

Due to the highly complex nature of the present problem, assumptions such as those made by Goldstein and Lu are needed for
theoretical modelling. Hence, the following constraints are imposed on the model:

1. Drops are uniformly distributed in the free stream and have a velocity \( U_\infty \), the free stream velocity of air and a velocity \( V_T \) in the negative y-direction.
2. Flow is steady and two-dimensional.
3. The transport properties of air and raindrop mixture are that of air alone.
4. Both phases, air and water, are incompressible.
5. Effects of surface tension, wave formation and cratering are not considered in the film thickness model.
6. Gravity effects are considered only in the drop trajectory model.
7. Air flow is unaffected by the presence of raindrops.
8. Boundary layer approximations apply to the water film.
9. Raindrops are spherical and achieve terminal velocity.
10. Entrainment effects are neglected.
A mathematical model is developed in the present section to estimate the thickness of the waterfilm on the upper surface of the airfoil.

Consider a two-dimensional airflow consisting of raindrops flowing over the surface of the airfoil. The raindrops impinge on the surface of the airfoil. They are flattened by the shearing action of the airflow and, as a result, a thin waterfilm is formed on the surface of the airfoil. Due to the mass transferred by the drops, the thickness of the waterfilm is increased. Finally, the waterfilm achieves steady state. The steady state flow around the airfoil is considered in the present model (Fig. 5). The purpose of the present section is to develop a mathematical model to calculate the steady state waterfilm thickness on the upper surface of the airfoil. A typical control volume consists of raindrops, airflow and the waterfilm (Fig. 6).

The motion of the raindrops, air and the waterfilm is considered separately and modeled. The interfacial conditions at the air/waterfilm interface are established by the conservation of mass and momentum. The step-by-step formulation is as follows:

1. Equations of motion of the raindrop.
2. Equations describing the changes in the airflow.
3. Equations describing the changes in the motion of the waterfilm.
Fig. 5. Physical Model
Fig. 6. Control Volume at Air/Waterfilm Interface
4. Equations describing the changes at the air/waterfilm interface.

Since the raindrops are discrete particles, the motion of the raindrops is described in a Lagrangian reference frame. The rest are expressed in an Eulerian reference frame. The mass and momentum transfer due to raindrops at the interface in the Eulerian frame is related to the free stream liquid volume fraction and the rate of impingement at the interface. This will become evident in later sections.

**Equation of Motion of a Raindrop**

Consider a raindrop far upstream of the airfoil. It is assumed that the raindrop has achieved terminal velocity. Therefore the raindrop has a velocity \( U_a \) in the X-direction and \( V_r \) in the Y-direction (Fig. 7).

The motion of the raindrop can be described by Newton's second law of motion as follows:

\[
\vec{F} = m_d \vec{a} = m_d \frac{d\vec{v_d}}{dt}
\]

[1]

The left-hand side of Eq. [1] contains the forces acting on the raindrops. The right side represents the product of the mass of the raindrop and the acceleration of the raindrop. The forces acting on the raindrop are the drag forces, lift forces and the force due to
Fig. 7. Velocity Components of a Raindrop in the Free Stream
gravity. As a first order analysis, the lift forces on the raindrop are neglected. Eq. [1] is written as

\[ m_d \frac{dv_d}{dt} = \vec{F}_{\text{drag}} + \vec{F}_{\text{gravity}} \]  

[2]

The drag force on the raindrops is dependent on the magnitude and direction of the relative velocity of air with respect to the drop. This is illustrated in Fig. 8. The magnitude of the drag force on the drop is written as

\[ |\vec{F}_{\text{drag}}| = C_d (|\vec{W}_R|) \frac{\pi d^2}{8} \rho_a |\vec{W}_R| \theta^2 \]  

[3]

Eq. [2] is resolved in X and Y directions as

\[ m_d \frac{dU_d}{dt} = F_{\text{Xdrag}} \]  

[4]

\[ m_d \frac{dV_d}{dt} = F_{\text{Ydrag}} - F_{\text{gravity}} \]  

[5]

The drag force on the rain drop in the X-direction is written as (see Fig. 8)

\[ F_{\text{Xdrag}} = |\vec{F}_{\text{drag}}| \cos \theta \]

\[ = C_d (|\vec{W}_R|) \frac{\pi d^2}{8} \rho_a |\vec{W}_R| (U_a - U_d) \]  

[6]
Fig. 8. Resolution of Drag Components
The drag force on the raindrop in Y-direction from Fig. 8 is given by

\[ F_{\text{drag}} = |F_{\text{drag}}| \sin \theta = C_d |\vec{W}_R| \frac{\pi d^2}{8} \rho_a |\vec{W}_R| (V_a - V_d) \]  \[ 7 \]

The force on the raindrop due to gravity is given by

\[ F_{\text{gravity}} = m_d g \]  \[ 8 \]

Therefore the final equations describing the motion of the raindrop are as follows:

\[ \frac{dU_d}{dt} = \frac{3\rho_a}{4\rho_w d} C_d (|\vec{W}_R|) |\vec{W}_R| (U_a - U_d) \]  \[ 9 \]

\[ \frac{dV_d}{dt} = \frac{3\rho_a}{4\rho_w d} C_d (|\vec{W}_R|) |\vec{W}_R| (V_a - V_d) - g \]  \[ 10 \]

The initial condition is as follows:

At \( t = 0, x = -\alpha, U_d = U_\alpha, V_d = -V_T \)

The terminal velocity of \( V_T \) of the raindrop is estimated by the following formula (11):

\[ V_T(d) = 0.58 \left[ 1 - e^{-\frac{-d}{0.885}} \right]^{1.147} \]
Equations of the Airflow

The Navier Stokes equations govern the motion of air around the airfoil. The equations for a steady and incompressible flow, in terms of the surface coordinates of the airfoil (12) are as follows (Fig. 9):

The conservation of mass is given by:

\[
\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} = 0 \tag{[11]}
\]

The conservation of momentum is given by:

\[
\rho_a \left( u_a \frac{\partial u_a}{\partial x} + v_a \frac{\partial u_a}{\partial y} \right) = -\frac{\partial p_a}{\partial x} + \nu_a \left( \frac{\partial^2 u_a}{\partial x^2} + \frac{\partial^2 u_a}{\partial y^2} \right) \tag{[12]}
\]

\[
\rho_a \left( u_a \frac{\partial v_a}{\partial x} + v_a \frac{\partial v_a}{\partial y} \right) = \frac{\partial p_a}{\partial y} + \nu_a \left( \frac{\partial^2 v_a}{\partial x^2} + \frac{\partial^2 v_a}{\partial y^2} \right) \tag{[13]}
\]

Using boundary layer approximations (12), Eqs. [11], [12] and [13] are written as follows:

\[
\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} = 0 \tag{[14]}
\]

\[
\rho_a \left( u_a \frac{\partial u_a}{\partial x} + v_a \frac{\partial u_a}{\partial y} \right) = -\frac{dp_a}{dx} + \nu_a \frac{\partial^2 u_a}{\partial y^2} \tag{[15]}
\]
Fig. 9. Surface Coordinates of the Airfoil
Equation of Motion of the Waterfilm

The motion of the waterfilm is also governed by the Navier-Stokes equations. Since it is assumed that the flow in the waterfilm is of boundary layer type, the equations are as follows:

The conservation of mass is given by:

\[
\frac{\partial u_w}{\partial x} + \frac{\partial v_w}{\partial y} = 0 \tag{16}
\]

The equation of motion is given by:

\[
\rho_w \left( u_w \frac{\partial u_w}{\partial x} + v_w \frac{\partial u_w}{\partial y} \right) = -\frac{\partial p_w}{\partial x} + \mu_w \frac{\partial^2 u_w}{\partial y^2} \tag{17}
\]

The pressure difference across the air/waterfilm interface is governed by the effect of surface tension (13). Since it is assumed that the effects of surface tension are negligible, the pressure gradient in the waterfilm flow is the same as that in the air flow.

\[
\frac{dp_a}{dx} = \frac{dp_w}{dx}
\]

The pressure gradient in the air boundary layer is linked to the potential flow distribution of air around the surface of the airfoil by:

\[
-\frac{dp_a}{dx} = \rho_a \frac{dU_e}{dx}
\]
The same pressure gradient as given by the above equation is also transmitted to the waterfilm. Therefore:

$$-\frac{dp_w}{dx} = \rho_a \frac{dU_e}{dx}$$

**Interface Conditions**

The previous sections have considered the raindrops, air and waterfilm separately. A common link has to be established between these equations. The conservation of mass and momentum at the air/waterfilm interface connects the boundary layer equations of air/waterfilm and the impingement of the drops at the interface.

Consider a control volume bound by A1, A2, A3 and A4 (Fig. 10). The control volume shown in Fig. 10 is a magnified version of Fig. 6. Raindrops and air enter the control volume through A1 and A2. Water flows through A3 and A4. The approach followed in this section is similar to that used by Lu (10) in his model.

Let

$$\vec{V}_d = u_d \hat{i} + v_d \hat{j}$$

$$\vec{V}_a = u_a \hat{i} + v_a \hat{j}$$

$$\vec{V}_w = u_w \hat{i} + v_w \hat{j}$$

Assuming unit depth perpendicular to the plane of the paper, the surface area of the faces A1, A2, A3 and A4 are as follows:
Fig. 10. Magnified Control Volume at Air/Waterfilm Interface
Surface area of face A1 = $\Delta \delta$
Surface area of face A2 = $\Delta x$
Surface area of face A3 = $\Delta \delta$
Surface area of face A4 = $\Delta x$

Conservation of Mass

For a steady flow through a fixed and nondeformable control volume, the conservation of mass is given by (14),

$$\int_{\mathcal{S}} \rho \vec{v} \cdot \hat{n} \, ds = 0 \quad \text{[18]}$$

Mass flow through A1 = \[ \int_{A1} (\rho_d \vec{V}_d + \rho_a \vec{V}_a) \cdot (-\hat{i}) \, dS \]

= \(- (\rho_d \vec{u}_d + \rho_a \vec{u}_a) \, \Delta \delta \)

Mass flow through A2 = \[ \int_{A2} (\rho_d \vec{V}_d + \rho_a \vec{V}_a) \cdot (\hat{j}) \, dS \]

= \((\rho_d \vec{v}_d + \rho_a \vec{v}_a) \, \Delta x \)
Mass Flow through $A_3 = \int_{A_3} \rho W \vec{V}_W \cdot (\hat{i}) \, dS$

$= \rho W u_W \Delta \delta$

Mass Flow through $A_4 = \int_{A_4} \rho W \vec{V}_W \cdot (-\hat{j}) \, dS$

$= -\rho W v_W \Delta \chi$

Substituting the above expressions in Eq. [18], it is written as:

$$-(\rho d u_d + \rho a u_a) \Delta \delta + (\rho d v_d + \rho a v_a) \Delta \chi + \rho w u_w \Delta \delta - \rho w v_w \Delta \chi = 0 \quad [19]$$

Dividing Eq. [19] by $\Delta \chi$ and approximating $\frac{\Delta \delta}{\Delta \chi} \approx \frac{d \delta}{d \chi}$, the conservation of mass at the interface is given by:

$$\rho d (v_d - u_d \frac{d \delta}{d \chi}) + \rho a (v_a - u_a \frac{d \delta}{d \chi}) - \rho w (V_w - u_w \frac{d \delta}{d \chi}) = 0 \quad [20]$$
Conservation of Momentum

The conservation of momentum in the x-direction is needed to link the boundary layer equation of air with the waterfilm. The conservation of momentum for a steady flow through a fixed and non-deformable control volume is given by (14),

\[ \int_{S} \rho \vec{V} \cdot \vec{n} \, dS = F \quad [21] \]

The forces acting on the control volume are the surface forces and gravity. Since the gravity effect is neglected, the right hand side of Eq. [21] contains only the surface forces.

Eq. [21] is rewritten as follows:

\[
\begin{align*}
\int \left( \rho_d \vec{V}_d \cdot (-\vec{i}) + \rho_a \vec{V}_a \cdot (-\vec{i}) \right) \, dS &+ \int \left( \rho_d \vec{V}_d \cdot \vec{j} + \rho_a \vec{V}_a \cdot \vec{j} \right) \, dS \\
A4 &+ A2 \\
&+ \int (\rho_w \vec{V}_w \cdot \vec{i}) \, dS + \int (\rho_w \vec{V}_w \cdot (-\vec{j}) \, dS + \int \vec{n} \cdot \vec{o}_w \, dS \\
A3 &+ A4 \\
\end{align*}
\]
The conservation of momentum at the interface in the x-direction is the x component of the Eq. [22], which is as follows:

\[-(\rho_d u_d^2 + \rho_a u_a^2) \Delta \delta - \sigma_{xxa} \Delta \delta \]
\[+ (\rho_d u_d v_d + \rho_a u_a v_a) \Delta x + \sigma_{yx a} \Delta x \]
\[+ (\rho_w u_w^2) \Delta \delta + \sigma_{xxw} \Delta \delta \]
\[-(\rho_w u_w v_w) \Delta x - \sigma_{yxw} \Delta x \]  

[23]

The component of the stress tensor, under boundary layer approximations, reduce to:

\[\sigma_{xxa} = - \rho_a \]
\[\sigma_{yx a} = \mu_a \frac{\partial u_a}{\partial y} \]
\[\sigma_{xxw} = - \rho_w \]
\[\sigma_{yxw} = \mu_w \frac{\partial u_w}{\partial y} \]

Dividing Eq. [23] by \( \Delta x \), substituting for the components of stress tensor and rearranging gives,

\[\rho_d u_d (v_d - u_d \frac{\Delta \delta}{\Delta x}) + \rho_a u_a (v_a - u_a \frac{\Delta \delta}{\Delta x}) = -\mu_a \frac{\partial u_a}{\partial y} \]
\[-\rho_w u_w (v_w - u_w \frac{\Delta \delta}{\Delta x}) + (\rho_a - \rho_w) \frac{\Delta \delta}{\Delta x} + \mu_w \frac{\partial u_w}{\partial y} \]

[24]
Since the effect of surface tension is neglected, there is no pressure differential across the interface. Therefore $P_a = P_w$. The final form of the conservation of momentum at the interface in the $x$-direction is given by:

$$\rho_d u_d (v_d - u_d \frac{d\delta}{dx})$$

$$+ \rho a u_a (v_a - u_a \frac{d\delta}{dx}) = \mu_a \frac{du_a}{\delta y} - \nu_w \frac{du_w}{\delta y}$$  \[25\]

$$-\rho_w u_w (v_w - u_w \frac{d\delta}{dx})$$

The quantities $\rho_d (v_d - u_d \frac{d\delta}{dx})$ in Eq. [20] and $\rho_d u_d (v_d - u_d \frac{d\delta}{dx})$ in Eq. [25] are linked to the trajectories of the raindrops as follows:

Consider a control volume as shown in Fig. 11. $Y_{dmin}$ refers to the point in the free stream from where the drop hits the stagnation point of the airfoil. Raindrops beyond $Y_{dmax}$ cannot strike the control volume. Therefore, the surfaces S2 and S5 of the control volume conform to the limiting trajectories of the raindrop. Mass flows through S1, S3 and S4 only. S3 and S4 in Fig. 11 refer to A1 and A2 in Figure 10, respectively.

Mass flow through S1 = \[
\int_{S1} \left( \rho_w \frac{4}{3} \pi r_d^3 N_0 \right) (V_d \cdot \hat{n}) \, dS
\]

\[= -\rho_w \frac{4}{3} \pi r_d^3 N_0 \alpha u \Delta Y_d\]
Fig. 11. Control Volume to Link the Mass Transfer at the Interface to the Free Stream Liquid Water Content
where, $\Delta Y_d = Y_{dmax} - Y_{dmin}$

Let $x = \frac{4}{3} \pi r_d^3 N_0$

Mass flow through $S_1 = -\rho_w x_e U_x \Delta Y_d$ \hspace{1cm} [26]

Mass flow through $S_4 = \int_{S_4} \rho_w x_e v_d \cdot \vec{i} \, ds = \rho_w x_e u_d \Delta \delta$ \hspace{1cm} [27]

Mass flow through $S_3 = \int_{S_3} \rho_w x_e v_d \cdot (-\vec{j}) \, ds = -\rho_w x_e v_d \Delta x$ \hspace{1cm} [28]

For the control volume shown in Fig. 11, the mass balance reduces to,

Mass flow through $S_4$

+ Mass flow through $S_1$ $= 0$

+ Mass flow through $S_3$

Substituting Eqs. [26], [27] and [28] in the above expression, dividing by "$\Delta x$" and rearranging gives,

$\rho_w x_e (v_d - u_d \frac{d\delta}{dx}) = -\rho_w x_e U_x \frac{dY_d}{dx}$ \hspace{1cm} [29]
Since $\rho_{wx}^{x} = \rho_{d}$, substituting equation 29 in equation 20 gives the conservation of mass at the interface as follows:

$$\rho_{a} (v_{a} - u_{a} \frac{d\delta}{dx}) - \rho_{w} (v_{w} - u_{w} \frac{d\delta}{dx}) = \rho_{x}^{x} \frac{dV_{d}}{dx}$$  \[30\]

The term $\rho_{d} u_{d} (v_{d} - u_{d} \frac{d\delta}{dx})$ in Eq. [25] is replaced as follows:

Consider the coordinate systems shown in Fig. 12. The coordinate "x" is tangential to the surface of the airfoil and "y" is normal to it. The coordinates $X, Y$ are regular rectangular cartesian coordinates. Let $\phi$ be the angle made by $x$ with $X$.

$$\hat{i} = \cos \phi \hat{I} + \sin \phi \hat{J}$$

$$\hat{j} = -\sin \phi \hat{I} + \cos \phi \hat{J}$$

The velocity of the drop in the $x, y$ system is

$$\vec{v}_{d} = u_{d} \hat{i} + v_{d} \hat{j}$$  \[31\]

The velocity of the drop in the $X, Y$ system is

$$\vec{V}_{d} = U_{d} \hat{I} + V_{d} \hat{J}$$  \[32\]
Fig. 12. Angle Between the Surface Coordinate x and the Regular Coordinate X
Eq. [31] transformed into \(x, y\) system is as follows:

\[
\hat{V}_d = (u_d \cos \phi - v_d \sin \phi) \hat{I} + (u_d \sin \phi + v_d \cos \phi) \hat{J} \quad [33]
\]

Comparing Eqs. [32] and [33],

\[
u_d \cos \phi - v_d \sin \phi = U_d \quad [34]\]

\[
u_d \sin \phi + v_d \cos \phi = V_d \quad [35]\]

Solving [34] and [35] gives

\[
u_d = U_d \cos \phi + V_d \sin \phi \quad [36]\]

Therefore the term \(\rho_d u_d (v_d - u_d \frac{d\phi}{dx})\) is replaced as follows:

\[
\rho_d u_d (v_d - u_d \frac{d\phi}{dx})
\]

\[
= \rho_w x \nu_d (v_d - u_d \frac{d\phi}{dx}) \quad [37]
\]

\[
= -\rho_w x \nu \frac{dY_d}{dx} (U_d \cos \phi + V_d \sin \phi)
\]

In the above equation, \(\rho_w x (v_d - u_d \frac{d\phi}{dx})\) is replaced by Eq. [29] and \(u_d\) by Eq. [36].
Therefore, the conservation of momentum in x direction at the interface is given by

$$\rho_a u_a (v_a - u_a \frac{d\delta}{dx}) - \rho_w u_w (v_w - u_w \frac{d\delta}{dx}) = \rho_w x_e U_e \frac{dy_d}{dx} \quad [38]$$

$$+ u_w \frac{\partial u_w}{\partial y} - u_a \frac{\partial u_a}{\partial y} \quad x (U_d \cos \phi + V_d \sin \phi)$$

Another interfacial condition, namely the "no-slip" condition is introduced, i.e.,

$$u_a = u_w \quad [39]$$

In summary, the interfacial conditions consist of Eqs. [20], [38] and [39].

Eq. [39] implies that there is no discontinuity in the x component of the flow. Eq. [20] represents the conservation of mass at the interface. Eq. [38] represents the conservation of momentum at the interface.
Boundary Conditions

The boundary condition for the motion of the waterfilm is the no-slip condition at the surface of the airfoil, i.e.,

At \( y = 0 \), \( u_w = 0 \), \( v_w = 0 \)

The boundary condition for the boundary layer of air is that the velocity component of air in x direction i.e., \( u_a \) at the edge of the boundary layer or at \( y = \alpha \), should match the potential flow distribution, i.e.,

As \( y \to \alpha \) \( u_a \to U_e(x) \)

Non-dimensionalization

The governing equations of the airflow, waterfilm flow and the interface conditions are non-dimensionalized as follows:

\[
\begin{align*}
U_d^* &= \frac{U_d}{U_0} \\
V_d^* &= \frac{V_d}{U_0} \\
\rho_a^* &= \frac{\rho_a}{\rho_0} \\
\rho_w^* &= \frac{\rho_w}{\rho_0} \\
\mu_a^* &= \frac{a}{c} \\
\mu_w^* &= \frac{\mu_w}{c} \\
U_e^* &= \frac{U_e}{U_0} \\
u_a^* &= \frac{u_a}{U_0} \\
u_w^* &= \frac{u_w}{U_0} \\
x^* &= \frac{x}{c} \\
Y_d^* &= \frac{V_d}{c} \\
y^* &= \frac{y}{c} \sqrt{Re_o} \\
\delta^* &= \frac{\delta}{c} \sqrt{Re_o} \\
v_a^* &= \frac{V_a}{U_0} \sqrt{Re_o} \\
v_w^* &= \frac{V_w}{U_0} \sqrt{Re_o} \\
Re_o &= \frac{\rho_o U_0 c}{\mu_o}
\end{align*}
\]
The reference values $\rho_0$, $U_0$, etc. are enclosed in Appendix I.

The governing equations in non-dimensional form (dropping the asterisks for the sake of convenience) are as follows:

**Airflow:**

\[
\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} = 0 \tag{40}
\]

\[
u_a \frac{\partial u_a}{\partial x} + \nu a \frac{\partial u_a}{\partial y} = \frac{\rho a}{\rho e} \frac{\partial U_a}{\partial x} + \nu_a \frac{\partial^2 u_a}{\partial y^2} \tag{41}
\]

**B.C.:** As $y \to x$, $u_a \to U_e(x)$

**Waterfilm Flow:**

\[
\frac{\partial u_w}{\partial x} + \frac{\partial v_w}{\partial y} = 0 \tag{42}
\]

\[
u_w \frac{\partial u_w}{\partial x} + \nu \frac{\partial u_w}{\partial y} = \frac{\rho a}{\rho e} \frac{\partial U_e}{\partial x} + \nu_w \frac{\partial^2 u_w}{\partial y^2} \tag{43}
\]

**B.C.:** At $y = 0$, $u_w = 0$, $v_w = 0$
Interface Conditions:

\[ u_a = u_w \]  \[ 44 \]

\[ \rho_a (v_a - u_a \frac{d\delta}{dx}) - \rho_w (v_w - u_w \frac{d\delta}{dx}) = \sqrt{Re_0} \rho_w x_e U_e \frac{dY_d}{dx} \]

\[ \rho_a u_a (v_a - u_a \frac{d\delta}{dx}) \]

\[ -\rho_w u_w (v_w - u_w \frac{d\delta}{dx}) = \sqrt{Re_0} \rho_w x_e U_e \frac{dY_d}{dx} (U_d \cos \phi + V_d \sin \phi) \]

\[ -\mu_a \frac{\partial u_a}{\partial y} + \mu_w \frac{\partial u_w}{\partial y} \]  \[ 46 \]

Coordinate Transformation

The non-dimensional equations for airflow and waterfilm flow, coupled by the interface equations, can be solved in principle by any of the standard finite difference methods. This requires the setting
up of a finite difference grid to solve the equations. The interface in the x-y coordinate system doesn't coincide with the grid points (Fig. 13). The finite difference equations are applicable at the discrete grid points. These equations should account for the interface also.

Two choices are available. One is to devise special numerical schemes near the interface (15). This leads to complicated programming tasks. On the other hand, the interface can be made to coincide with the grid points. Hsu et al (16), Srinivasan and Rao (17), Duda et al (18) and Gaddis (19) have used coordinate transformations to immobilize the interface in their studies. A coordinate transformation $\xi = x$, $\eta = y/\delta$ is introduced in the present equations to align the interface to the grid system (Fig. 14). The computational plane is the $\xi$, $\eta$ plane.

The coordinate transformation is given by:

$$\xi = x$$

$$\eta = y/\delta$$
Fig. 13. Physical Space Consisting of the interface

Fig. 14. Computational Space Consisting of the Interface After Transformation
Therefore,

\[
\frac{d}{dx} ( ) = \frac{d}{d\xi} ( )
\]

\[
\frac{\partial}{\partial x} | y = \frac{\partial}{\partial \xi} | \eta \times \frac{\partial \zeta}{\partial x} + \frac{\partial}{\partial \eta} | \zeta \times \frac{\partial \eta}{\partial x}
\]

\[
\frac{\partial}{\partial y} | x = \frac{\partial}{\partial \zeta} | \eta \times \frac{\partial \zeta}{\partial y} + \frac{\partial}{\partial \eta} | \zeta \times \frac{\partial \eta}{\partial y}
\]

Eqs. [40-46] in the $\xi, \eta$ plane are as follows:

**Airflow:**

\[
\frac{\partial \nabla u_a}{\partial \xi} - \frac{n}{\delta} \frac{d \delta}{d \xi} \frac{\partial \nabla u_a}{\partial \eta} + \frac{\partial \nabla v_a}{\partial \eta} = 0
\]

\[\text{[47]}\]

\[
U_a \frac{\partial \nabla u_a}{\partial \xi} - \frac{n}{\delta} \frac{d \delta}{d \xi} \frac{\partial \nabla u_a}{\partial \eta} \cdot U_a + \frac{\partial \nabla v_a}{\partial \eta} \cdot U_a = U_e \frac{d U_e}{d \xi} + \frac{\partial^2 \nabla u_a}{\partial \eta^2} \frac{\partial^2 u_a}{\partial \eta^2}
\]

\[\text{[48]}\]

**B.C.:** As $n \to \infty$, $U_a + U_e(\xi)$
Waterfilm Flow:

\[ \frac{\partial u_W}{\partial \xi} - \frac{n}{\delta} \frac{d\delta}{d\xi} \frac{\partial u_W}{\partial n} + \frac{1}{\delta} \frac{\partial v_W}{\partial n} = 0 \tag{49} \]

\[ u_W \frac{\partial u_W}{\partial \xi} - \frac{n}{\delta} \frac{d\delta}{d\xi} u_W \frac{\partial u_W}{\partial n} + v_W \frac{\partial u_W}{\partial n} = \rho_w \rho e \frac{dU_e}{d\xi} + \frac{\nu_W}{\delta^2} \frac{\partial^2 u_W}{\partial n^2} \tag{50} \]

**B.C.:** At \( \eta = 0, u_w = 0, v_w = 0 \)

**Interface Conditions:**

\[ u_a = u_w \tag{51} \]

\[ \rho_a (v_a - u_a \frac{d\delta}{d\xi}) - \rho_w (v_w - u_w \frac{d\delta}{d\xi}) = \sqrt{Re_0} \rho_w \varepsilon \frac{dU_d}{d\xi} \tag{52} \]
\[ \rho_a u_a (v_a - u_a \frac{d\delta}{d\xi}) \]

\[ -\rho_w u_w (v_w - u_w \frac{d\delta}{d\xi}) = \text{Re} \rho_w e^{U_*} \frac{dY_d}{d\xi} x (U_d \cos \phi + V_d \sin \phi) \]

\[ + \frac{1}{\delta} (u_w \frac{\partial u_w}{\partial \eta} - u_a \frac{\partial u_a}{\partial \eta}) \]
SOLUTION METHODOLOGY

The equations of airflow and the waterfilm flow can be solved if, the pressure gradient parameter $U_e \frac{dV_e}{d\xi}$, the quantities $dY_d/d\xi$ and the velocity of components of the drop $(U_d, V_d)$ at the edge of the film are known. The pressure gradient parameter $U_e \frac{dV_e}{d\xi}$ is obtained from the potential flow analysis of the air flow around the airfoil. The quantities $dY_d/d\xi$, $U_d$ and $V_d$ are obtained by solving the raindrop trajectory equation in the $x$-$y$ plane and converting to the $\xi$, $\eta$ plane, after non-dimensionalization. In sequential order, the solution is as follows:

1. For various positions of the raindrop along $Y$ in the free stream, solve the trajectory equation of the raindrop and obtain the various points of impact on the surface of the airfoil and the velocity components of the raindrop $U_d$, $V_d$ in $X$, $Y$ directions respectively. The position of the raindrop in the free stream is shifted on the $y$-axis in the positive $Y$ direction until the drop is unable to strike the upper surface of the airfoil. The values $Y$, $(X, Y)$, the coordinates of the surface of the airfoil where the drop strikes, and $U_d$, $V_d$ are tabulated.

2. Using the distance $\Delta x$ between successive impacts on the surface of the airfoil and the initial vertical distance, $\Delta Y_d$
between the same raindrops, $\Delta Y_d/\Delta x$ is estimated. Since $\frac{dY_d}{dx} = \frac{\Delta Y_d}{\Delta x}$, the values generated for $\Delta Y_d/\Delta x$ are used for $dY_d/dx$. It is noted that $dY_d/d\xi = dY_d/dx$.

3. Using $\frac{dY_d}{d\xi}$, $U_d$, $V_d$ and the pressure gradient parameter obtained from the potential flow analysis, the equations of air/waterfilm flow can be solved with the boundary and interface conditions.

The later sections of this chapter explain the computational procedures in detail.

**Trajectory Analysis of a Raindrop**

Eqs. [9] and [10] are first-order differential equations with a prescribed initial condition. The Eqs. [9] and [10] can be written as follows:

\[
\frac{dU_d}{dt} = F_1 (X,Y,U_a, U_d, C_d, d) \tag{54}
\]

\[
\frac{dX_d}{dt} = U_d \tag{55}
\]

\[
\frac{dV_d}{dt} = F_2 (X,Y,V_a, V_d, C_d, d) \tag{56}
\]

\[
\frac{dY_d}{dt} = V_d \tag{57}
\]
Initial condition: At \( t = 0 \) \( X = - \infty \)

\[
U_d = U_a, \quad V_d = -V_T
\]

If the functional form of \( F_1 \) and \( F_2 \) on the right-hand side of the Eqs. [54] and [56] is known, the differential equation can be solved by marching forward in time, using the initial condition. The quantities appearing on the right-hand side of Eqs. [9] and [10] are obtained as follows:

The values \( \rho_w, \rho_a, g \) and \( d \) are prescribed as an input to the equations. The value of \( U_a \), the velocity of air in the flow field around the airfoil, is obtained by using a Kutta Joukowsky transformation (Appendix II). Hence, a Joukowsky airfoil is used in the present analysis. The details of the transformation are discussed in Appendix II. The potential flow distribution so obtained is used for the values of \( U_a, V_a \) in Eqs. [9] and [10]. However, any type of airfoil can be used and, accordingly, the potential flow around the airfoil can be simulated and used in Eqs. [9] and [10].

The magnitude of the relative velocity of air with respect to the moving raindrop is obtained by using

\[
\left| \vec{W}_R \right| = \sqrt{(U_a - U_d)^2 + (V_a - V_d)^2}
\]

The drag coefficient of the drop is estimated from the Stokes law variation of drag coefficient on a sphere (Fig. 15). A curve fit for Fig. 15 was obtained by Morsi and Alexander (20) to study particle trajectories in the two-phase flow systems. The same curve fit is used in the present analysis. The details are given in Appendix III.
Fig. 15. Experimental Curve for $C_d V_s Re$ (21)
The numerical solution of Eqs. [54-57] is obtained by using a fourth order Runge Kutta method (21). The drop position is incremented by a "small" time step. The "small" time step was obtained by a trial and error process so that instabilities are avoided. The velocity of air at this position must be known to calculate the relative velocity of air and coefficient of drag on the drop.

The airfoil used in the present study is a Joukowsky airfoil. The airfoil surface is generated by using a cylinder in the X', Y' plane and transforming it into the X, Y plane by using the Joukowsky transformation. The method of generating a Joukowsky airfoil is explained in Appendix II.

The raindrop trajectory analysis is carried out in the X, Y plane, consisting of the raindrop. To obtain the air velocity at the new position of the raindrop the coordinates (X, Y) of the new position of the raindrop are transformed back into the cylinder plane (x', y') coordinate system (Fig. 16). This is done as follows:

The Joukowsky transformation is given by

\[ Z = Z' + b^2 / Z^1 \]  \[58\]

which has the following inverse as follows:

\[ Z^1 = \frac{Z \pm \sqrt{Z^2 - 4b^2}}{2} \]  \[59\]
Fig. 16. Reverse Joukowsky Transformation
Since the position of the rain drop in the X, Y plane is known, the roots of Eq. [59] can be estimated. The root which occupies the same quadrant as occupied by the raindrop in the X, Y plane, is chosen and substituted into the complex potential obtained for the circular cylinder (see Appendix II). This gives the velocity of air in the cylinder plane.

By using the formula
\[ U_a - iV_a = \frac{d\omega}{dz} = \frac{d\omega}{dz^1} \times \frac{dz^1}{dz}, \]

the velocity of air at the new position of the drop is computed. The relative velocity of air with respect to the drop is used to compute the drag coefficient on the drop. \( C_D \) is obtained by using the curve fit derived by Morsi and Alexander (20). Using the fourth order Runge Kutta formulae (21) repeatedly, the above process is carried out a number of times, until the raindrop strikes or misses the airfoil surface.

The analysis starts from a position in the free stream from where the drop can hit the leading edge stagnation point of the airfoil. The particular position of the drop in the free stream, from where it strikes the stagnation point is determined by a trial and error process. The position the raindrop in the free stream is incremented by a small distance \( \Delta Y_d \) in the positive direction of the Y-axis and the calculations are repeated. The coordinates of the surface of the airfoil where the drop strikes, are noted. Likewise, the trajectory equation is solved for various positions in the free stream by marching in the positive Y-direction. When the raindrop can
not strike the airfoil, the calculations are stopped.

The above numerical procedure is flow charted in Fig. 17. The distance between the successive impact of raindrops on the surface of the airfoil is also calculated. This is denoted by \( \Delta x \). The initial distance between two such drops in the free stream is \( \Delta Y_d \), which is a known value. By using

\[
\frac{dY_d}{dx} = \frac{\Delta Y_d}{\Delta x}
\]

the value \( \frac{dY_d}{dx} \) is calculated. Since \( \frac{dY_d}{d\xi} = \frac{dY_d}{dx} \), the value of \( \frac{dY_d}{dx} \) can be used in the interface conditions developed for air/waterfilm flow in the \( \xi, n \) plane. The computer program of the trajectory analysis is enclosed in Appendix IV.

**Finite Difference Equations**

The governing equations of air boundary layer and the waterfilm flow, Eqs. [47-53] are formulated in terms of finite difference equations.

A Crank Nicholson Scheme (22) is used to formulate the finite difference equations. The finite difference grid is shown in Fig. 18. Since it is not possible to go up to infinity in the \( n \)-direction, a finite value of \( n \) is chosen for numerical solution. Values of \( u_a/u_w \) are determined at the intersection of the grid lines and the values of \( v_a/v_w \) are determined at the points designated by open circles. The
Initialize All Variables

Map Joukowsky Airfoil

Set Initial Conditions

Map Drop Position into Cylinder Plane

Obtain Velocity Components of Air in Airfoil Plane

Increment Drop Position by Runge-Kutta Method

Out of Range

Drop Hits Airfoil

STOP

$Y_d = Y_d + \Delta Y_d$

Fig. 17. Flowchart of Trajectory Analysis
finite difference equations of Eqs. [48], [50] and [53] are centered at the points denoted by open circles (Fig. 18). The equations expressing mass balance, [47], [49] and [52], are centered at the points denoted by crosses (Fig. 18).

The solution of Eqs. [47-53] is possible if an initial profile for \( u_a/u_w \) at some \( \xi \) is specified. The initial profile can be specified close to the leading edge so that marching downstream is possible. The initial profile for \( u_a/u_w \) at the initial station near the leading edge of the airfoil is obtained by iterating the finite difference form of the Eqs. [47-53], without the \( \xi \) derivatives. Eqs. [47-53], without the \( \xi \) derivatives are as follows:

**Airflow:**
\[
\begin{align*}
\frac{dv_a}{d\eta} &= 0 \quad [60] \\
\frac{v_a}{\delta^2} \frac{d^2u_a}{d\eta^2} - \frac{v_a}{\delta} \frac{du_a}{d\eta} + U_e \frac{dU_e}{d\xi} &= 0 \quad [61]
\end{align*}
\]

B.C.: As \( \eta \to \infty \), \( u_a \to U_e(\xi) \)

**Waterfilm Flow:**
\[
\begin{align*}
\frac{dv_w}{d\eta} &= 0 \quad [62] \\
\frac{v_w}{\delta^2} \frac{d^2u_w}{d\eta^2} - \frac{v_w}{\delta} \frac{du_w}{d\eta} + \frac{\rho_a U_e}{\rho_w} \frac{dU_e}{d\xi} &= 0 \quad [63]
\end{align*}
\]

B.C.: At \( \eta = 0 \), \( u_w = 0 \), \( v_w = 0 \)

**Interface Conditions:**
\[
\begin{align*}
u_a &= u_w \quad [64] \\
\rho_a v_a - \rho_w v_w &= \sqrt{Re_0} \rho_w \frac{dY_d}{d\xi} \quad [65]
\end{align*}
\]
Fig. 18. Grid for the Finite-Difference Equations

Fig. 19. Grid for Estimating the Initial Profile
The continuity equations of air and water are centered at points designated by "X" symbols. The Eqs. [61], [63] and [66] are centered at the intersections of grid lines (Fig. 19). Eq. [63] forms finite difference equations, which have a tridiagonal structure as follows:

\[
A_j u_{w,j-1} + B_j u_{w,j} + C_j u_{w,j+1} = D_j \quad 1 < j < N
\]  

At the interface, \( n = 1 \), i.e., \( J = N \), the following equation links the velocity components of waterfilm in the \( \xi \)-direction, i.e., \( u_w \), the velocity component of air, \( u_a \), the shear at the interface and the momentum exerted by the drops.

\[
A_N^m u_{wN-1}^m + B_N^m u_{aN}^m + C_N^m u_{aN+1}^m = D_N^m
\]  

The Eq. [61] forms finite difference equations, which have the tridiagonal structures as follows:

\[
A_j^m u_{aj-1}^m + B_j^m u_{aj}^m + C_j^m u_{aj+1}^m = D_j^m \quad N < j < N1
\]  

Eqs. [67], [68] and [69] are solved by the Gaussian elimination technique. An initial profile of the form \( u_w = u_w^e \), 1 < j < N
and \( u_a = U_e, j > N \), is chosen. An initial film thickness of arbitrary value is chosen. The finite difference Eqs. [67], [68] and [69] are solved. The finite difference form of Eqs. [60], [62] and [65] are solved for the component of velocity in the \( \eta \) direction, \( v_a/v_w \). These values are used in the finite difference form of Eq. [66] to produce a new value of film thickness. The non-linear coefficients in Eqs. [61], [63] and [66] are taken as the previous iteration values and the finite difference equations of [60-66] are solved again. Likewise, equations are iterated, until the computed values converge. The convergence criteria is specified as an input to the numerical procedure. Thus, the initial velocity profile in the \( \xi \) direction is \( u_a/u_w \), velocity profile in the \( \eta \) direction, i.e., \( v_a/v_w \) and the thickness of waterfilm \( \delta \) is established at the initial station near the leading edge.

After the initial profile is established at \( i = 1 \), the full form of the equations including \( \xi \) derivatives, i.e., Eqs. [47-53] are considered to obtain the profile at next the \( \xi \) downstream of the initial station \( i = 1 \). This is obtained as follows:

The initial guess for the velocity profile in the \( \xi \) direction at \( i = 2 \) is the same as that obtained for \( i = 1 \). The initial profile in the \( \eta \) direction at \( i + 1/2 \) is the same as that obtained for \( i = 1 \). The analogs to Eqs. [48], [50] and [53] are centered at the points
designated by open circles (Fig. 18). These analogs are centered at \( m + 1/2 \), where \( m \) is the previous iteration and \( m + 1 \) is the present iteration.

The finite difference form of Eq. [50] has the tridiagonal structure as follows:

\[
A_j^m \ u_{i+1,j-1}^m + B_j^m \ u_{i+1,j}^m + C_j^m \ u_{i+1,j+1}^m = D_j^m \quad 1 < j < N
\]  

[70]

The finite difference form of Eq. [48] also has the same tridiagonal structure as follows:

\[
A_j^m \ u_{i+1,j-1}^m + B_j^m \ u_{i+1,j}^m + C_j^m \ u_{i+1,j+1}^m = D_j^m \quad N < j < N+1
\]  

[71]

The Eqs. [70] and [71] are linked by the finite difference form of the interface Eq. [53] as follows:

\[
A_N^m \ u_{i+1,N-1}^m + B_N^m \ u_{i+1,N}^m + C_N^m \ u_{i+1,N+1}^m = D_N^m
\]  

[72]

From the interface condition, it is to be noted that

\[
u_{i+1,N}^m = u_{i+1,N}^{m+1}\]

Eqs. [70], [71] and [72] form a tridiagonal matrix. The tridiagonal matrix is solved by the Gaussian elimination technique.

The nonlinear coefficients at the center points for the finite difference Eqs. [70], [71] and [72] are approximated as the average of the four values surrounding it at the same \( n \) level.
For example, the nonlinear term $u_a$ corresponding to $\mathbf{m+1/2}$ in Eq. [48] is averaged as follows:

$$u_{ai+1/2, j}^{m+1} = \frac{1}{4} (u_{ai+1, j}^{m+1} + u_{ai, j}^{m+1} + u_{ai, j}^m + u_{ai+1, j}^m)$$

Since the tridiagonal matrix is to be solved for $u_{ai+1, j}^{m+1}$, $j = 2, N1 - 1$, the nonlinear term $u_{ai+1/2, j}^{m+1}$, should not contain values such as $u_{ai+1, j}$. Therefore, the non-linear terms containing the value of $u_a$ at $i + 1$ and $m + 1$ level are taken as the same as that of $u_a$ at $i = 1$ and of previous iteration $m$. The velocity component in the $\xi$ direction, $u_a/u_w$ at $j = 1, N1 - 1$, is the result obtained after solving the Eqs. [70], [71] and [72].

Now the finite difference form of the mass balance Eqs. [47], [49] and [52] are solved to obtain the component of velocity in the $\eta$-direction, $v_a/v_w$. Using these values in the finite difference form of Eq. [53], the new thickness of the waterfilm at $i = 2$ is obtained. The new values for $u_a/u_w$, $v_a/v_w$ and $\delta$ are used to improve the non-linear coefficients in the Eqs. [47-53] and the finite difference equations are solved again. The iterations are repeated until convergence is obtained. The final values of $u_a/u_w$, $v_a/v_w$ and $\delta$ at $i = 2$ are used as an initial guess at $i = 3$ and iterations are repeated again, until convergence is obtained. Likewise, we proceed downstream along the surface of the airfoil until separation is encountered.

The numerical procedure is flowcharted in Figure 20. The computer program is enclosed in Appendix IV.
Initialize All Variables and Assign Data

Read Impingement Factor

Map Joukowsky Airfoil and Find the Velocity of Air at the Surface of the Airfoil

Identify Points on the Surface of the Airfoil, with a Spacing of 0.005 in Between Them

Assign Velocity Components of the Drop

Non-Dimensionalize All Variables

Continue
Calculate Mass and Momentum Transfer due to Drops

Calculate Velocity Gradient of Air at the Nodes on the Surface of the Airfoil

Initialize Velocity Components of Air and Water

Assume Initial Profile for Air and Water

Set up the Tridiagonal Matrix at Initial Station

Solve the Tridiagonal Matrix to Obtain the Component of Velocity of Air/water Along the Surface of the Airfoil
Reassign the Variables

1. Solve Continuity Equation

2. Calculate New Film Thickness

3. Compare the Variables

4. $I = I + 1$

5. Use the Previous Station Values to Start the Iterations

6. Set up Tridiagonal Matrix According to Crank-Nicholson Scheme

7. Solve the Tridiagonal Matrix

8. Solve Continuity Equation

9. F

10. T

11. $I = I + 1$
Fig. 20. Flowchart for Solving the Finite Difference Equations
RESULTS AND DISCUSSION

The numerical procedures developed to solve the trajectory equation and waterfilm thickness are tested in this chapter. The results are discussed below. The procedures are enclosed in Appendix IV.

The trajectory analysis of a 1 mm diameter raindrop is performed as a trial run. A time step of 0.0005 was chosen. No numerical instabilities were encountered for the above time step. The complete trajectory of a raindrop of diameter 1 mm is plotted in Fig. 21. The drop follows a straight line path to impact on the surface of the airfoil. This is because of the effect of gravity. Gravity pulls down the drop and the drop tends to follow a straight line path compared to the airflow.

The trajectory analysis is performed for various positions of the raindrop in the free stream. The position of impact on the surface of the airfoil in the X, Y coordinates, the component of drop velocity in the X direction, i.e., $U_d$, and the component of drop velocity in the Y-direction, $V_d$, are tabulated in Table 1. The velocity components of the drop at impact on the surface of the airfoil are almost constant.

The impingement factor $\frac{dY_d}{dx}$ is calculated from Table 1. The impingement factor $\frac{dY_d}{dx} = \frac{\Delta Y_d}{\Delta x}$ and the distance on the surface of the airfoil is given in Table 2. These are plotted in Fig. 22.
Fig. 21. Trajectory of a Raindrop of Diameter 1MM
<table>
<thead>
<tr>
<th>$Y_{ds}$ Position of Drop in the Freestream, m</th>
<th>Coord. of the Impact on the Airfoil, $x, m$</th>
<th>Coord. of the Impact on the Airfoil, $y, m$</th>
<th>Vel. Comp. of Drop in x-Direction $u_d, \text{m/s}$</th>
<th>Vel. Comp. of Drop in y-Direction $v_d, \text{m/s}$</th>
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<td>(x), Distance on the Surface of Airfoil, m</td>
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Fig. 22. Impingement Factor $V$ Distance on the Surface of the Airfoil
The impingment factor is high, close to the leading edge. Since a large number of raindrops strike the region near the leading edge, the impingement factor is high in this region. As we traverse downstream on the surface of the airfoil a lesser number of raindrops strike the surface of the airfoil. Therefore, the impingement factor reduces downstream of the surface of the airfoil.

To solve the finite difference form of Eqs. [61-67], the values \( \rho_w x_e \frac{dY_d}{d\xi} \), \( \rho_w x_e u_\infty \frac{dY_d}{d\xi} \) \( (U_d \cos \phi + V_d \sin \phi) \) and the pressure gradient are needed.

A step size of \( \Delta \xi = 0.005 \) was chosen in the \( \xi \) direction. The properties of air and water, the volume fraction of water content \( x_e \) and the free stream velocity of air is supplied as a part of the data to the numerical procedure. The impingement factor \( \frac{dY_d}{d\xi} \) at the discrete locations on the surface of the airfoil, is in the \( \xi/x \)-direction, is interpolated from Fig. 22. The velocity components of the drops, \( U_d \) and \( V_d \) respectively, at the discrete locations on the surface of the airfoil in the \( \xi \) direction are also supplied as data to the numerical procedure. The angle \( \phi \) at the grid points in the \( \xi \)-direction is obtained from the surface geometry of the airfoil. The pressure gradient parameter \( \frac{dU_e}{d\xi} \) is obtained from the potential flow distribution (Appendix 1). The velocity gradient \( \frac{dU_e}{d\xi} \), at the grid points in the \( \xi \)-direction is estimated by fitting a parabola to the velocity distribution, \( U_e \) and differentiating the expression.
Until now, the input to the numerical procedure to solve the Eqs. [47-53] has been prepared. The finite difference equations are to be solved as follows:

A step size of $\Delta n = 0.025$ chosen in the $n$-direction. Just away from the stagnation point of the airfoil, an initial station, say, $\xi_0$ is chosen. The Eqs. [60-66], which do not consist of $\xi$ derivatives are considered at $\xi = \xi_0$. An initial profile of the form $u = u_e^n$ when $n < 1$ and $u = U_e$ when $n > 1$ is chosen. An initial thickness of waterfilm of the order 0.1 mm in the non-dimensional plane is chosen. The finite difference form of the Eqs. [61, 63 and 66], namely Eqs. [67-69], are iterated until convergence is obtained. The convergence criteria is that the difference of the sum of the velocity component in the $\xi$ direction, $u_a/u_w$, at all grid points in the present iteration and the sum of the velocity component in the $\xi$ direction, $u_a/u_w$ at all the grid points in the previous iteration is less than 0.3. The error, 0.3 distributed over 400 points in the $n$-direction is quite small.

Similar convergence criteria were chosen for $u_a/u_w$ and the thickness of the waterfilm (see Appendix IV). About 24 iterations were needed to obtain convergence. The film thickness was about 0.01 mm.

Based on the film thickness, the velocity profile $u_a/u_w$ in the $\xi$ direction at $\xi_0$, the thickness of waterfilm and the velocity profile at the next station downstream of $\xi_0$ was established. The finite difference form of Eqs. [47-53], were iterated at $\xi_1$, where $\xi_1$ is the station downstream of $\xi_0$.

Numerical instabilities are encountered in the procedure, while
iterating at $\xi_1$. The same convergence criteria as above is used. The error in the computed values is growing with the number of iterations. The error in the computed values of the velocity component $u_a/u_w$ is tabulated in Table 3. It can be seen that convergence is not obtained.
<table>
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<tr>
<th>Iteration Number</th>
<th>Difference in $u_w$ Between M, M+1 Iterations</th>
<th>Difference in $u_a$ Between M, M+1 Iterations</th>
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</thead>
<tbody>
<tr>
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<td>-0.1182688</td>
<td>8.204650</td>
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<tr>
<td>2</td>
<td>-0.1347039</td>
<td>275.535100</td>
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<td>3</td>
<td>-6.8693620</td>
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</table>
A theoretical study to estimate the thickness of the waterfilm on the upper surface of the airfoil has been completed. Scattered throughout the text material are the results, conclusions and recommendations of this study. They are summarized in this chapter.

1. The literature search has indicated that the study of the effect of rainfall on aircraft performance has often been a neglected area, until recently.

2. Based on a two-phase boundary layer approach, a mathematical model to calculate the thickness of the waterfilm on the upper surface of the airfoil is proposed. The model includes the effect of the curvature of the surface of the airfoil. The effect of surface tension on the thickness of the coated film was not considered in this study. Future modelling efforts can consider the effects of surface tension. This would mean that the magnitude of pressure gradients in the waterfilm would be less than that existing in the boundary layer of air.

3. Based on a finite control value, interface equations are derived to link the equations describing the changes in the air flow and the waterfilm flow on the surface of the airfoil. The analysis is based on a differential approach.
4. The interface of air/waterfilm is curved in the physical plane \((x,y)\). Special numerical procedures have to be developed to accommodate the curved interface. By using a simple coordinate transformation, the equations are transformed into a computational plane \((\xi,\eta)\). This transformation forces the interface to be a part of the grid system for the difference equations. In effect, simple numerical procedures were adequate to form the computational model.

5. A numerical program, using a fourth-order Runge Kutta method, was developed to trace the trajectory and predict the position of the impact of raindrops on the surface of the airfoil. The velocity components of air to be used in the above procedure were obtained from the potential flow analysis around the airfoil. The point of introduction of the raindrop in the free stream affects the position of impact on the surface of the airfoil. Gravity effects tend to straighten the path of the raindrop by pulling it down during the course of the trajectory.

6. The impingement factor, \(\frac{dV}{dx}\), is high near the stagnation point of the airfoil. It decreases rapidly along the surface of the airfoil towards the rear. Hence, large numbers of drops strike the region, close to the leading edge.

7. In this study, the angle of attack is zero. Further studies should consider small angles of attack. This will change the potential flow distribution around the airfoil,
in turn affecting the impingement factor. However, the present model is capable of accommodating the angle of attack.

8. Since the governing equations of air and waterfilm flow on the surface of the airfoil are parabolic in nature, a marching technique was proposed to solve these equations. A Crank Nicholson scheme was used to set up the finite difference equations. The convergence of velocity profiles and waterfilm thickness at the initial station were achieved after about 24 iterations. The thickness of the waterfilm at the initial station is about 0.01 mm. Further calculations downstream of the initial station were not possible on account of numerical instabilities. Even though the Crank Nicholson scheme is unconditionally stable, convergence is not obtained in the present numerical procedure. This may be due to the fact that, the equations are highly nonlinear in the computational phase \((\xi, \eta)\). Future studies should remedy this problem.
APPENDIX I

Properties of Water: @ 293 K

Density of Water, \( \rho = 998.2 \text{ Kg/M}^3 \)
Absolute Viscosity, \( \mu = 10.03 \times 10^{-4} \text{ N.Sec/M}^2 \)
Kinematic Viscosity, \( \nu = 10.05 \times 10^{-7} \text{ M}^2/\text{Sec.} \)

Properties of Air: @ 293 K

Density, \( \rho_0 = 1.2047 \text{ Kg/M}^3 \)
Absolute Viscosity, \( \mu_0 = 18.17 \times 10^{-6} \text{ N.Sec/M}^2 \)
Kinematic Viscosity, \( \nu_0 = 15.08 \times 10^{-6} \text{ M}^2/\text{sec.} \)

Acceleration Due to Gravity = 9.8 M/sec\(^2\)

Length of Chord of

Juokowsky Airfoil, \( C = 1.01\text{M} \)

Liquid Water Content of Raindrops in Free Stream, \( \rho_{lw} = 5 \text{ gm/M}^3 \)

Liquid Volume Fraction, \( X_e = \frac{\rho_{lw}}{\rho_w} \)

Free Stream Velocity, \( U_0 = 50 \text{ m/s} \)

The reference values \( \rho_0, \mu_0, \nu_0, \) are taken as the properties of air at 293 K.
Consider a circular cylinder with its center coinciding with the origin of the x" and y" coordinate system (Fig. 23). Let "a" be the radius of the cylinder.

A lifting potential flow around the cylinder can be constructed by superimposing

1. A uniform stream
2. A doublet
3. A line vortex

By using an appropriate transformation, the flow about a cylinder can be transformed into flow around an airfoil. The transformation is named after Joukowsky. The airfoil generated by the transformation is called a "Joukowsky Airfoil." The Joukowsky transformation is given by (Ref. 22),

\[ Z = Z' + \frac{b^2}{Z'} \]  

[1]

The complex plane Z consists of the airfoil and plane Z' consists of the circular cylinder (Fig. 24). It is to be noted that the origin of the Z' plane doesn't coincide with the center of the circular cylinder. It is offset by a small distance Z_p'. This can be explained as follows:
Fig. 23. Complex Plane Consisting of the Cylinder
Fig. 24. Joukowsky Airfoil
Consider the Joukowsky transformation

\[ Z = Z'' + \frac{b^2}{Z''} \]  

The center of the circle coincides with the origin of the \( Z'' \) plane and "a" is the radius of the circle. The points on the surface of the circular cylinder are mapped onto an ellipse in the \( Z \) plane (Fig. 25). As the radius of the circle is decreased, ellipses smaller in size are formed. Eventually, for a particular value of the radius "a," (say \( a=b \)), the transformation produces a strip of length (Fig. 25). The points \( 2b, -2b \) are singular. If somehow a circle of radius "a" is described in the \( Z'' \) plane such that the image of it in the \( Z \) plane lies between the strip and ellipse, then an airfoil-like shape can be produced in the \( Z \) plane.

To produce such an airfoil, a circular cylinder with its center offset from the origin of the coordinate system has to be considered (Ref. 22). Thus, we consider a circular cylinder whose center coincides with the \( Z'' \) plane and describe the same cylinder with respect to another coordinate system \( Z' \). The center of the circular cylinder, \( P \), is offset by a distance \( Zp' \) from the origin of the coordinate system \( z' \). This distance is denoted by \( Zp' \) (Fig. 26).

The singular points \(-2b, 2b\) should not lie in the flow field in the \( Z \) plane. This difficulty is avoided by describing the circle with respect to the \( Z \) system in such a fashion that the rearward stagnation point of the circular cylinder passes through the point \(+b\) in the \( Z' \) plane (Fig. 26). Thus, the singular point \( 2b \) in the \( Z \) plane becomes
Fig. 25. General Joukowsky Transformation
Fig. 26. Joukowsky Transformation
the rearward stagnation point of the airfoil. By doing this, the other singular point, \(-b\), is mapped into the surface of the airfoil.

To obtain potential flow around the Joukowsky airfoil, a lifting flow around the circular cylinder is considered. As mentioned before, the superposition of a uniform stream, a doublet and a line vortex, will produce a lifting flow. The complex potential for such a flow is

\[
\omega = U_\infty \left[ Z'' + \frac{a^2}{Z''} + 2i \frac{\Gamma}{4\pi U_\infty} \ln \left( \frac{Z''}{a} \right) \right]
\]

The circular cylinder and the lifting flow are to be described with respect to the \(Z'\) system. Let the center of the cylinder be offset by a distance \(Z_p'\) from the origin of the \(Z'\) plane (Fig. 4).

\[
Z_p' = X_p' + iY_p'
\]

\[
Z' = Z'' + Z_p'
\]

According to the Joukowsky transformation

\[
Z = Z' + b^2/Z'
\]

\[
(b - X_p')^2 + Y_p'^2 = a^2
\]

\(X_p', Y_p'\) are the coordinates of the center of the circular cylinder with respect to the \(Z'\) plane. "a" is the radius of the circular cylinder. By varying the coordinates \(X_p', Y_p'\) and the radius of the cylinder, various shapes of airfoils are generated.

Since the derivative of the complex potential, \(\omega\), yields the velocity components of the flow, by using the formula

\[
\frac{d\omega}{dz} = \frac{d\omega}{dz''} \times \frac{dz''}{dz'} \times \frac{dz'}{dz}
\]
the velocity components of the flow in the airfoil plane are obtained.

The magnitude of the velocity vector is given by

\[ |\frac{d\omega}{dz}| = \sqrt{u^2 + v^2} \] [6]

The normal component of the velocity at the body surface vanishes as a boundary condition for the potential flow. The velocity of air at the body is determined by using

\[ V = |\frac{d\omega}{dz}| = |\frac{d\omega}{dz}| / |\frac{dZ}{dz}| \] [7]

A numerical procedure developed by Chow (20) has been used for potential flow calculations. A Joukowsky airfoil of chord length approximately 1M and 15% thickness has been used in this study. The airfoil is symmetric. The computer program is shown in Appendix IV.
APPENDIX III

Expressions to approximate the experimental drag curve versus Reynolds number, have been developed by Morsi and Alexander (19). These are as follows:

\[ C_d = \frac{24.0}{Re} \quad \text{Re} < 0.1 \]

\[ C_d = \frac{22.73}{Re} + \frac{0.0903}{Re^2} + 3.69 \quad 0.1 < \text{Re} < 1.0 \]

\[ C_d = \frac{29.167}{Re} - \frac{3.8889}{Re^2} + 1.222 \quad 1.0 < \text{Re} < 10.0 \]

\[ C_d = \frac{46.5}{Re} - \frac{116.67}{Re^2} + 0.6167 \quad 10.0 < \text{Re} < 100.0 \]

\[ C_d = \frac{98.33}{Re} - \frac{2778}{Re^2} + 0.3644 \quad 100.0 < \text{Re} < 1000.0 \]

\[ C_d = \frac{148.62}{Re} - \frac{4.75 \times 10^4}{Re^2} + 0.357 \quad 1000.0 < \text{Re} < 5000.0 \]

\[ C_d = \frac{490.546}{Re} + \frac{57.87 \times 10^4}{Re^2} + 0.46 \quad 5000.0 < \text{Re} < 10000.0 \]

\[ C_d = \frac{-1662.5}{Re} + 5.4167 \times 10^6 \frac{1}{Re^2} + 0.5191 \quad 10000.0 < \text{Re} < 50000.0 \]
APPENDIX IV
NUMERICAL PROCEDURES

*JOB
C*******CONSTRUCTION OF JOUKOWSKY AIRFOIL AND THE POTENTIAL FLOW
C*******AROUND THE AIRFOIL
C*******INITIALIZATION AND DATA INPUT
COMMON Cl,CZ,Z1,Z2,ZP1
COMMON Ms,Ns,DX,Y1/GROUP/A,B,ZP1
DIMENSION X(300),Y(300),X1(121),Y1(101),PSI(121,101)
&APH1(200)*AP(200)*CP(200)*CONST(19)*ANG(200)*ANGLE(200),DKSI(200)
&X0(200),V(200),SLOPE(200)
DATA XMAX,XMIN,YMAX,YMIN/7.0,-17.0,19.0,0.0/
DATA XMAX1,XMIN1,YMAX1,YMIN1/7.0,-17.0,19.0,0.0/
U=5.00
RH1=1.0
FPRES=101325.0
A=0.788867513
R=0.75
C1=C*MPLX(0.0,J+1.0)
YP1=0.0
XP1=-0.028867513
DO 1 I=1,18
1 READ(5*,2) CONST(I1)
2 FORMAT(F8.1)
WRITE(6,3)
3 FORMAT('15E15.10/50X,*THE STREAMLINES TO BE PLOTTED ARE')
DO 5 K1=1,18
WRITE(6,4) CONST(K1)
4 FORMAT(50X,*PSI=',F9.1)'
5 CONTINUE
C*******SETUP THE GRID IN CYLINDRICAL PLANE AND COMPUTE THE VALUE OF THE
C*******STREAM FUNCTION AT THE GRID POINTS
ZP1=C*MPLX(XP1,YP1)
DX1=0.2
DY1=0.18
UX=(XMAX1-XMIN1)/DX1+1
UY=(YMAX1-YMIN1)/DY1+1
Y1(1)=XMIN1
DO 6 I=2,M
6 X1(I)=X1(I-1)+DX1
Y1(I)=YMIN1
DO 7 J=2,N
7 Y1(J)=Y1(J-1)+DY1
DO 8 I=1,M
8 DO 9 J=1,N
Z1=C*MPLX(X1(I),Y1(J))
Z2=Z1-ZP1
8 PSI(I,J)=U*AIMAG(Z2+AO/B2/Z2+CI*Z0*YP1*CLOG(Z2/A))
9 CONTINUE
C*******SEARCH FOR POINTS LYING ON A SPECIFIC STREAM LINE
CALL SEARCH(X1,Y1,PSI,CONST(K)*X,Y,KMAX)
C*******MAP THOSE POINTS TO AIRFOIL PLANE
CALL MAPPIN(X,Y,KMAX,XMIN,XMAX,YMIN,YMAX,NPINT)
C*****PRINT THE POINTS
  WRITE(6,...) CONST(K)
  9 FORMAT(*11,...) THE POINTS(X,Y) ON THE STREAMLINE PSI=*,F9.1)
  DU 10 K1=1,...NP0INT
  10 WRITE(6,...) X(K1),Y(K1)
  11 FORMAT(50X,...,F8.4,...,F8.4,3X,...,F9.4)
  12 CONTINUE
C*****SELECT A CUT 200 POINTS LYING ON THE CYLINDER AND MAP THESE
C*****POINTS INTO A JOUKOWSKY AIRFOIL USING JOUKOWSKY TRANSFORMATION
C*****COMPUTE THE VELOCITY OF AIR AT THE SURFACE OF THE AIRFOIL
  WRITE(6,...)
  14 FORMAT(*11,...) THE AIRFOIL CO-ORDINATES ARE*)
    PI=4*ATAN(1.)
    DPHI=PI/100,0
  DU 22 L=1,...200
  IF(L.EQ.1) GO TO 21
  APHI(L)=APHI(L-1)*DPHI
  GO TO 17
  21 APHI(L)=0,0
  V(L)=0,0
  GO TO 18
  17 ZZ=ACOS*(C1*APHI(L))
  ZL=ZL+ZZ/71
  X(L)=REAL(Z)
  Y(L)=AIMAG(Z)
  X(L)=X(L)+O,.5106195
  ANG(L)=(APHI(L)/PI)*180
  W=CAH(B(1.0-...(A/ZZ)**2+CT*2*YPL1/ZZ))
  R=CAH(B(1.0-(Z/ZZ)**2))
  D=W/R
  V(L)=0
  GO TO 15
  18 WRITE(6,...) APHI(L)
  19 FORMAT(*50X,...,F9.4)
  GO TO 22
  15 WRITE(6...) X(L),Y(L)
  16 FORMAT(25X,...,VELOCITY=*,F9.5)
  22 CONTINUE
  STOP
END

C*****SUBROUTINE SEARCH SEARCHES FOR POINTS LYING ON A SPECIFIC
C*****STREAMLINE
  SUBROUTINE SEARCH(X,Y,...,XX,YY,...,POINT)
  DIMENSION X(121),Y(101),PSI1,...,YY(30),XX(30)
  COMMON H,N,6Y
  K=0
  I=0
  J=1
  I=I+1
  IF(I.EQ.1) GO TO 7
  2 P=PSI(I,J)-PSIA
  IF(ABS(P).LE.7.00001) GO TO 6
  3 J=J+1
  IF(J.EQ.1) GO TO 1
  4 P=Q
  GO TO 3
  5 K=K+1
  XX(K)=X(I)
  YY(K)=Y(J)-DY*ABS(Q)/(ABS(P)+ABS(Q))
  P=0
  GO TO 3
  6 K=K+1
  XX(K)=X(I)
  YY(K)=Y(J)
  J=J+1
  IF(J.EQ.1) GO TO 1
GO TO 2
7 KMAX=K
RETURN
END

C******MAPS THE POINTS IN THE CYLINDER PLANE TO AIRFOIL PLANE BY
C******JOUKOWSKY TRANFORMATION

SUBROUTINE MAPPIN(X1,Y1,K1,XMIN,XMAX,YMIN,YMAX,NPOINT)
DIMENSION X1(K1),Y1(K1)
COMMON/GROUP/A,B,ZP1
COMPLEX Z,Z1,Z2,ZP1
NPOINT=0
DO 1 I=1,K1
Z1=COMPLEX(X1(I),Y1(I))
Z2=Z1-ZP1
R=ABS(Z2)
IF(R.LT.A) GO TO 1
Z=R**2/Z1
X=REAL(Z)
Y=IMAG(Z)
IF(X.LT.XMIN .OR. X.GT.XMAX .OR. Y.LT.YMIN .OR. Y.GT.YMAX) GO TO 1
NPOINT=NPOINT+1
X1(NPOINT)=X
Y1(NPOINT)=Y
1 CONTINUE
RETURN
END

ENTRY
1.0
2.0
3.0
4.0
6.0
10.0
14.0
20.0
25.0
35.0
50.0
65.0
80.0
100.0
125.0
150.0
175.0
200.0
DATA INPUT AND CALCULATION OF THE COORDINATES OF THE SURFACE

EXTERNAL FX, FY

COMPLEX CI, Z1, Z2, ZP1

DATA RHJ, RHOF, G/998, Z1, 1.2047, 9.8/

CI = COMPLX(0, 0, 1.0)

C = 0.278667513

B = 0.25

SI = 0.0

X8 = 0.0

YP1 = 0.0

XP1 = -0.028967513

ZP1 = CMPLX(XP1, YP1)

ANU = 1.508E-5

R = 0.00025

U1 = 50.0

T0 = 0.0

XO = 5.5

Y0 = 0.33

D = 2.0 * R

RHOBAR = RHOF / RHJ

A = (3.0 * RHOBAR) / (4.0 * 0.0)

PI = 4.0 * ATAN(1.0)

DPHI = PI / 1250

APHI(1) = 0.0

DO 1 I = 2, 2500

1 APHI(I) = APHI(I - 1) + DPHI

DO 2 L = 1, 2500

Z2 = C * EXP(CI * APHI(L))

Z1 = Z2 + ZP1

Z = Z1 + 300 * Z1

XX(L) = REAL(Z)

YY(L) = AIMAG(Z)

C 2 CONTINUE

REARRANGEMENT OF ARRAYS CONTAINING COORDINATES OF THE SURFACE

DO 16 I = 1, 1251

K = 1251 - I + 1

GX(K) = XX(I)

GY(K) = YY(I)

16 CONTINUE

C 16 PRINT, K, GX(K), GY(K)

DO 50 J = 1, 1251

S(J) = SORT((GX(J))**2 + (GY(J))**2)

ANGLE(J) = (ATAN(GY(J) / GX(J))) * 180 / PI

50 CONTINUE

C 50 PRINT, J, GX(J), GY(J), S(J), ANGLE(J)

MCOUNT = 1

BEGIN TRAJECTORY ANALYSIS FROM A SPECIFIC POINT IN THE FREE STREAM

T = T0

X = XO

Y = Y0

DT = 0.0005
K1=0
VT=TRMWEL(R)
V=-VT
U=50.0
WRITE(6,4)
4 FORMAT(//50X/*TRAJECTORY ANALYSIS BEGIN*/)
WRITE(6,5) T,XT,Y
5 FORMAT(//50X/*INITIAL TIME T=*,F8.4,*,/)
50X/*INITIAL POSITION*=*X=*,F8.4,*Y=*,F8.4)
C***MAP THE POSITION OF THE RAINDROP TO CYLINDER PLANE BY USING AN
C***INVERSE JOUKOWSKY TRANSFORMATION. COMPUTE THE VELOCITY OF AIR
C***IN THE CYLINDER PLANE AND USING JOUKOWSKY TRANSFORMATION COMPUTE
C***THE VELOCITY OF AIR IN THE AIRFOIL PLANE AT THE CORRESPONDING
C***POINT. THE AIR VELOCITY IS USED TO COMPUTE THE RELATIVE VELOCITY
C***AND DRAG COEFFICIENT
6 CALL MAPPI(X,Y,UF,VF,C,B,YP1,XP1,U1)
C***USING FOURTH ORDER RUNGE-KUTTA METHOD INCREMENT THE DROP POSITION
C***AND VELOCITY
C 22 FORMAT(//25X/*AFTER TIME T=*,F8.4,*RAINDROP POSITION=*,F8.4,*F3*4)
C***DETERMINE THE POSITION OF IMPACT IF THE RAIN DROP IS SUFFICIENTLY
C***CLOSE TO THE AIRFOIL.
IF(ABS(X).LE.0.53.AND.Y.LT.0.075)
& CALL SEARCH(X1,Y1,R1,K1,DT)
C***IF THE RAINDROP FALLS IN NEGATIVE Y-AXIS REGION STOP ITERATION
IF(R1.NE.0.0 .AND. Y1.LT.0.0) GO TO 12
C***RAINDROP HITS THE UPPER SURFACE OF THE AIRFOIL.
IF(R1.NE.0.0 .AND. Y1+GT.0.0) GO TO 7
C***RAINDROP CANNOT HIT THE AIRFOIL.
IF(X1.GT.0.1 .AND. ABS(Y1).GT.0.0) GO TO 9
GO TO 6
7 B1=X8
DEF=GX(1251)-0.0
XT=X1+DEF
WRITE(6,8) R,X,Y,X1,Y1,U,Y
8 FORMAT(//50X/*RAINDROP HITS THE AIRFOIL*/)
50X/*RADIUS=*,F6.4,*DROPP LOCATION X=*,F8.4,*Y=*,F8.4)
50X/*IMPACT POSITION X=*,F8.4,*Y=*,F8.4)
50X/*DROP VELOCITY UD=*,F8.4,*VD=*,F8.4)
GO TO 11
9 NCOUNT=NCOUNT+1
WRITE(6,10) R,X,Y
10 FORMAT(//50X/*RAINDROP CANNOT HIT THE AIRFOIL*/)
50X/*RADIUS=*,F6.4,*DROPP X=*,F8.4,*Y=*,F8.4)
C*** IF RAINDROP CANNOT HIT THE AIRFOIL TWICE CONSECUTIVELY STOP
C*** THE ITERATIONS
IF(NCOUNT.EQ.2) GO TO 14
C***DROP POSITION IN THE FREESTREAM IS INCREMENTED BY 0.005 AND
C***AGAIN THE EQUATION OF MOTION OF THE DROP IS SOLVED TO DETERMINE
C***THE NEW IMPACT POSITION ON THE SURFACE OF THE AIRFOIL
11 YD=Y0+0.005
GO TO 3
12 WRITE(6,13) R,X,Y
13 FORMAT(//50X/*RAINDROP FALLS IN THE -VE Y AXIS REGION*/
50X/*RADIUS=*,F6.4,*DROPP X=*,F8.4,*Y=*,F8.4)
14 STOP
END
C***SUBROUTINE MAPPI USES THE INVERSE JOUKOWSKY TRANSFORMATION TO
C***MAP THE RAIN DROP POSITION IN THE AIRFOIL PLANE TO THE CYLINDER
C****PLANE AND RETURNS THE VELOCITY OF AIR AT THE SAME POINT IN THE
C****AIRFOIL PLANE
SUBROUTINE MAPPI(X,Y,Uf,Vf,C,YPl,XP1,U1)
COMPLEX CI,2ZP1,2Z4,2Z8,2Z9,2Z12,2Z16,2Z13,2Z17,2Z20,2Z21
CI=CMPLX(0.0,1.0)
ZPI=CMPLX(XPI,YPI)
Z=CMPLX(X,Y)
Z3=Z**2
Z4=Z3-4.0*Z**2
YTEMP=AIMAG(Z4)
IF(X.LT.0.0 .AND. YTEMP.LE.0.0) GO TO 1
IF(X.GE.0.0 .AND. YTEMP.LE.0.0) GO TO 2
IF(X.LT.0.0 .AND. YTEMP.GT.0.0) GO TO 3
IF(X.GE.0.0 .AND. YTEMP.GT.0.0) GO TO 4
1 Z5=CMPLX(Z4)
Z6=SQRT((Z5+REAL(Z4))/2.0)
Z7=SQR((2*Z6-Z5))/2.0)
Z8=CMPLX(Z6+Z7)
Z9=Z-Z4
Z6=REAL(Z9)
Z7=AIMAG(Z9)
CALL VEL(X,Y,Z61,Z71,Uf,Vf,XPI,YPI,C,U1,B)
GO TO 5
2 Z2500=CMPLX(Z4)
Z10=SQRT((Z2500+REAL(Z4))/2.0)
Z11=SQRT((2*Z10-Z10))/2.0)
Z12=CMPLX(Z10+Z11)
Z13=Z-Z12
Z151=REAL(Z13)
Z111=AIMAG(Z13)
CALL VEL(X,Y,Z151,Z111,Uf,Vf,XPI,YPI,C,U1,B)
GO TO 5
3 Z131=CMPLX(Z4)
Z14=SQRT((Z131+REAL(Z4))/2.0)
Z15=SQRT((2*Z14-Z14))/2.0)
Z16=CMPLX(Z14+Z15)
Z17=Z-Z16
Z151=REAL(Z17)
Z161=AIMAG(Z17)
CALL VEL(X,Y,Z151,Z161,Uf,Vf,XPI,YPI,C,U1,B)
GO TO 5
4 Z171=CMPLX(Z4)
Z18=SQRT((Z171+REAL(Z4))/2.0)
Z19=SQRT((2*Z18-Z18))/2.0)
Z20=CMPLX(Z19+Z19)
Z21=Z-Z20
Z181=REAL(Z21)
Z211=AIMAG(Z21)
CALL VEL(X,Y,Z181,Z211,Uf,Vf,XPI,YPI,C,U1,B)
GO TO 5
5 RETURN
C****SUBROUTINE VEL COMPUTES THE VELOCITY OF AIR IN THE CYLINDER
C****PLANE AT THE POINT GIVEN BY SUBROUTINE MAPPIN AND USES
C****JOUKOWSKY TRANSFORMATION TO COMPUTE THE VELOCITY OF AIR
C****AT THE CORRESPONDING POINT IN THE AIRFOIL PLANE
SUBROUTINE VEL(X,Y,ZP,XPY,Uf,Vf,XPI,YPI,C,U1,B)
COMPLEX Z,ZP,ZPP,ZPI,TERMT,STERMT,TERMT,FTERMT,SITERMT
Z=CMPLX(X,Y)
ZPI=CMPLX(XPI,YPI)
ZP=CMPLX(XPY,ZPY)
C*****SUBROUTINE KUTTA USES A FOURTH ORDER RUNGE-KUTTA METHOD TO
C*****SOLVE THE DIFFERENTIAL EQUATION DEVELOPED FOR THE MOTION OF
C*****THE RAINDROP

SUBROUTINE KUTTA(X,Y,U,V,T,H,F1,F2,F3,F4,F5,F6)

COMMON XX(2500),YY(2500),A,D,ANU,RHOBAP,PI,VT,UF,VF,G,C,B,U1,V,
APHI(2500),S(1251),GX(1251),GY(1251),ANGLE(1251)

DIX=H*U
D1U=H*F1(X,Y,U,V,T)
D1V=H*F2(X,Y,U,V,T)
D2X=H*(U*DIU/2+0)
D2Y=H*(V*DIV/2+0)
KZ=X*DIX/2+0
KY=Y*D1Y/2+0
CALL MAPPIN(X2,Y2,UF,VF,C,B,YP1,XP1,U1)
D2U=H*F1(X+D1X/2+0,Y+D1Y/2+0,U+D1J/2+0,V+DIV/2+0+S+T+H/2)
D2V=H*F2(X+D1X/2+0,Y+DIV/2+0,U+D1J/2+0,V+DIV/2+0+S+T+H/2)
D3X=H*(U+D2U/2+0)
D3Y=H*(V+DIV/2+0)
X3=X+D2X/2+0
Y3=Y+D2Y/2+0
CALL MAPPIN(X3,Y3,UF,VF,C,B,YP1,XP1,U1)
D3U=H*F1(X+D2X/2+0,Y+D2Y/2+0,U+D2J/2+0,V+DIV/2+0+S+T+H/2)
D3V=H*F2(X+D2X/2+0,Y+DIV/2+0,U+D2J/2+0,V+DIV/2+0+S+T+H/2)
D4X=H*(U+D3U)
D4Y=H*(V+D3V)
X4=X+D3X
Y4=Y+D3Y
CALL MAPPIN(X4,Y4,UF,VF,C,B,YP1,XP1,U1)
D4U=H*F1(X+D3X,Y+D3Y,U+D3J,V+D3V+T+H)
D4V=H*F2(X+D3X,Y+D3Y,U+D3J,V+D3V+T+H)
T=T+H
X=X+(DIU+2.0*D2U+2.0*D3U+D4U)/6.0
Y=Y+(D1Y+2.0*D2Y+2.0*D3Y+D4Y)/6.0
U=U+(DIJ+2.0*D2J+2.0*D3J+D4J)/6.0
V=V+(DIV+2.0*DIV+2.0*D3V+D4V)/6.0
RETURN

END

FUNCTION FX(X,Y,U,V,T)

RETURN

FUNCTION FY(X,Y,U,V,T)

RETURN

END
VR=VF-V
WR=SQRT((UF-U)**2+(VF-V)**2)
FY=G*AVR*CD(WR)*WR
RETURN
END
C****FUNCTION CD ESTIMATES THE DRAG COEFFICIENT ON THE DROP BY USING
C****THE CURVE FIT DEVELOPED BY MORSICA.alexander
FUNCTION CD(V)
COMMON XX(2500),YY(2500),AX,DX,ANU,RHOBAR,PI,VT,UF,VF,G,C,9,u1,
APHI(2500),S(1251),GX(1251),GY(1251),ANGLE(1251)
RE=ABS(V)**0/ANU
C PRINT,RE
IF(RE.EQ.0.0) CX=0.0
IF(RE.GT.0.0.AND.RE.LE.1.0) CX=24.0/PE
IF(RE.GT.1.0.AND.RE.LE.10.0) CX=(22.73/RE)*0.903/(RE**2)*3.69
IF(RE.GT.10.0.AND.RE.LE.100.0) CX=(29.1667/RE)-3.869/(RE**2)+1.222
IF(RE.GT.100.0.AND.RE.LE.1000.0) CX=(4.65/RE)-116.67/(RE**2)+0.167
IF(RE.GT.1000.0.AND.RE.LE.5000.0) CX=(-490.546/RE)+57.87E04/(RE**2)
*0.47
IF(RE.GT.5000.0.AND.RE.LE.10000.0) CX=(-1662.5/RE)+(5.4167E04/RE**2)
*0.5191
CD=CX
C PRINT,CX
RETURN
END
C****FUNCTION TRMVEL ESTIMATES THE TERMINAL VELOCITY OF THE RAINDROP
FUNCTION TRMVEL(V)
COMMON XX(2500),YY(2500),AX,DX,ANU,RHOBAR,PI,VT,UF,VF,G,C,9,u1,
APHI(2500),S(1251),GX(1251),GY(1251),ANGLE(1251)
EX=(D*100.0/895)**1.147
TRMVEL=9.58*(1-EXP(-EX))
RETURN
END
C****SUBROUTINE SEARCH DETECTS THE POINT OF IMPACT OF THE RAINDROP
C****ON THE UPPER SURFACE OF THE AIRFOIL
SUBROUTINE SEARCH(X,Y,X1,Y1,E1,E,TP)
COMMON XX(2500),YY(2500),AX,DX,ANU,RHOBAR,PI,VT,UF,VF,G,C,9,u1,
APHI(2500),S(1251),GX(1251),GY(1251),ANGLE(1251)
DT=0.00005
ANG=(ATAN(Y/X))*180/P1
EPS=4.0*P1
EPS1=1.5
IF(X*LT.0.0.AND.Y*GT.0.0) GO TO 1
IF(X*GT.0.0.AND.Y*GT.0.0) GO TO 3
IF(X*LT.0.0.AND.Y*LT.0.0) GO TO 4
1 DO 5 I=1,667
DIFF=SQRT(X**2+Y**2)
ERR=DIFF-S(I)
ERRI=ANG-ANGLE(I)
IF(ABS(ERR)+LE.EPS+AND+ABS(ERRI)+LE.EPS1) B1=X*Y
IF(B1*NE.0.0) PRINT,T,ERR,ERR1
IF(B1*EQ.0.0.AND.I.EQ.667) GO TO 2
IF(B1*EQ.0.0.AND.I.EQ.667) GO TO 4
5 CONTINUE
3 DO 6 I=568,1251
DIFF=SQRT(X**2+Y**2)
ERR=DIFF-S(I)
ERRI=ANG-ANGLE(I)
\[ \text{IF}(\text{ABS}(\text{ERR}) \leq \text{EPS} \land \text{ABS}(\text{ERR1}) \leq \text{EPS1}) \quad B1 = X \times Y \]
\[ \text{IF}(B1 \neq 0.0) \quad \text{PRINT} \times I \times \text{ERR} \times \text{ERR1} \]
\[ \text{IF}(B1 \neq 0.0) \quad \text{GO TO 2} \]
\[ \text{IF}(B1 = 0.0 \land I = 1.0251) \quad \text{GO TO 4} \]

1. CONTINUE
2. \( X1 = Gx(I) \)
3. \( Y1 = Gy(I) \)
4. RETURN

ENTRY
**$JOB**

**CGAAMAA4,T=25**

**CALCULATION OF THE THICKNESS OF THE WATERFILM USING THE**

**FINITE DIFFERENCE FORM OF THE TWO PHASE BOUNDARY LAYER**

**EQUATIONS C**

**INITIALIZE AND DATA INPUT**

REAL NUMER,NUMER1,NUMER2,NUMER3,NUMER4,NUMERS,NUMER6,NUMER7
REAL NUMER,A NUMERB,NUMERd,NUMERG,NUMERF,NUMERG,NUMERH
DIMENSION XX(3000),YY(3000),APH1(3000),G(3001),GY(3001)
& TX(1501),TY(1501),X9(1501),AKSI(1245),XCOR(1245),YCOR(1245),
& BETA(1245),ANG$(1245),VE(3000),VEL(1245),SOURCE(1245),ETA(401)
DIMENSION VGRAD(1245),UOLW(35+401),VOL0M(35+401),U(401),
& UNEW(35+401),VNEW(35+401),UOLDA(35+401),VOLDA(35+401),
& UNEW(35+401),VNEW(35+401),DELTO(35),DELT1(35),C(401,4),
& VNOLW(35+401),VMUW(35+401),VMOLW(35+401),VMOU(35+401),
& LV(1501),SLOPE(1245),UD(1245),VD(1245)
DIMENSION U1NLOW(35+401),UIOLDA(35+401),VIOLW(35+401),
& U1OLV(35+401)
COMPLEX CI,2,ZZ,ZZ,ZP1

**DATA INPUT**

DKSI=0.0005
ETA=0.0
ETA1NF=10.00
ETA=0.025
N=41
IETA=401
RHOA=1.2047
AMUA=1.817E-6
ANUA=1.508E-5
RHOW=9.82
AMUW=1.003E-4
ANW=1.003E-7
XE=0.00005900
IFR=10
IT=30

**ANALYSIS**

DO 200 I=1,IT
READ(5*201) BETA(I)
201 FORMAT(F52)
C GO TO 200
C 202 BETA(I)=0.0
200 CONTINUE

**CONSTRUCTION OF JUOKOSKY AIRFOIL AND THE POTENTIAL FLOW VELOCITY**

**DISTRIBUTION ON THE SURFACE OF THE AIRFOIL**

CI=COMPLX(0.0,1.0)
A=0.278365513
B=0.25
U1=50.0
YP1=0.0
XP1=-0.028865513
ZP1=COMPLX(XP1,YP1)
P1=4.0*ATAN(1.0)
PH1=P1/1500
APHI(1)=0.0
DO 200 I=1,1245
IF(I*GT.30) GO TO 202
DO 1 I=2,3000
1 APHI(I)=APHI(I-1)+PH1
DO 2 L=1,3000
2 Z2=A*CEXP(CI*APHI(L))
\[ Z = Z_1 + Z_2/2 \]

\[ XX(L) = \text{REAL}(Z) \]

\[ YY(L) = \text{IMAG}(Z) \]

\[ W = \text{COS}((\pi - (A/2)*2) * \text{CPI} * (Z1/2)) \]

\[ R = \text{COS}((\pi - (D/1)*2)) \]

\[ W^R \]

\[ \text{VE}(L) = \text{DEG} \]

\[ \text{GO TO 2} \]

\[ \text{WRITE}(6,5) \text{ APHT(L)} \]

\[ \text{FORMAT}(2X,V) \text{ VELOCITY NOT DETERMINED AT TRAILING EDGE, ANGLE=8.3.4) VE(L)=0.0} \]

\[ \text{CONTINUE} \]

**REASSIGNMENT OF ARRAYS CONSISTING OF THE AIRFOIL SURFACE**

**COORDINATES**

\[ \text{DO 6 I=1,1501} \]

\[ K=1501-I+1 \]

\[ GX(K)=XX(I) \]

\[ GY(K)=YY(I) \]

\[ V(K)=\text{VE}(I) \]

\[ \text{CONTINUE} \]

\[ \text{DIFF=ABS}(GX(1)-0.0) \]

\[ \text{GO 7 I=1,1501} \]

\[ GX(1)=GX(1)+\text{DIFF} \]

\[ GY(1)=GY(1) \]

\[ \text{CONTINUE} \]

**SEARCH FOR POINTS LYING ON THE SURFACE OF THE AIRFOIL. THE DISTANCE BETWEEN THE POINTS IS APPROXIMATELY 0.005**

\[ XB(1)=0.0 \]

\[ YCOR(1)=GX(1) \]

\[ YCOR(1)=GY(1) \]

\[ \text{DO 8 J=2,1501} \]

\[ X=(J-1)+5.9T((5X(J)-5X(J-1))**2*(GY(J)-GY(J-1))**2) \]

\[ N1=2 \]

\[ J=1 \]

\[ K=2 \]

\[ AKSI(1)=XB(1) \]

\[ VEL(1)=V(1) \]

\[ \text{TEMP}=XB(J) \]

\[ \text{TEMP1}=XB(N1) \]

\[ \text{TEMP2}=GX(N1) \]

\[ \text{TEMP3}=GY(N1) \]

\[ \text{VTEMP}=V(N1) \]

\[ \text{DIFF}=\text{TEMP1}-\text{TEMP} \]

\[ \text{EPS}=\text{ABS}(\text{DIFF}-0.005) \]

\[ \text{IF}(\text{ABS}(\text{EPS}) \leq 0.0005) \text{ AKSI(K)}=\text{TEMP1} \]

\[ \text{IF}(\text{ABS}(\text{EPS}) \leq 0.0005) \text{ VEL(K)}=\text{VTEMP} \]

\[ \text{IF}(\text{ABS}(\text{EPS}) \leq 0.0005) \text{ XCOR(K)}=\text{TEMP2} \]

\[ \text{IF}(\text{ABS}(\text{EPS}) \leq 0.0005) \text{ YCOR(K)}=\text{TEMP3} \]

\[ \text{IF}(\text{ABS}(\text{EPS}) \leq 0.0005) K=K+1 \]

\[ \text{IF}(\text{ABS}(\text{EPS}) \leq 0.0005) J=N1 \]

\[ \text{IF}(N1=\text{EQ.1501}) \text{ GO TO 10} \]

\[ N1=N1+1 \]

\[ \text{GO TO 9} \]

\[ \text{ANGLE(I)=0.0} \]

\[ \text{DO 11 K=2,K} \]

\[ \text{SLOPE(K1)=(YCOR(K1)-YCOR(K1-1))/(XCOR(K1)-XCOR(K1-1))} \]

\[ \text{ANGLE(K1)=ATAN(SLOPE(K1))} \]

\[ \text{ANGLE(1)=0.0} \]

\[ \text{GO TO 11} \]
11 CONTINUE
UO=50.0
C****ASSIGN THE VELOCITY COMPONENTS OF THE DROP BY USING THE RESULTS
C****OF TRAJECTORY ANALYSIS
DO 250 J=1,K
UO(J)=69.9
250 VD(J)=-5.5
C*****NONDIMENSIONALIZATION
RHOO=RHOAD
ANO=ANUA
AMU=AMUA
CHORD=GX(1501)-GX(1)
RED=RHOO*UO*CHORD/AMUO
RHOAN=RHOA/RHOO
AMUAN=AMUA/AMUO
ANUAN=ANUA/ANUO
RHOAN=RHOAM/RHOO
AMUAN=AMUAM/AMUO
AMUAN=ANUAM/ANUO
UINFN=U1/UO
DO 13 J=1,K
UO(J)=UD(J)/UO
VD(J)=VD(J)/UO
13 VEL(J)=VEL(J)/UO
C*****CALCULATION OF THE QUANTITIES PERTAINING TO DROPS IN THE
C*****INTERFACE CONDITIONS
DO 14 J=1,K
ETA(J)=ETA(J)*SQRT(RED)*UINFN*XE
SOURCE(J)=ETA(J)*U(J)*COS(ANGLE(J))*VD(J)*SIN(ANGLE(J))
C PRINT* ,ETA(J)*SOURCE(J)
14 CONTINUE
C*****CALCULATION OF VELOCITY GRADIENT OF AIR AT THE SURFACE OF THE
C*****AEROFOIL, OBTAINED FROM POTENTIAL FLOW
V1=VEL(3)
X1=AKSI(3)
V2=VEL(1)
X2=AKSI(1)
VEL(K+1)=VEL(K-2)
AKSI(K+1)=AKSI(K-2)
DO 15 I=1,K
V3=V1
X3=X1
V1=V2
X1=X2
V2=VEL(K-2)
IF(I.LT.K) V2=VEL(I+1)
X2=AKSI(I+1)
FACT=(X3-X1)/(X2-X1)
VGRAD(I)=((V2-V1)*FACT-(V3-V1)/FACT)/(X3-X2)
C PRINT* ,V1,VEL(I),VGRAD(I)
15 CONTINUE
C*****INITIAL GUESS FOR THE THICKNESS OF THE WATERFILM
DO 230 J=2,IT
DELTO(J)=0.173261399
230 CONTINUE
C*****INCORPORATE THE NO-SLIP BOUNDARY CONDITION AT THE SURFACE OF
C*****THE AEROFOIL
J=1
ETA(J)=0.0
DO 16 I=1,IT
UOLJW(I,J)=0.0
UNEW(I,J)=0.0
VOLDW(I,J)=0.0
VHEW(I,J)=0.0
VMNWU(I,J)=0.0
VMOLW(I,J)=0.0
VMOLA(I,J)=0.0

16 CONTINUE
C*****INITIALIZE VELOCITY COMPONENTS OF AIR AND WATER
DO 161 I=2*IT
   DO 161 J=2*IETA
      UNEW(I,J)=0.0
      VOLDW(I,J)=0.0
      VHEW(I,J)=0.0
      UNFWA(I,J)=0.0
      VOLDA(I,J)=0.0
      VNFWA(I,J)=0.0
      EHTA(J)=EHTA(J-1)+DETA
      PRINT*,J,EHTA(J)
   161 CONTINUE
C*****INTEGRATE THE BOUNDARY CONDITION AT INFINITY FOR AIR
DO 17 I=2*IT
   UOLJA(I,IETA)=VEL(I)
   UNEWA(I,IETA)=VEL(I)
C PRINT*,I,VEL(I)
   UOLJA(I,J)=UOLJA(I,J-1)
   UNEWA(I,J)=UNFWA(I,J-1)
C PRINT*,I,SEWA(I,J)
17 CONTINUE
C*****CHOSE AN INITIAL STATION AWAY FROM THE STAGNATION POINT AND
C*****ASSUME A LINEAR PROFILES FOR THE COMPONENTS OF VELOCITY ALONG
C*****THE SURFACE OF THE AIRFOIL
C I=10
   IETA1=IETA-1
   DO 18 J=2,IETA1
      IF(J-N)260,260,261
260 UOLJW(I,J)=UOLJA(I,IETA1)*EHTA(J)
      IF(J-NE.F.N) GO TO 18
      GO TO 10
261 UOLJW(I,J)=UOLJA(I,N)
18 CONTINUE
C*****CALCULATE THE *W* FROM THE CONTINUITY EQUATION
DO 22 J=2,IETA
   IF(J-N) 19,19,21
19 VOLDW(I,J)=VOLDW(I,J-1)
   IF(J-NE.F.N) GO TO 22
   VOLDA(I,J)=(EHTA(I)/RHOAN)+(RHOW/RHOAN)*VOLW(I+N)
C PRINT*,I,J,VOLDA(I,J),VOLDA(I,J+1)
   GO TO 22
21 VOLDA(I,J)=VOLDA(I,J-1)
C PRINT*,I,J,VOLDA(I,J),VOLDA(I,J+1)
22 CONTINUE
ICOUNT=0
C*****START ITERATION AT INITIAL STATION LIMIT ITERATIONS TO 50
100 ICOUNT=ICOUNT+1
   IF(ICOUNT .GE.50) GO TO 35
C*****SETUP THE TRI DIAGONAL MATRIX ACCORDING TO THE FINITE DIFFERENCE
C*****FORM OF THE EQUATIONS, NOT CONTAINING KSI*DERIVATIVES
DO 25 J=2,IETA
  IF(J.EQ.0) GO TO 23
  IF(J.EQ.IETA) GO TO 24
  A1=(DELTOL(I))*DETA*2
  A2=ANUWN/A1
  A3=VOLDN(I,J)/(2.0*DETA)
  C(J,J)=A2*A3
  C(J,J+2)=-2.0*ANUWN/A1
  C(J,J+3)=A2-A3
  C(J,J+4)=-VGRAD(I)*VEL(I)*RHOAN*DELTOL(I)/RHOVN
  GO TO 25
23

C(J11)=ANUWN/A1
C(J11)=VOLDN(I,J)/(2.0*DETA)
C(J12)=C(J11-CJ11A
C(J13)=2.0*DETA*DELTOL(I)*CJ12
C(J13)=C(J11+CJ11A
C(J13)=-ANUWN*CJ15/CJ13A
C(J14)=ANUWN/(2.0*DETA*DELTOL(I))
C(J14)=C(J13-CJ14A
C(J,J)=CJ16
C(J21)=RHOAN*VOL*AN(I,J)
C(J22)=RHOVN*VOLW(I,J)
A4=ANUAN/A1
A5=VOLCA(I,J)/(2.0*DETA)
A6=2.0*DELTOL(I)*(DETA)**2
A7=2.0*ANUAN/A6
A8=A4*A5
A9=ANUAN*A7/A8
C(J3)=A9
A10=2.0*ANUWN/A6
A11=ANUWN/A1
A12=VOLWN(I,J)/(2.0*DETA)
A13=A11-A12
C(J24)=ANUWN/A10/A13
C(J,J)=CJ21-CJ22+CJ23+CJ24
C(J31)=ANUAN/(2.0*DETA*DELTOL(I))
C(J32)=A4-A5
C(J33)=A4-A5
C(J35)=CJ31+(C(J32+CJ31/CJ33)
C(J,J)=CJ35
C(J36)=VGRAD(I)*VEL(I)*ANUW*RHAN*DELTOL(I)/(RHOAN*CJ12)
C(J37)=VGRAD(I)*VEL(I)*ANUAN*DELTOL(I)/CJ33
C(J38)=CJ36+CJ37
C(J,J)=SOURCE(I)*(C(J38/(2.0*DETA*DELTOL(I))
GO TO 25
24

A14=ANUAN/A1
A15=VOLDA(I,J)/(2.0*DETA)
C(J,J)=A14+A15
C(J,J)=2.0*ANUAN/(DELTOL(I)*(DETA)**2)
C(J,J)=A14-A15
C(J,J)=-VGRAD(I)*VEL(I)*DELTOL(I)
25 CONTINUE

C** INCORPORATE BOUNDARY CONDITIONS
C(2,J)=(C(2,J)-C(2,J)*UNEWN(I,J)
C(2,J)=0.0
C(IETA1,J)=C(IETA1,J)-C(IETA1,J)*VEL(I)
C(IETA1,J)=0.0
C DO 235 J=2,IETA
235 PRINT J,C(J,J)+C(J,J)+C(J,J)+C(J,J)
C** SOLVE THE TRIDIAGONAL MATRIX BY THOMAS ALGORITHM
CALL TRIO(C1U1!ETAl)
DO 27 J=2*IETA1
IF(J.GT.N) GO TO 26
IF(J.EQ.N) UNEWA(I+N)=U(J)
UNEW(I+N)=U(J)
C PRINT I,J,UNEW(I,J)
GO TO 27
26 UNEWA(I+N)=U(J)
C PRINT I,J,UNEW(I,J)
27 CONTINUE
IF((ICOUNT/IFPRT)*IFPRT.NF..ICOUNT) GO TO 500
PRINT I,ICOUNT
C DO 327 J=2*IETA1
C IF(J.GT.N) GO TO 326
C PRINT I,J,UNEW(I,J)
C GO TO 327
C 326 PRINT I,J,UNEW(I,J)
C 327 CONTINUE
C*****CALCULATE "W" FROM THE CONTINUITY EQUATION
500 DO 30 J=2,IETA
IF(J.EQ.1) VMNUM(I,J)=UNEW(I,J)
IF(J.NE.1) VMNUM(I,J)=VMNUM(I,J-1)
29 VMNUM(I,J)=VNEW(I,J-1)
C PRINT I,ICOUNT I,J,VMNEW(I,J)
VMNUM(I,J)=VNEW(I,J)
IF(J.NE.N) GO TO 30
VNEW(I,J)= (BETA(I)/RHOA)+(RHOA*VNEW(I,J)/RHOA)
C PRINT I,ICOUNT I,J,VNEW(I,J)
VNUMA(I,J)=VNEW(I,J)
GO TO 30
29 VNEW(I,J)=VNEW(I,J-1)
C PRINT I,ICOUNT I,J,VNEW(I,J)
VNUMA(I,J)=VNEW(I,J)
30 CONTINUE
C*****CALCULATE THE NEW FILM THICKNESS FROM THE INTERFACE MOTHERN
C*****EQUATION
NUMERA=2.0*ANUN
NUMEB=VNEW(I+N)*DELTOL(I)*DETA
NUMERC=NUMERA-NUMEB
NUMER=4.0*ANUN/NUMERC
NUMER2=(NUMERA+NUMER)/(NUMERA-NUMER)
NUMERD=2.0*ANUN
NUMERE=VNEW(I+N)*DELTOL(I)*DETA
NUMER3=(4.0*ANUN)/(NUMERD+NUMERE)
NUMER4=(NUMERD+NUMERE)/(NUMERD+NUMERE)
NUMERP=-CJ36
NUMER5=ANUAN*(NUMERD*UNEW(I,N)-NUMER2*UNEW(I+N-1)-UNEW(I+N-1)
&   -CJ36)
NUMER5=NUMERD/(2.0*DETA)
NUMER6=ANUAN*(UNEW(I+N)+NUMER3*UNEW(I,N)+NUMER4*UNEW(I+N+1)
&   +CJ37)
NUMER6=NUMERG/(2.0*DETA)
NUMER5=NUMER5-NUMER6
DENOM=RHOA*VNEW(I+N)*VNEW(I,N)*RHOA*VNEW(I+N)*VNEW(I+N)
&   *SOURCE(I)
DELTINU(I)=NUMER/DENOM
C*****COMPARE THE VALUES OF PRESENT ITERATION TO THAT OF PREVIOUS
C*****ITERATION
SUM=0.0
SUM=SUM+1.0
SUM2=0.0
SUM3=0.0
DO 31 J=2,N
SUM=SUM+VNEWW(I,J)
SUM1=SUM1+VOLDW(I,J)
SUM2=SUM2+UOLDW(I,J)
SUM3=SUM3+UOLDW(I,J)
31 CONTINUE
SUM5=0.0
SUM6=0.0
SUM7=0.0
SUM8=0.0
DO 32 J=N,TETA
SUM5=SUM5+VNEWW(I,J)
SUM6=SUM6+VOLDW(I,J)
SUM7=SUM7+VNEWW(I,J)
SUM8=SUM8+UOLDW(I,J)
32 CONTINUE
DO 33 J=1,N
UOLDW(I,J)=UNEWW(I,J)
VOLDW(I,J)=VNEWW(I,J)
VMOLW(I,J)=VMNWJW(I,J)
33 CONTINUE
DO 34 J=N,TETA
UOLDW(I,J)=UNEWW(I,J)
VOLDW(I,J)=VNEWW(I,J)
VMOLW(I,J)=VMNWJW(I,J)
34 CONTINUE
EPS=SUM1-SUM
EPS1=SUM2-SUM3
EPS2=SUM5-SUM6
EPS3=SUM7-SUM8
EPS5=DELTNU(I)-DELTOL(I)
PRINT,ICOUNT, EPS1,EPS2,EPS4,EPS5,DELTNU(I),DELTOL(I)
C*****IF THE COMPARISON IS SATISFIED PROCEED TO THE NEXT STATION
IF(ABS(SUM-SUM1)<=0.01 .AND. ABS(SUM2-SUM3)<=0.01 .AND.
ABS(SUM5-SUM6)<=0.01 .AND. ABS(SUM7-SUM8)<=0.01)
GO TO 35
DELTO1(I)=DELTNU(I)
GO TO 100
35 CONTINUE
PRINT,ICOUNT
DO 537 J=2,N
IF(J>4) GO TO 536
PRINT,I,J,UNEWW(I,J)
537 CONTINUE
DELTO1(I)=DELTNU(I)
C*****AT THE NEXT STATION TO START THE IERATIONS ASSUME THE VALUES
C*****OF THE PREVIOUS STATION AS AN INITIAL GUESS
I=I+1
DELTO1(I)=DELTO1(I-1)
DO 37 J=1,N
VMOLW(I,J)=VMNWJW(I-1,J)
UOLDW(I,J)=UNEWW(I-1,J)
VMNWJW(I,J)=VMOLW(I,J)
UNEWW(I,J)=UOLDW(I,J)
PRINT,I,J, UOLDW(I,J)
CONTINUE
38 J=N+1,IETA
VMOLA(I,J)=VMNUA(I-1,J)
UGLOA(I,J)=UNEWA(I-1,J)
VMNUA(I,J)=VMOLA(I-1,J)
UNEWA(I,J)=UGLOA(I,J)

C
PRINT*+J,UGLOA(I,J)

38 CONTINUE
DELTNU(I)=DELTOL(I)
UGLOA(I+1,IETA)=VEL(I)
UNEWA(I+1,IETA)=VEL(I)
ITER=0
PRINT=2

110 ITER=ITER+1
C** IF IT IS THE FIRST ITERATION, ASSUME THE FILM THICKNESS AT THE PREVIOUS ITERATION AS THE INITIAL GUESS
IF(ITER.EQ.1) DLTMU(I)=DELTOL(I)
TF(ITER.EQ.1) GO TO 111
C** CALCULATION OF THE NEW FILM THICKNESS FROM THE INTERFACE CHARGE EQUATION, AFTER THE FIRST ITERATION
TEMP1=0.5*(SOURCE(I)+SOURCE(I-1))
TEMP2=(AMUNA*(UNEW(I+1)-UNEW(I+1-1)))/DELT
TEMP3=AMUNA*(UNEW(I+1-1)-UNEW(I+1))/DELT
TEMP4=(2.*DELMP+ELMP3))/DELTOU(I)+DELTMU(I-1))
TEMP5=(AMOON*VMJUA(I+1)*VMJUA(I+1-1))/2.*
TEMP6=(AMOON*VMJUA(I+1)*VMJUA(I+1-1))/2.*
NUMER=TEXP1-TEXP4+TEXP5-TEMP6
TEMP7=(JNEW(I,N)*JNEW(I-1,N))/2.*
TEMP8=RHOHU*TEMP7/4.*
NUMER=TEMP7-TEMP9
DELTMU(I)=DELTMU(I)*{OKSI*NUMER/DENOM}
PRINTNUM*DENOM
PRINT=DELTOU(I)+DELTMU(I)
IF(ITER.EQ.5) GO TO 116
C** SETUP THE TRIDIAGONAL MATRIX, BASED ON THE CRANK-NICHOLSON SCHEME. FINITE DIFFERENCE EQUATIONS INCLUDE "KSI" DERIVATIVES
111 DO 43 J=2+1,IETA
IF(J-N) 40,41,42
40 AINU=(ANUN)/
& ((JETA=*2)*2*((DELTNI(I)+DELTNI(I-1))**2))
AILD=(ANUN)/
& ((JETA=*2)*2*((DELTOL(I)+DELTOL(I-1))**2))
3INU=VMNUN(I,J)/
& ((*DETA+DELTNI(I)+DELTNI(I-1)))
3IOL=VMOLD(I,J)/
& ((*DETA+DELTOL(I)+DELTOL(I-1))
C1NU=(EHTA(I)*JNEW(I,J)+JNEW(I-1,J))/
& (DELTNI(I)-DELTNI(I-1))/
& (16.*DETA*KSI*(DELTNI(I)*DELTNI(I-1)))
C1OL=(EHTA(I)*JOLD(I,J)+JOLD(I-1,J))/
& (DELTOL(I)-DELTOL(I-1))/
& (16.*DETA*KSI*(DELTOL(I)*DELTOL(I-1)))
DINU=(1.+ANUN)/
& ((JETA=*2)*((DELTNI(I)+DELTNI(I-1))**2))
DICL=(1.+ANUN)/
& ((JETA=*2)*((DELTOL(I)+DELTOL(I-1))**2))
A1=AILD+AINU
B1=AILD+AINU
C2=C20t+C2'U
D2=D0t+D2'u
F2=(U1NEWA(I,J)*U1NEWA(I-1,J)+U1OLDA(I,J)*U1OLDA(I-1,J))/(3.*DKSI)
C(J,J+1)=A2*B2-C2
C(J,J+2)=D2-E2
C(J,J+3)=A2-B2+C2
EMP1=(-A2-B2+C2)*(U1NEWA(I-1,J)+U1OLDA(I-1,J)+U1OLDA(I,J))
EMP2=(-D2-E2)*(U1NEWA(I,J-1)+U1OLDA(I,J)+U1OLDA(I,J+1))
EMP3=0.25*((VEL(I)*VEL(I-1))*VGRAD(I)+VGRAD(I-1))
C(J+1,J+2)=EMP1+EMP2-EMP3
43 CONTINUE
*****INCORPORATE BOUNDARY CONDITIONS
C(2,J)=C(2,J)-C(2,J)*UNNEW(I,J)
C(2,J+1)=0.0
C(IETA1+4)=C(IETA1+4)-C(IETA1+3)*VEL(I)
C(IETA1+3)=0.0
*****STORE THE PREVIOUS ITERATION VALUES SO THAT THE NONLINEAR
*****COEFFICIENTS CAN BE IMPROVED
DG=0.0
J=1*IETA
IF(J-N) 601 601 603
601 UNOLW(I,J)=UNW(I,J)
VIOLW(I,J)=VMJW(I,J)
IF(J=NE.N) GO TO 600
603 UNOLDA(I,J)=UNEW(I,J)
VIOLDA(I,J)=VMJN(I,J)
500 CONTINUE
DMOL=DELTNU(I)
*****SOLVE THE TRIDIAGONAL MATRIX EY THOMAS ALGORITHM
CALL TPID(C,U,IETA)
DG 45 J=2*IETA
IF(J=GT.N) GO TO 44
TF(J=EQ.N) U1NEWA(I,J)=U(J)
UNFMW(I,J)=U(J)
C PRINT(I,J,UNFMW(I,J)
GO TO 45
44 UNNEW(I,J)=U(J)
C PRINT(I,J,UNNEW(I,J)
45 CONTINUE
IF((ITER/IPRT)*IPRT < NE*ITER) GO TO 531
PRINT(ITER)
DG 502 J=2*IETA
IF(J=GT.N) GO TO 503
PRINT(I,J,UNFMW(I,J),VMJW(I,J)
GO TO 502
503 PRINT(I,J,UNNEW(I,J),VMJN(I,J)
502 CONTINUE
*****CALCULATE VM FROM THE CONTINUITY EQUATION
501 DO 48 J=2*IETA
IF(J-N) 48 48 48
46 TEMP1=0.25*(IETA(J)*IETA(J-1))
TEMP2=UNEW(I+1,J)-UNEW(I+1,J-1)+U1OLDA(I,J)+U1OLDA(I,J-1)
TEMP3=(DELNU(I)*DELNU(I-1)*
& (UNNEW(I,J)+UNEW(I+1,J-1)-UNEW(I+1,J-1)-UNEW(I,J-1))
& (4.*DGKSI))
VMJN(J+J)=VMJN(J+J)-TEMP3*TEMP2*(DELNU(I)-DELNU(I-1))
& /DKSI)
IF(J=NE.N) GO TO 48
TEMP1=PHOWN*VMJN(I,J)/RHOAN
TEMP2=(U1NEWA(I,J)*U1NEWA(I,J)*DELNU(I)-DELNU(I-1))
C (2*0*DKS1)
TEMP3=DELTNU(I+J)+UNEWW(I+J)+UNEWW(I-1+J) *(DELTNU(I)-DELTNU(I-1))} \n(2*0*HOAN*DKS1)
TEMP4=(BETA(I)+BETA(I-1))*0.5
VMVNUA(I+J)=TEMP1+TEMP2+TEMP3+TEMP4
GO TO 48
47 TEMP1=0.25*(EHTA(J)+EHTA(J-1))
TEMP2=UNEWW(I-1+J)+UNEWW(I-1+J-1) *UNEWW(I+J)-UNEWW(I+J-1)
TEMP3=0.25*(DELTNU(I)+DELTNU(I-1))/(4.0*DKS1)
TEMP4=UNEWW(I+J)+UNEWW(I-1+J)-UNEWW(I-1+J-1)
VMVNUA(I+J)=VMVNUA(I+J-1)-{TEMP3+TEMP4} *
(TEMP1+TEMP2)*(DELTNU(I)-DELTNU(I-1))/DKS1}
48 CONTINUE
C****COMPARISON ARE THE VALUES OF PRESENT ITERATION TO THAT OF PREVIOUS
C****ITERATION
SUM=0.0
SUM2=0.0
SUM3=0.0
DO 49 J=2,N
SUM=SUM+VMVNUW(I,J)
SUM2=SUM+VMVNUW(I,J)
SUM3=SUM+VMVNUW(I,J)
49 CONTINUE
SUM5=0.0
SUM6=0.0
SUM7=0.0
SUM8=0.0
DO 50 J=2,N,IE
SUM3=SUM3+VMVNUA(I,J)
SUM4=SUM4+VMVNUA(I,J)
SUM5=SUM5+VMVNUA(I,J)
SUM6=SUM6+VMVNUA(I,J)
SUM7=SUM7+UNEWW(I,J)
SUM8=SUM8+UNEWW(I,J)
50 CONTINUE
EPS1=SUM1-SUM2
EPS2=SUM2-SUM3
EPS3=SUM3-SUM4
EPS4=SUM4-SUM5
EPS5=DELTNU(I)-DELTOL(I)
C****IF THE COMPARISON IS SATISFIED PROCEED TO THE NEXT STATION
C****ALONG THE SURFACE OF THE AIRFOIL IF NOT ITRATE AGAIN
IF(ABS(SUM-SUM1)+LE0.01) AND ABS(SUM2-SUM3)+LE0.01
AND ABS(SUM4-SUM5)+LE0.03
AND ABS(SUM6-SUM7)+LE0.03
AND ABS(DELTNU(I)-DELTOL(I))+LE0.005) GO TO 43
DO 60 J=2,IETA
IF(J=N) 51,51,52
51 UGLOW(I,J)=U1GLOW(I,J)
YMOLW(I,J)=VIOWL(I,J)
U1OLDA(I,J)=VIUOLA(I,J)
60 CONTINUE
DELOL(I)=VMV
GO TO 110
53 PRINTITER
DO 54 J=2,IETA
IF(J.GT.N) GO TO 60
PRINT,ITER
DO 54 J=2,IETA
IF(J.GT.N) GO TO 55
PRINT,ITER
UNEWW(I,J)+VMVNUW(I,J)
GO TO 54
55 PRINT*I,J,UNEA(I,J)+VMUA(I,J)
54 CONTINUE
C*****PROCEED TO NEXT STATION
IF(I.LE.IT) GO TO 36
56 STOP
END
C*****SUBROUTINE TRIO SOLVES A TRIODIAGONAL MATRIX BY THOMAS
C*****ALGORITHM
SUBROUTINE TRIO(CC,UC,M)
DIMENSION CC(4*1+4),UC(401),A(401),
(401),C(401),D(401)
MM=4-1
DO 225 J=2,M
A(J)=CC(J+1)
C(J)=CC(J+2)
D(J)=CC(J+3)
225 CONTINUE
DO 1 I=3,M
TEMP1=A(I)
TEMP2=D(I-1)
TEMP3=TEMP2/TEMP2
B(I)=3(I)-TEMP3+C(I-1)
1 CONTINUE
C PRINT*,J,A(J),B(J),C(J),D(J)
C 225 CONTINUE
C PRINT*,I,D(I)
C ENTRY
1*0.5
0.98
0.95
0.87
0.79
0.66
0.65
0.63
0.59
0.56
0.52
0.49
0.46
0.43
0.41
0.40
0.35
0.37
0.33
0.31
0.31
0.31
0.31
0.29
105
0.29
0.27
0.25
0.24
0.24
0.23
REFERENCES


