Radar: Theory and Insight

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RADAR: THEORY AND INSIGHT

BY

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RESEARCH REPORT

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ABSTRACT

Radar principles and design considerations are reviewed, using both analytical and heuristic methods. Emphasis is placed on achieving physical insight while maintaining technical rigor. Waveform properties and signal detectability maximization through matched filtering is discussed in detail. The Moving Target Indicator radar system is presented as an illustration of digital signal processing technology applied to modern radar systems.
# TABLE OF CONTENTS

## INTRODUCTION
- History of Radar ........................................ 1
- Common Radar Applications ............................... 2
- Radar Environments ...................................... 4

## RADAR FUNDAMENTALS
- Radar System Architecture ............................... 7
- Wave Properties .......................................... 8
- Radar Parameters ......................................... 8

## ANTENNAS
- Parabolic Antennas ....................................... 15
- Plane Antennas ........................................... 15
- Array Radars ............................................. 16

## DETECTION OF SIGNALS IN NOISE
- Types of Noise ........................................... 25
- The Matched Filter ....................................... 25
- Detection Schemes ....................................... 26

## WAVEFORM DESIGN
- Rectangular Pulse ......................................... 33
- Pulse Compression ........................................ 33
- Discrete Coded Waveforms ............................... 34
- The Ambiguity Function .................................. 40

## APPLICATIONS
- Radar Altimeters ......................................... 47
- Over-the-Horizon Radars ................................. 47
- Synthetic Aperture Radars ............................... 48

## MOVING TARGET INDICATOR
- MTI History ................................................ 52
- System Architecture ...................................... 53
- MTI Return Signal Analysis ............................. 54
- Airborne MTI .............................................. 55
- Delay Line Cancelers .................................... 56
- MTI Doppler Filters ...................................... 57
- The Moving Target Detector ............................. 62

## SUMMARY .................................................. 68

## LIST OF REFERENCES ...................................... 69
# LIST OF FIGURES

1. Radar System Components ................................................. 7
2. Rectangular Pulse ........................................................ 9
3. RCS of a Sphere ............................................................ 12
4. Radar Range Equation Geometry ......................................... 14
5. Parabolic Antenna .......................................................... 15
6. Flat Aperture Antenna ..................................................... 16
7. Far-Field Antenna Pattern ................................................. 18
8. Comparison of Far-Field Antenna Patterns .............................. 20
9. Time Delay Beam Steering .................................................. 22
10. Phase Shift Beam Steering ................................................ 23
11. Noise Probability Density Functions .................................... 30
12. Rectangular Pulse Characteristics ...................................... 33
13. Linear FM Chirp ............................................................ 37
14. LFM Matched Filter Transfer Function .................................. 37
15. LFM Matched Filter Impulse Response .................................. 38
16. LFM Matched Filter Output ............................................... 38
17. LFM Progression Through Matched Filter ............................... 38
18. LFM Matched Filter Response to Noise and Signal ..................... 39
19. LFM Matched Filter Response to Simultaneous Echos ................ 39
20. Pulse Burst ................................................................. 40
21. Random Pulse Burst ....................................................... 41
22. Ambiguity Functions ...................................................... 43
23. LFM Matched Filter Response to Doppler Shift ....................... 46
24. Response of V-FM Chirp to Doppler Shift ............................. 46
25. Over-the-Horizon Radar ................................................... 48
26. Coherent MTI Block Diagram ............................................. 55
27. MTI Signal After Demodulation ......................................... 56
28. Delay Line Canceler Network Representation .......................... 57
29. Single Delay Line Canceler Transfer Function ........................ 57
30. Cascaded Canceler Transfer Function .................................. 58
31. Doppler Ambiguity in MTI ............................................... 59
32. Composite Transfer Function for Two PRFs ........................... 60
33. Composite Transfer Function for Four PRFs ........................... 61
34. In Phase and Quadrature Phase Detection ............................. 62
INTRODUCTION

This research report is about radar. It covers enough essential radar principles to provide the reader with an overview of the field. Furthermore, it reviews the applications of Digital Signal Processing (DSP) techniques to radar signal processing technology. All of the material presented here is written with the goals of being easily readable and readily understandable. It is not like a typical textbook, since it attempts to explain material on both a rigorous and a heuristic level. Examples and diagrams assist in giving the reader a physical feel for the subject. Conversely, this paper is not superficial. The first section attempts to distill the very complex technology of radar into its basic principles. It assumes the reader has a background in physics, signal and system analysis and communications. The second section overviews the features of the Moving Target Indicator (MTI). This radar has grown in popularity in recent years, primarily because of advances in the field of digital processing. It assumes the reader has a working knowledge of DSP. This section serves as an excellent example of how DSP can be exploited.
History of Radar

RADAR stands for Radio Detection And Ranging. Radar had its beginnings in 1903 when Hulsmeyer, a German scientist, experimented with the detection of radio waves reflected from ships. He applied for and received patents in several countries for an "...Apparatus Adapted to Indicate or Give Warning of the Presence of a Metallic Body, Such as a Ship or a Train..." (Hulsmeyer 1904). However, the state of technology at the time severely hampered the application of his ideas and the technique was dismissed as being no better than a visual observer.

In 1922, Taylor and Young of the Naval Research Laboratory (NRL) detected a wooden ship using a continuous wave radar. Unfortunately, their proposal for further research was disapproved. In 1930, Hyland of NRL was experimenting with a direction finding device which used a continuous wave beam. He noticed that aircraft flying through the beam increased his reception. This accident spurred further investigation into the applications of continuous wave signals to detect objects. These early radars were incapable of measuring the distances to targets, but merely indicated their presence. The research was classified three years later.

In April of 1936, the first pulse radar was successfully demonstrated at NRL. Its range was only 2.5 miles,
but the pulse principle had been proven to be feasible. By 1939 the Army had developed the SCR-270, a long-range radar for early warning. This radar detected the impending Japanese attack on Pearl Harbor, but the significance of the blips was not fully realized until it was too late (Skolnik 1980).

During and after WWII large radar systems were developed to guard against aerial bombing. These radars covered large areas and required an all-out effort to accelerate and integrate the available technology. The necessary devices for the new radars were created. Radar applications entered a new era when the ICBM was developed. Now the task was not so much to detect and track large aircraft, but to detect small, fast moving targets at long range in a minimum amount of time. These drivers caused radar technology to expand forcefully, and into diverse areas (Toomay 1982).

Today, for the first time, radar research may be decelerating. The SALT treaty may curtail the development of the next generation of military radars, those used for Anti-Ballistic Missile (ABM) Defense. Furthermore, airports are leaning more towards beacon systems, curtailing research in the civil area (Rabiner and Gold 1975).
Common Radar Applications

While the military establishment is certainly the largest user of radar and the possessors of the most advanced systems, there are many civilian applications.

Air Traffic Control

This is one of the most important of the civilian applications of radar. By use of high-resolution radars, air traffic controllers can monitor up to several hundred aircraft simultaneously. Typically, the antenna rotates mechanically every 4 to 12 seconds, and the aircraft are illuminated several times by a pulse of energy as the beam sweeps past. Additionally, radar is invaluable in assisting aircraft to safe landings in poor weather.

Weather Forecasting

Satellites and ground-based radars are integral parts of modern meteorology. Rain and clouds reflect energy in a pattern that can be interpreted and used to map the prevailing cloud and storm conditions.

Ship Applications

Many ships employ radar to enhance safety by warning of other ships or obstacles. Radar is a primary tool of navigating during adverse weather. By comparing the radar
returns with navigational charts an experienced navigator can guide a ship safely despite poor visibility.

Law Enforcement

The use of radar to detect the speed of vehicles has proven to be an effective deterrent to speeding on public roadways.

Radar Environments

The central problem in radar is the detection of signals returning from the target in the presence of system noise and background clutter. System noise, the noise introduced into the signal in the process of receiving and processing the reflected pulses can be reduced through appropriate engineering design and high quality components. Clutter, the extraneous signals produced by uninteresting objects, can be alleviated by proper design and through signal processing technologies. Any design which fails to take into account these two effects will undoubtedly perform well below its potential or not at all. For example, Air Traffic Control (ATC) Radars which are not tilted slightly upward will illuminate and produce large returns from uninteresting ground objects. The size and proximity of this clutter could overwhelm the receiver and mask any aircraft in the sector. Rain and flocks of birds, however, cause unwanted clutter even in properly pointed ATC radars.
Furthermore, their speed makes Doppler discrimination difficult.
RADAR FUNDAMENTALS

This section overviews the basic characteristics of radar systems including system architecture, wave properties, and applicable radar parameters.

Radar System Architecture

The basic components of a radar system are shown in Figure 1. The main computer controls all of the radar system functions: the signal generator, the antenna, the receiver, the signal processor, and the display. The main computer commands the signal generator to create the signal which will return the most possible information. It

![Radar System Components](image)

**Figure 1:** Radar System Components
controls the antenna, scheduling antenna positions as necessary. It controls the filters in the signal processor, varying its strategies based upon the expected return signal. The computer also processes the extracted information and formats it prior to sending it to the display (Toomay 1980).

Wave Properties

Many radars make use of the phenomenon of the Doppler effect. If the target and the radar are in motion with respect to each other, the reflected frequency will not be the same as the transmitted frequency. The received frequency will be higher if the distance is decreasing and lower if the distance is increasing. This shift can be detected and used to estimate the radial speed of the target. The Doppler shift is given by the equation

$$f = \frac{2v}{\lambda} = \frac{2v}{c} f_0$$  \hspace{1cm} (1)

where $v$ is the target's radial speed and $f_0$ is the frequency of the transmitted wave.

Radar Parameters

One of the simplest radar waveforms is a string of narrow rectangular pulses that are multiplied by a sine wave carrier (Figure 2). This is known as a rectangular pulse. The time between the initiation of each pulse, $T$, is
called the interpulse period, and the number of pulses sent per time interval is the pulse repetition frequency, PRF. T and PRF are reciprocal. The duration of the pulse is $T$. The maximum amplitude of the signal is denoted by $A$, and is limited by the radar's peak power output.

The range, $R$, to the target, can be found by measuring the time delay of the returning pulse. The time that the wave takes to propagate to the target and back, $\Delta t$, is related to $R$ by the equation

$$R = \frac{(c \Delta t)}{2} \quad (2)$$

The factor of two appears in the denominator due to the two-way propagation of the signal.

The interpulse period, $T$, determines the maximum unambiguous range, $R_{\text{max}}$. $R_{\text{max}}$ is defined as the largest range of the target so that the echo returning from it will be received prior to the next pulse transmission. If the target is beyond $R_{\text{max}}$, then the returning echo will be

![Figure 2: Rectangular Pulse](image)
received after the next pulse has been transmitted. Signals returned from a target outside $R_{\text{max}}$ are called "second time around echos," since the return is after the second pulse.

The maximum unambiguous range is given by the equation

$$R_{\text{max}} = \frac{c \cdot T}{2}$$

The "dead zone" is that range interval where targets cannot be detected since the receiver has been disabled. Since most radars operate with the same antenna for receiving as for transmitting, the receiver must be disabled while the transmitter is on. Violation of this constraint would damage the receiver's sensitive detectors. If a target is inside the range $(cT)/2$, then the pulse is received while the transmitter is on and the receiver off. Hence, the radar will only detect part of the signal. Additionally, there is a dead zone around each integer multiple of $R_{\text{max}}$.

The range resolution, $\Delta R$, is a measure of how closely spaced two targets can be and still be resolved as independent targets instead of one extended target. Given a sinusoidal pulse of constant frequency (as shown in Figure 1), if the targets are closer than $(cT)/2$ the two signal returns will interfere and may be interpreted as one return from an extended target. Narrowing the pulse width improves the range resolution, but also decreases the total amount of
energy radiated. Therefore, close targets may be resolved better, but targets at large ranges may be lost due to the lower energy. The technique of pulse compression, discussed later, is a method of achieving good range resolution while maintaining signal energy.

Radar Cross Section (RCS) is a measure of the amount of signal energy which is intercepted by the target and is reradiated at the same wavelength. A target with a large RCS will be more easily detected. The RCS of an object does not usually bear a simple relationship to the physically illuminated area, except that larger targets generally have a larger RCS. Surprisingly, the RCS of a target may be larger than its actual physical area. The RCS depends upon the shape, size, and material composition of the target, as well as the wavelength of the signal. Generally, a target will have a large RCS if it is large in size, concave in shape, made of a highly conductive material or is approximately the size of an integer number of wavelengths. Targets that are large in size will have a larger physical area and tend to intercept and reflect more energy. Targets that are concave instead of convex will tend to reradiate energy back towards the radar. Convex targets tend to reradiate energy isotropically, that is, they tend to scatter the energy in all directions. Targets that are good conductors
will absorb and reradiate more of the incident radiation than poor conductors.

The ratio between the wavelength of the signal and the size of the target is an important parameter. For spherical targets with a circumference substantially smaller than a wavelength, the RCS drops off according to the Rayleigh curve. When the circumference of the target is approximately one wavelength, the currents induced on the target's surface constructively interfere with the reflected wave and the RCS increases. As the circumference of the sphere increases to a bit more than 3/2 of a wavelength, the radiation from the induced currents interferes destructively and tends to cancel the reflected radiation, thus decreasing

![Figure 3: RCS of a Sphere](image-url)
the RCS. As the sphere’s size continues to increase the RCS tends to peak at circumferences somewhat larger than an exact multiple of the wavelength and dip in between. The magnitude of the variations decrease as the object grows in size compared to the wavelength. This variation is depicted in Figure 3. Although this curve is valid only for a sphere, other objects exhibit the same general characteristics.

The radar range equation is one of the most important of the system design equations. It can be derived from fundamental principles (Figure 4). Assume that there is a radar operating with peak power $P$ and pulse duration $t$. We further consider that an antenna focuses the energy into some solid angle $\theta$. Since there are $4\pi$ steradians in a sphere, the gain, $G_t$, that this transmitting antenna creates over an isotropic radiator is given by $G_t \theta/(4\pi)$. The energy density received at range $R$ is given by $rP0G_t/(4\pi R^2)$.

Assuming that the target has an RCS of $\sigma$, then the energy reflected from the target back towards the radar is $\tau P0G_t \sigma/(4\pi R^2)$. The returning signal energy will be decreased by a factor of $A/(4\pi R^2)$ where $A$ is the effective area of the receiving antenna. Hence, the amount of energy which is received at the antenna $(S)$ is

$$S = \frac{\tau P0G_t A}{(4\pi)^2 R^4}$$

(4)
Unfortunately, there is always some noise contaminating the signal that arrives at the receiver. There are several different sources of noise, many of which are dependent on factors exterior to the system. A convenient measure of noise, that the antenna inputs to the signal prior to detection, is given by the equation

\[ N = k T_0 B \overline{NF}_0 \] (5)

where \( N \) is the received noise, \( k \) is Boltzmann’s constant, \( T_0 \) is 290° K, \( B \) is the equivalent noise bandwidth of the receiver in Hz and \( \overline{NF}_0 \) is the Noise Figure for the antenna at \( T_0 \).

Figure 4: Radar Range Equation Geometry
ANTENNAS

An antenna is the mechanism by which the radar signal is transmitted and received. An antenna has three roles: to be a contributor to the radar's sensitivity, to provide the required surveillance, and to allow precise angle measurements to the target.

Parabolic Antennas

A parabolic antenna is frequently used in transmitting and receiving radar signals. The parabola has the characteristic that signals emanating at its focus will reflect off its surface and generate a plane wave parallel to the

![Parabolic Antenna Diagram]

Figure 5: Parabolic Antenna
parabolas axis (Figure 5). Conversely, plane waves arriving at the parabola will be concentrated at the focus if they are entering the parabola parallel to its axis. Two conditions are sufficient to ensure that these requirements are met: The waves must reflect off of the reflector at the incident angle, and the distance that each wave travels from the focus to the parabola’s exterior is the same. Snell’s law assures us of the first condition, and the characteristics of a parabola assure us of the second. If the parabola is generalized to three dimensions its shape is called a paraboloid.

**Plane Antennas**

The plane wave created by the parabola could have been generated by a series of infinitesimal sources on a line, all oscillating in-phase. The radiation pattern in the "far-field," a position sufficiently distant from the antenna, is

\[
E(l) \propto \sin \theta \frac{\sin \theta}{\theta}
\]

*Figure 6: Flat Aperture Antenna*
important. To find the far-field antenna pattern we find the contribution of each infinitesimal source along the continuum and sum their effects in the far-field. Figure 6 illustrates this process. The effect on an arbitrary point \( p \) in the far-field from a source at the center point of the antenna is

\[
v = \frac{E \sin(\omega t)}{k} \, dl
\]

where \( E \) is the amplitude of the radiating sinusoid, \( dl \) is the length along the aperture of the infinitesimal source, \( k \) is a constant which is due to losses as the wave travels to \( p \) and \( \omega \) is the frequency of the sinusoid. The total phase angle of the sinusoid arriving at \( p \) is dependent on the distance from \( d \) to \( p \) and the wavelength of the signal. The phase angle is calculated by finding the number of wavelengths between \( d \) and \( p \) and multiplying by \( 2\pi \). The distance from \( d \) to \( p \) is given by \( ct \) where \( t \) is the time for propagation. Hence, the total phase angle is \( 2\pi ct/\lambda \).

The energy arriving at \( p \) from other points along the antenna have a phase difference with respect to the center point. Call this angle \( \beta_n \), where \( n \) indicates the angle for the \( n \)th element. \( \beta_n \) depends on how far from the center of the antenna the radiating increment is. Referring to Figure 6, it is easy to see that the energy from the increment of concern is related to the additional distance that the
energy must travel. This can be converted into the correct phase angle in radian form by dividing by the wavelength and multiplying by $2\pi$. Therefore

$$\beta_n = \frac{2\pi}{\lambda} l_n \sin \theta$$

(7)

where $l_n$ is the distance from the center of the antenna.

The expression for the contribution of each source is then

$$V_p(n) = \oint_{l_n} E_k \sin [2\pi ft + \frac{2\pi}{\lambda} (l_n \sin \theta)] \, dl$$

(8)

The total signal at point $p$ is the sum of all these sources.

$$V_p = \int_{-D/2}^{D/2} \oint_{l_n} E_k \sin [2\pi ft + \frac{2\pi}{\lambda} (l_n \sin \theta)] \, dl_n$$

(9)

Figure 7: Far-Field Antenna Pattern
Integrating equation 9 over the range -D/2 to D/2 results in

\[ V_p = \frac{E_D}{k} \frac{\sin \left( \frac{\pi D}{\lambda} \sin \theta \right)}{\pi D \sin \theta} \sin (2\pi ft) \]  

This is of the form \((\sin x)/x\) and is the Fourier Transform of a uniform illumination of the antenna. The graph of the far-field antenna pattern is shown in Figure 7. The case of the uniform illumination function is not a special one. In general, the far-field antenna pattern is the Fourier Transform of the illumination function. That is,

\[ G(\phi) = \frac{1}{2\pi} \int_{-D/2}^{D/2} f(l) e^{-j\phi l} dl \]  

\[ f(l) = \int_{-\infty}^{+\infty} G(\phi) e^{j\phi l} d\phi \]

where \(G(\phi)\) is the far-field antenna pattern and \(f(l)\) is the antenna illumination function.

One very important consideration in antenna design and illumination function design is that of sidelobe levels. In the previous example the main beam of the antenna contains about 90 percent of the antenna power. This is found by squaring the far-field antenna pattern and integrating the function from null to null and from \(-\infty\) to \(+\infty\). However, 10 percent of the power is still in the sidelobes beyond the
first null. If an object with a high RCS is illuminated by the sidelobe the resulting return could mask an object with a lower RCS than that which occurs in the mainlobe. Hence, the smaller object may go undetected. The usual method of combating this problem involves changing the illumination function of the antenna to reduce sidelobe level. This always results in the widening of the main lobe. This in turn results in a higher probability of detecting the small target close to the large target, but the radar also loses some of its angular precision since the beam is wider. The antenna patterns for both uniform and cosine illumination functions are illustrated in Figure 8. Note that the first sidelobe of the cosine illumination is 23 decibels (dB) down from the main lobe, compared to 13.4 dB down for the

![Figure 8: Comparison of Far-Field Antenna Patterns](image-url)
uniform illumination pattern. However, the mainbeam is almost 50% wider.

**Array Radars**

An important class of radars that does not use parabolic dishes for antennas is called array radars. These radars are made up of many point emitters. The advantage of these radars is that they are electronically steerable. That is, scanning is accomplished by electronic means without any required motion of the antenna. The resulting system is much more sophisticated than a mechanically rotated radar. Despite its cost, array radars are used when it is important to track many targets in different directions at the same time and when large angles must be searched. As a result, array radars are used in ICBM warning and tracking, Naval battlegroup defense, and in the world’s largest airports. In all of these applications it is important to track up to hundreds of targets at a time, and electronic beam steering is really the only practical method.

Three methods of steering array radar beams will be considered here: the time delay, the frequency change, and the phase shift. The time delay approach is illustrated in Figure 9. The leftmost element begins emitting first and then the other elements across the array join in sequentially so that a beam in the appropriate direction is generated. Elementary trigonometry will convince the
reader that the time delay between neighboring elements must be

\[ \Delta t = \frac{d}{c} \sin \theta \]  

A large number of time-delay networks are required because there is a one-to-one correspondence between the time delay and the beam direction \( \theta \). While true time-delay radars have been built, it is more economical to use either phase or frequency shifting.

Figure 10 illustrates the method of steering a beam electronically by means of phase shifting. The phase relationship between each of the sources is calculated so that the radiation patterns of the sources will add up to produce the main beam at the appropriate angle. The relative phase

![Figure 9: Time Delay Beam Steering](image-url)
shift between two neighboring elements for an arbitrary angle $\theta$, is given by

$$\beta = \frac{2 \pi d \sin \theta}{\lambda}$$  \hspace{1cm} (14)

Beam steering by frequency shifting is derived from the result in phase shifting with the exception that the beam is not steered by varying $\lambda$. In this case the variable which changes is the frequency of the sources. As the frequency changes, the wavelength changes and $\theta$ is shifted. One typical method of steering beams by frequency shifting is the Huggins Beam Steering Approach (Toomay 1982).

Specialized illumination functions to reduce sidelobes for array radars can be accomplished in two ways. One method requires the magnitude of the signal to be varied as a function of the position. Typically, the equipment necessary to process all of the appropriate changes in point emitter magnitudes every few milliseconds is uneconomical.

\[d \sin \theta \]

\[\theta\]

\[\lambda\]

\[\beta\]

\[\lambda\]

\[\theta\]

\[d\]

**Figure 10:** Phase Shift Beam Steering
Instead of this complex method, the array can be "thinned" by not illuminating some of the elements during transmission. Several of the emitters are turned off in regions where the magnitude of the illumination function is small. The thinning of the array will be done so that the illumination function approximates the desired weighting function. The elements which are disabled must be thinned randomly in the areas where the desired illumination function is small or high sidelobes could appear.
DETECTION OF SIGNALS IN NOISE

The first problem in designing any radar system is that of noise. Types of noise were discussed briefly in connection with the range equation and shall not be discussed here further. However, we shall consider the characteristics of signals in combination with noise. In particular, we shall examine how to correctly design the signal processor to yield the maximum signal-to-noise ratio. The resulting signal processor is called the "matched filter."

Types of Noise

Three types of noise are typically considered in radar systems: white noise, colored noise, and jamming. White noise is called "white" because it contains all frequency components, much as white light contains all of the visible frequencies. The magnitude of its spectral density is a constant $N_0$. All frequencies from $-\infty$ to $+\infty$ are represented, and all contribute equally to the signal. Hence, the signal is uncorrelated and its autocorrelation function is represented by an impulse at $t=0$.

Colored noise is noise in which some spectral components are more heavily weighted than others. Colored noise is usually dealt with by sending the signal through a
"prewhitening" filter that will attenuate the stronger spectral components of the colored noise and emphasize the weaker components. The resulting filter output will be white noise, along with whatever signal is present. This assumes that the spectral components of the noise are known ahead of time, which is often possible for fixed radars.

Jamming of signals occurs most frequently in military applications. A barrage jammer obscures reflections by radiating continual wide band noise to prevent the radar from fixing the position of the target. The radar will detect the noise, but will be unable to distinguish the target's range for some time period. A sophisticated jammer may have a system which mimics the radar signal causing one or more false target returns in his sector.

The Matched Filter

The matched filter is the filter which will maximize the Signal-to-Noise Ratio (SNR) at the output of the filter. The matched filter is dependent only on the received waveform and the spectral density of the noise.

Let the received signal, \( x_1(t) \), be made up of two portions, \( s_1(t) \) and \( n_1(t) \). \( s_1(t) \) is the transmitted signal except that it may be frequency shifted due to the target's motion. \( n_1(t) \) is an additive noise signal and is corrupting the returning target signal. The output of the linear filter is \( x_0(t) \) and is made up of two portions: \( s_o(t) \) which
is the portion of the signal that is from the targets echo and $n_0(t)$ which is the output of the filter due to the noise input. $s_0(t)$ and $n_0(t)$ are related to their input signals and the assumed filter impulse response, $h_{\text{opt}}(t)$, by equations 15 and 16.

$$s_0(t) = s_i(t) \ast h_{\text{opt}}(t) = \int_{-\infty}^{+\infty} s_i(\tau) h_{\text{opt}}(t - \tau) \, d\tau$$  \hspace{1cm} (15)$$

$$n_0(t) = n_i(t) \ast h_{\text{opt}}(t) = \int_{-\infty}^{+\infty} n_i(\tau) h_{\text{opt}}(t - \tau) \, d\tau$$  \hspace{1cm} (16)$$

The frequency domain representation of $n_0(t)$ is $N_0(\omega)$, $s_0(t)$ is $S_0(\omega)$, $n_i(t)$ is $N_i(\omega)$, $s_i(t)$ is $S_i(\omega)$, and the transfer function of $h_{\text{opt}}(t)$ is $H_{\text{opt}}(\omega)$. Since convolution in the time domain is equivalent to multiplication in the frequency domain, equations 17 and 18 apply.

$$S_0(\omega) = S_i(\omega) H_{\text{opt}}(\omega)$$  \hspace{1cm} (17)$$

$$N_0(\omega) = N_i(\omega) H_{\text{opt}}(\omega)$$  \hspace{1cm} (18)$$

The time domain signal can then be found by taking the inverse Fourier transform of the frequency domain representation shown in equations 19 and 20.

$$s_0(t) = \int_{-\infty}^{+\infty} S_0(\omega) e^{jwt} \, dw$$  \hspace{1cm} (19)$$
The output which we wish to maximize is

\[
\frac{\lvert s_o(t)_{\text{max}} \rvert^2}{N}
\]

where \( s_o(t)_{\text{max}} \) is the maximum value of the output signal voltage and \( N \) is the mean noise power at the receiver output. Assuming the input noise is white, and has a value \( N_o \) for its constant spectral density, the mean output noise power from the filter is

\[
N = k \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega
\]

Substituting appropriately, the value we wish to maximize is

\[
\frac{\int_{-\infty}^{+\infty} S_i(\omega) H_{\text{opt}}(\omega) e^{j\omega t} dw}{k \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega}
\]

By Schwartz's inequality, the complex integral in the numerator will be a maximum whenever \( H_{\text{opt}}(\omega) \) is proportional to the complex conjugate of \( S_i(\omega) \). Hence, \( H_{\text{opt}}(\omega) = a S_i^*(\omega) \). Mathematically, this will result in an entirely real positive argument prior to integration. Therefore, all of the
terms will add up with each other in the best possible fashion. Taking the complex conjugate of a signal in the frequency domain corresponds to reversing time in the time domain. Hence, \( h_{opt}(t) = s_i(-t) \). This function is physically unrealizable since the impulse response of the filter will have output prior to \( t = 0 \). As a result, a time delay must be added to the system which gives \( h_{opt}(t) = s(T_d - t) \). In the frequency domain this means adding the phase shift \( e^{(j\omega T_d)} \) to the transfer function \( H_{opt}(\omega) \). Hence, the transfer function of the physically realizable filter which maximizes SNR at a point in time is \( H_{opt}(\omega) = s_i^*(\omega) e^{(j\omega T_d)} \). Examples will be shown in the section on waveform design.

**Detection Schemes**

There are two types of errors that the radar system can make in the process of detecting a target: not detecting a target that exists or giving a false detection on a nonexistent target. By setting the threshold of the signal detector low we can detect small spikes from the matched filter output, so targets may be detected at longer ranges. Conversely, the lower the detection threshold is set, the more frequently random noise can exceed the threshold. These occurrences give false alarms.

If the noise input to the receiver has known characteristics, then we can calculate the probability that the detection threshold will be exceeded by noise. We will
consider the case of white noise, but other spectral densities may be considered. If detection is based on power detection, i.e., square law, then the probability density function will be exponential. If envelope detection is used, then the probability density function is Rayleigh distributed. These distributions are shown graphically in Figure 11.

There are several methods of maximizing the probability of successful detection and minimizing the probability of false detection. Three of these methods will be considered here: M out of N detection; coherent integration; and non-coherent, or envelope, integration.

In M out of N detection schemes, the radar design assumes that the target will be pulsed several times by the radar antenna while scanning. In so doing, an in range target should return several signals to the radar at

![Noise Probability Density Functions](image)

**Figure 11:** Noise Probability Density Functions

- a. Gaussian (Input Noise Voltage)
- b. Exponential (Input Noise Power)
- c. Rayleigh (Envelope Detection)
sufficient level to cause a detection to occur. The computer that controls the radar’s signal processing strategies will be set so that a target will be identified if at least M out of N pulses are returned above the detection threshold. In this way the probability of false detections can be reduced dramatically. For example, assume the probability of noise exceeding the detection threshold once is .05. By assuming a three out of five detection criteria, the probability of a false alarm is reduced from .05 to .0025. If it is unnecessary to have this low of false detection level, the detection threshold may be reduced and the radar’s range increased.

Another method of detecting targets is to use multiple pulses and sum the energy prior to thresholding. The energy received is stored in appropriate range and angle bins. Any signals arriving from a particular range and angle are added together. When the additions are complete a decision is made as to whether or not a target was detected based on the amount of energy in the bin. If the phase and amplitude of the signal are retained then the system is called "coherent integration." The voltage gain of coherent integration over a single pulse is n, where n is the number of pulses that are coherently integrated. If envelope detection is used, the phase of the signal is discarded and the gain is between n and $\sqrt{n}$ over a single pulse. The falloff from n to
\( \sqrt{n} \) for envelope detection is relatively slow as \( n \) increases. This falloff is associated with the time that the integration takes to occur and the possible motion of the target during that time (Toomay 1982).

Theoretically, there should be no difference in detection probability between sending many small pulses and sending one large pulse with the same total energy. However, the additional signal processing necessary to perform integration is usually less expensive to implement than to upgrade the components in the radar to handle the higher power.
In order to maximize the amount of information which can be received from the returning signal the waveform must be designed intelligently. Of prime importance is the ability to extract the signal from the noise, but the signal must also be designed to yield the information required. Many thousands of practical signal types can be developed, but we will only discuss three that illustrate the fundamentals of signal design: rectangular pulse, linear FM chirp and the pulse burst. Additionally, the ambiguity function will be defined and discussed.

**Rectangular Pulse**

The chief advantage of this pulse is its ease of implementation. However, its signal detection capabilities make it inferior to other signals. The important

![Figure 12: Rectangular Pulse Characteristics](image)

- a. Rectangular Pulse
- b. Spectral Content
- c. Matched Filter Output
characteristics of the rectangular pulse are shown in Figure 12. The pulse as seen in the time domain is shown in a), the frequency representation is shown in b), and the output of the matched filter is shown in c). Note that the envelope of the matched filter output is diamond-shaped.

In order to achieve maximum signal detectability we wish to make the maximum height of the matched filter output as large as possible. In the case of the rectangular pulse, much of the energy is distributed away from the maximum. This energy is, in a sense, wasted. A better output would be a sharp impulse containing most of the returned signal's energy. This would maximize signal detectability for a given returned energy and allow for easy determination of the time that the signal crossed the detection threshold.

Pulse Compression

The liabilities of the rectangular pulse prompted radar designers to consider other waveforms which might keep range resolution while maintaining signal energy. The solution was found in the general class of signal design techniques known as pulse compression. This method required the radar to send out a long duration signal (large energy), but with a short duration correlation function. This short duration correlation function meant that a sharp pulse would occur at the matched filter output. There are several techniques for implementing these functions. One method involves
weighting the envelope of the transmitted signal with a sine function instead of a rectangular function. If this signal is put through a matched filter then a larger peak amplitude will be obtained. A second method consists of varying the frequency of the carrier wave throughout the pulse. The best-known analog example of a frequency changing signal is the "FM chirp."

The frequency modulated (FM) chirp is known as a chirp because of the constantly changing frequency of the sinusoid. It would make a chirping sound if it could be heard. The chirp's frequency begins at some value and varies during the duration of the signal. For a linear FM chirp the frequency increases linearly throughout the duration of the signal. Hence, the frequency of the waveform varies over the signal duration by the equation \( \omega = (\omega_0 + \mu t) \). By integrating the frequency with respect to time we can find the total phase angle which is \( \omega_0 t + \mu t^2/2 + \psi \). The general equation for the linear FM chirp is given in Equation 24.

\[
\begin{align*}
s(t) &= A \sin(\omega_0 t + \mu t^2/2 + \psi) & 0 \leq t \leq T \\
s(t) &= 0 & \text{otherwise}
\end{align*}
\]

The phase angle \( \psi \) is constant and can be taken to be zero without loss of generality. The time and frequency domain representation of the signal can be found in Figure 13.
Since it is impossible to integrate Equation 24 analytically, the sketch is done by numerical methods. The initial angular frequency of the sinusoid is $6.28 \pi$ radians per second, the rate of increase of the frequency is $6.28 \pi$ radians per square second, and the time duration of the signal is $1 \times 10^{-5}$ seconds. The time interval was subdivided 40,000 times for each of the 40 points plotted. Note the relatively constant magnitude for the frequencies in the range $[\omega_0, \omega_0 + \pi \tau]$. These are the frequencies the pulse takes on.

The matched filter transfer function can be derived from this representation and is plotted in Figure 14. It is found by keeping the magnitude the same, changing the sign of the phase, and adding a phase constant. Again this phase constant is necessary to make the filter implementation physically realizable. It is a frequent mistake of textbooks to neglect the phase shift entirely. This, of course, would result in a system that is not causal. Additionally, in practical implementations, the filter's phase must be zero for DC.

A conceptual sketch of the matched filters impulse response is shown in Figure 15. The time delay, $T_d$, must be longer than the signal duration. The output of the matched filter, when excited by its matched signal, is shown conceptually in Figure 16. The scales are expanded so that
the reader may see the oscillations which would normally be too fast to see. Figure 17 illustrates the signal shape as it passes through the matched filter. Note how the pulse is compressed due to the phase characteristics of the matched

Figure 13: Linear FM (LFM) Chirp  

a. Time Domain  
b. Frequency Domain

Figure 14: LFM Matched Filter Transfer Function
Figure 15: LFM Matched Filter Impulse Response

Figure 16: LFM Matched Filter Output

Figure 17: LFM Progression Through a Matched Filter
(Cook and Bernfield 1967)
filter so that almost all of the signal's energy is released at once to cause a high spike. This spike is an easily detected peak. The response of the matched filter to input noise and signal is seen in Figure 18. The reaction of two signals arriving simultaneously is shown in Figure 19. Much better range resolution can be obtained using this linear FM chirp than could be obtained using the rectangular pulse.

**Figure 18:** LFM Matched Filter Response to Noise and Signal

**Figure 19:** LFM Matched Filter Response to Simultaneous Echos
**Discrete Coded Waveforms**

Discrete Coded Waveforms are a group of signals which are derived from discrete codes. This group is different from analog codes which employ continuous amplitude, frequency, or phase modulation. In general, these discrete waveforms are ordered sequences which are impressed, at specific times, on the amplitude, frequency, or phase of some continuous carrier (Cook and Bernfield 1967).

The simplest discrete waveform is the pulse burst. A burst consists of a sequence of pulses spaced in time. The time representation of a uniformly spaced burst and the matched filter output are shown in Figure 20. Much of the burst’s energy is dispersed away from the largest pulse where detection is most likely to occur. This is similar

![Figure 20: Pulse Burst](image_url)

- a. Signal
- b. Matched Filter Output
to the problem that exists in the matched filter output of
the rectangular pulse. One method of decreasing these
sidelobes is to randomly space the pulses as shown in Figure
21a. The matched filter output of the random pulse burst is
shown in Figure 21b.

Polyphase codes, Barker codes, and Huffman codes are
all examples of discrete coded waveforms which are designed
to reduce the ambiguity associated with frequency shifts.
Cook and Bernfield (1967) give an excellent treatment of the
subject for the interested reader.

The Ambiguity Function

The ambiguity function is defined as the function which
describes the response of the matched filter to a frequency
shifted input signal. This function is useful in determin-
ing the response of the matched filter when the input sig-
nals represent returns from moving targets. The ambiguity

Figure 21: Random Pulse Burst (Cook and Bernfield 1967)
a. Signal
b. Matched Filter Output
function is a criterion that measures the usefulness of a particular signal for determining simultaneous, one pulse measurement of speed and range for moving targets.

Physically, this is illustrated easily using the characteristics of the linear FM chirp. The matched filter for the chirp achieves its pulse output by delaying the different frequency components of the chirp by different amounts. The filter is designed so that only small amounts of energy are allowed to emerge until time $T_d$ when the waves constructively interfere and a signal approximating an impulse emerges. The filter imparts a specific time delay for each signal component. If the frequency of the received signal is different from the expected matched signal, then the time delay of the pulse will be incorrect and will appear either early or late. Hence, the target's measured range is different from its actual range. The phenomenon where speed affects the range measurement and range affects the speed measurement is known as speed and range coupling. If the speed changes then the perceived range changes.

Analytically, the ambiguity function can be calculated by finding the squared magnitude of the function $g(\tau, \omega_d)$. This function is found by frequency shifting the expected returned signal. Using the frequency domain representation of the signal, $u(\omega + \omega_d)$, and the matched filter, $u^*(\omega)$, $g(t, \omega_d)$ is shown in Equation 25.
In the time domain, \( g(t, \omega_d) \) is calculated by convolving the impulse response with the input signal.

\[
g(t, \omega_d) = \int_{-\infty}^{+\infty} u(\tau) u(\tau - t) e^{j\omega_d \tau} d\tau
\]

The ambiguity function \( g(t, \omega_d) \) describes the response of the matched filter to an input signal which is frequency shifted by \( \omega_d \) from the matched signal. Examples of ambiguity functions for the rectangular pulse, the linear FM chirp and the uniformly spaced burst are shown in Figure 22.

- a. Rectangular Pulse
- b. LFM Chirp
- c. Uniform Pulse Burst

**Figure 22:** Ambiguity Functions
The real significance of the ambiguity function is this: The cross sections of the ambiguity function found by holding $\omega_d$ constant are the squared magnitudes of the matched filter output when the signal received is frequency shifted by $\omega_d$. That is, if a signal is transmitted, say $\cos(\omega_0 t)$, the ambiguity function will show us the response of the matched filter to inputs of the form $\cos[(\omega_0 + \omega_d)t]$. By taking the cross section of the ambiguity function along the line $\omega = \omega_d$ we find the squared output of the ambiguity function. The radar will determine the target’s range from the maximum magnitude of the cross section.

A great deal of insight can be extracted by observing the ambiguity function. If the ambiguity function is sharply peaked about the origin, then the signal has good simultaneous speed and range-measuring capability. It will only be excited by a small range of Doppler frequency shifted signals, and will output easily detected pulses. Other frequency shifted signals will not excite the filter. On the other hand, if the ambiguity function is spread broadly over the plane, then the matched filter responds to many Doppler shifted signals. The output of the filter is not a sharp pulse and detection will be difficult. Finally, a knife-edge ambiguity function along the t=0 axis would always measure range correctly, but would be useless for speed measurement.
Both range and speed can be measured accurately on the basis of one pulse if the ambiguity function approximates a thumbtack at the origin. This filter will respond only to the signal to which it is matched and will ignore all other signals. Hence, excellent speed and range measurement can be made on the basis of one pulse. However, these filters must still cover the entire range of Doppler shifts expected. Since each of the filters only respond to a narrow frequency band, many filters must be utilized.

Figure 23 gives examples of the outputs from a linear FM matched filter when the expected return signal is frequency shifted from the actual return signal. Note that the pulse is not as sharp and is shifted in time. This shift is the range and Doppler coupling appearing again, and will cause the target’s range to be measured erroneously.

The simplest signal which is used to detect signal speed and range accurately is the V-FM chirp, so called because the frequency of the chirp decreases linearly and then increases linearly, creating a V. The effect of a Doppler shift is to cause the output pulse to split into two smaller pulses, one that is shifted forward in time and one that is shifted backwards in time (Figure 24). The space between the pulses can be used to measure the frequency shift of the received signal and the pulse positions may be averaged to accurately detect the range to the target.
However, if there is more than one return, then the processing required to discriminate which pulses belong to each other will increase system complexity.

Figure 23: LFM Matched Filter Response to Doppler Shift

a. No Doppler Shift  
b. Doppler Shift  
c. Opposite Doppler Shift

Figure 24: Response of V-FM Chirp to Doppler Shift

a. No Doppler Shift  
b. Doppler Shift  
c. Opposite Doppler Shift
APPLICATIONS

In this section three applications of radar will be discussed: the radar altimeter, the over-the-horizon radar, and the synthetic aperture radar.

Radar Altimeters

The radar altimeter is a device used by a flying vehicle to sense its altitude by use of radar. The radar sends a signal down to the ground and measures the time for return. Since the earth is a big, nearby target the echoes are easily detected. Hence, the transmitter can be of low power, the receiver relatively noisy and an accurate estimate of the altitude will still be obtained. More of a problem for the engineer is to make the device economical, small, lightweight, dependable and able to fit in an aerodynamically designed carrier.

The signal of choice for the designer is typically a V-FM chirp. This signal can detect the range to the earth while the aircraft is climbing or diving. Modern radar altimeters are available that use less than 10 watts of power and have modest gains and noise figures (Toomay 1982). Special cases of particularly long distances to the ground (space vehicles) or particularly high accuracy with "look
ahead" capability (cruise missiles) require more complex signals, antennas and processing.

**Over-the-Horizon Radars**

The over-the-horizon radar uses the electromagnetic skip capability of the ionosphere. A radar wave is directed up to the ionosphere and is allowed to reflect off of it and illuminate targets over the horizon (Figure 25). The signal reflected off of the target is reflected back against the ionosphere and is received by the radar. Only those frequencies in the high-frequency range of the frequency spectrum are suitable for skipping.

Amateur radio operators knew early on that it was possible to send and receive signals at huge distances.

*Figure 25: Over-the-Horizon Radar*
because of the skip capability of the ionosphere. However, the idea of transmitting radar signals via the ionosphere did not seem credible until the 1960s, when coherent signal processing became practical. The last two decades have seen slow, but steady, progress as DSP has permitted arbitrarily long integration times. Typically these systems are used in military applications when it is important for the defender to recognize the presence of an attacker while he is still several hundred miles away.

The signal duration for an over-the-horizon radar is usually much longer than other radars. This is attributed to the tremendous losses associated with radiating over such large distances while reflecting the signal off of the ionosphere twice.

The processing associated with over-the-horizon radar is usually limited to Doppler processing. Essentially, this radar is a down-looking radar and the earth scatters back much more radar energy than the target. Hence, targets are detected based on the Doppler shift caused by their motion. Often, the long pulse periods (as much as 100% of the interpulse period) require that the receiver be located remotely from the transmitter.

**Synthetic Aperture Radars**

Possibly the most complex of the newest radars are the so-called Synthetic Aperture Radars (SARs). By virtue of
their own motion they can make very detailed maps of the target. These radars achieve range resolution by matched filtering similar to other radars. Extremely high angular resolution is accomplished by moving the radar with respect to the target so that the effective aperture of the radar is large. In the frequency representation the antenna’s motion effectively gives a different Doppler shift to each angle that the radar beam is scanning. The signal processor recognizes the returned Doppler shift and places the target in the appropriate bin. Appropriate processing allows for high-angle resolution.

SARs have several interesting characteristics. The best angle resolution in a SAR is obtained from the smallest antenna traveling over the longest distance. The maximum resolution is independent of distance and is equal to half the physical aperture of the radar. Secondly, motion of targets on the ground cause additional Doppler shifts, misleading the signal processor into placing the object into the wrong angle bin. Hence, cars will be displaced off of highways and trains off of tracks in the resulting picture. These peculiarities can usually be corrected by a radar interpreter. Displacement of vehicles which normally operate off of the road, e.g., a tank, can be a significant limitation. Finally, the signal processing requirements for a SAR are tremendous. Returns must be processed for
every range, angle, and frequency cell possible for a rapidly scanning beam.
MOVING TARGET INDICATOR

The MTI, or moving target indicator, is defined as a device which limits the display of radar information primarily to moving targets (IRE Dictionary of Electronic Terms and Symbols 1961). As such, it uses the Doppler frequency shift caused by moving targets to determine the relative speed of a target or to separate desired moving targets from undesired stationary objects (clutter) (Barton 1964). It is possible to separate, through appropriate filtering, the moving target returns from the clutter when the clutter returns are several orders of magnitude larger. Applications of this technology are diverse and could include locating trucks moving on roads in forests, airplanes flying in rain, or alligators swimming in a swamp.

MTIs typically operate with a low pulse repetition frequency. This yields a large unambiguous range, but introduces severe ambiguity in speed measurement. A sister radar to the MTI is the Pulse Doppler which operates on the same basic principles. However, the Pulse Doppler operates with a high PRF, yielding large unambiguous speeds, but introducing severe ambiguity in range measurement.
The original concept of MTI was introduced back in the mid 1950s. However, the materials and the signal processing technology required to make the system workable were not developed for about 20 years. An early concept of MTI was the so-called "area" MTI. The radar returns from one antenna revolution were stored in memory and then subtracted from the returns on the next revolution. The design predicted that the returns from stationary targets would be canceled leaving only signals from targets which had moved between scans. Hence, the system suppressed the numerous returns from stationary clutter and passed information about moving objects to the operator. This method is still used today, but is severely limited in its performance. The chief technical problem in this system is that cancellation is never complete due to radar instabilities, weather changes, or motion of the clutter.

Another early form of MTI operated with cancellation based on a pulse-to-pulse basis. One major technical problem which inhibited the implementation of such a system was the lack of an adequate delay tool. The returning signal needed to be delayed for a millisecond, or so, and then subtracted from the next returning signal. One proposed solution required converting the returned signals to audio waves, and then transmitting them along a fluid-filled
pipe prior to converting them back to electric signals. Such techniques were tried, but the delay medium contributed large amounts of noise, often degrading the signal as much as 50 decibels (Skolnik 1980). Advances in digital technology allowed for the easy storage and delay of these signals.

Theoretically, the analog processor could function as well as the digital processor. However, the availability, cheapness, and ease with which the filters can be designed and adjusted all point towards utilizing the digital signal processor.

There are two forms of modern MTIs, coherent and clutter coherent. The coherent MTI has an internal oscillator that is very stable and provides the steady phase reference necessary for proper demodulation. The clutter coherent MTI depends on returns from clutter in the area of the target to provide the coherent signal necessary for demodulation. If no clutter appears in the same range as the target, then the target will not be detected.

**System Architecture**

A typical block diagram of a coherent MTI is shown in Figure 26. Two very stable oscillators are used in generating and demodulating the transmitted and received pulse. The stable local oscillator, stalo, and coherent oscillator, coho, are multiplied together and filtered to generate the sinusoid of frequency $f_1 + f_c$. This sinusoid is then
pulse-modulated, amplified and sent to the transmitter. On receiving, the signal is multiplied by the stalo and the high frequency term discarded. This resulting signal will be amplified, multiplied by the coho and lowpass filtered. The resulting succession of pulses is fed into the signal processing unit.

![Coherent MTI Block Diagram](image)

**Figure 26: Coherent MTI Block Diagram**

**MTI Return Signal Analysis**

If we imagine that a continuous wave signal is transmitted it will have the form

\[
V_1 = A_1 \cos(\omega_0 t + \xi) \quad 0 \leq t \leq T
\]  

(27)

The returned signal will be of the form
\[ V_2 = A_2 \cos[(\omega_0 + \omega_d)t - \frac{2\omega_o R_o}{c} + \xi] \]  

(28)  

The reference signal and the target echo signal are heterodyned in the mixer stage of the receiver and the output is

\[ V_3 = A_3 \cos [\omega_d t - \frac{2\omega_o R_o}{c}] \]  

(29)  

This sinusoid is oscillating at the Doppler frequency. Since the MTI is only transmitting a sampling of the continuous wave, the received signal will only be a sampling of the Doppler frequency. If the Doppler frequency is small, compared to the reciprocal of the pulse duration, then the returns will look like a succession of pulses of varying heights. This situation is illustrated in Figure 27.

---

**Figure 27: MTI Signal After Demodulation**

**Airborne MTI**

If the MTI is Airborne then it is called an Airborne Moving Target Indicator (AMTI). Clutter presents a special
problem for this situation since all stationary objects have a motion relative to the radar. For this case the clutter rejection notch must be made adjustable in an adaptive manner (Short 1982), or the system must operate in a clutter coherent fashion.

**Delay Line Cancelers**

The MTI delay line canceler is shown in Figure 28 in its network representation. Essentially this filter will pass all frequencies except 0. The magnitude of the filter transfer function is shown in Figure 29. The transfer function has zero gain at DC, therefore canceling all target returns from stationary targets.

![MTI Delay Line Canceler Network Representation](image)

**Figure 28:** MTI Delay Line Canceler Network Representation

![Single Delay Line Canceler Transfer Function](image)

**Figure 29:** Single Delay Line Canceler Transfer Function
The band reject notch around zero is often too narrow for practical application. Foliage or wave motion due to wind, antenna motion due to mechanical scanning, or weather instabilities are effects which will cause the return from uninteresting objects to change and be detected. Hence, the clutter rejection notch around zero is usually made larger by cascading two or more delay line cancelers in sequence with each other. This is equivalent to squaring the magnitude of the single canceler's transfer function. Figure 30 shows the transfer characteristics when two or three delay line cancelers are cascaded. A significant side effect of this technique is the narrowing of the passband. Interesting, but slow moving, targets may fail to be detected.

Emerson (1954) researched methods of weighting the transfer function so that the maximum signal to clutter improvement was reached. This type of filter is called an

Figure 30: Cascaded Canceler Transfer Functions
"optimal" filter. For low order filters the optimum weights are close to the binomial weights. In fact, for the two pulse canceler the binomial weights given above are the same as the optimum weights assuming Gaussian noise. For a three pulse canceler, the largest difference between the optimal filter and the binomial filter is less than 2 dB (Skolnik 1980).

One of the major problems in MTI is the ambiguity of the target's speed. Due to the nature of MTI, the largest Doppler frequency which can be accurately measured is the PRF. All Doppler frequencies larger than the PRF will have integer multiples of the PRF subtracted, so that the measured frequency will be smaller than the PRF. If the PRF is 1000 Hz, then targets moving with speeds corresponding to frequency shifts of 500 Hz, 1500 Hz, or 2500 Hz will appear

Figure 31: Doppler Ambiguity in MTI
to have the same speed. This situation is illustrated in Figure 31 where the samples occur at 1000 Hz, and sinusoids corresponding to 500 and 2500 Hz are sketched. If the Doppler shift is an integer multiple of the PRF, then the phase of the pulse from the target will be the same at every return. Hence, the MTI will cancel the returns and the target will not be detected. These speeds, corresponding to those Doppler frequencies, are called "blind speeds" because the radar cannot differentiate these from clutter. These blind speeds are clearly seen in Figures 29 and 30.

One method of coping with both speed ambiguities and blind speeds is to transmit at alternating PRFs. By observing the change in a target's measured speed when the PRF is switched, the target's unambiguous speed can be determined unambiguously over a wide range. Furthermore, targets which are at a blind speed of one PRF may not be at a blind speed of the other PRF. Figure 32 shows how the transfer

![Composite Transfer Function for Two PRFs](image)

Figure 32: Composite Transfer Function for Two PRFs
characteristics of two PRFs can be combined to yield a much larger unambiguous range window and a larger first blind speed. Figure 33 shows the transfer characteristic of a four period stagger.

One final problem that occurs with regularly spaced pulse intervals is that of "blind phases." Blind phases occur when the signal returning from a moving target has a phase shift such that the signal is zero when it is sampled. This problem can be rectified by the network shown in Figure 34. When the signal is demodulated it is demodulated by an in-phase and a quadrature channel. The signals from both channels are squared and summed prior to detection.

To derive the maximum amount of information from the received signal, narrow band filtering is executed after the signal passes out of the canceler. These filters may be

$$\text{center frequency}$$

![Figure 33: Composite Transfer Function for Four PRFs](image)

$$T_1:T_2:T_3:T_4 = 25:30:27:31 \quad \text{(Skolnik 1980)}$$
Finite Impulse Response (FIR) or Infinite Impulse Response (IIR) filters. Alternatively, the signals spectral components may be determined by a Discrete Fourier Transform (DFT).

**MTI Doppler Filters**

A range-gated Doppler filter uses range gates which only allow a signal to pass at certain times after signal transmission. The number of range gates determines the range resolution of the signal. After the signal passes through a range gate it is fed into a set of narrowband bandpass filters. The number of filters employed will

![Diagram of MTI Doppler Filter](image)

**Figure 34:** In Phase and Quadrature Phase Detection
determine the speed resolution possible on the target. If a
signal contains a frequency which corresponds to a filter,
then that filter will pass the signal to the full-wave
linear detector. This detector squares the magnitude, which
is then fed into an integrator. If the amount of energy in
the integrator achieves some threshold value, then a contact
is reported. By correlating the time the signal was re-
ceived and the filter which was excited, the range and speed
of the target may be found.

Recursive (FIR) or Nonrecursive (IIR) filters may be
used in implementing the narrowband bandpass filters.
Nonrecursive filters are filters that contain no feedback
loops. If a nonrecursive filter has n delay lines, then it
will be in steady state after n samples. This gives the
filter a superior transient response. However, to achieve
highly shaped filter characteristics it is necessary to use
many delay lines which inhibit the filter’s superior tran-
sient response.

Recursive filters have feedback as well as feedforward
loops. This allows the designer to use poles in the design
of the filters as well as zeros. These filters can exhibit
very specialized characteristics, and the literature is full
of material on the design of Chebyshev, Elliptic, Bessel, or
Butterworth filters. Additionally, these filters will have
a superior steady state response compared with the
nonrecursive filters. However, the transient response is poor. If a large signal is applied to the filter, then the transient response could swamp the steady state response, thus making it undetectable until the transients have subsided. In the typical surveillance scenario, however, the filters only have a few pulses to gather data on each target so the filters are usually in a transient state.

One final method of detecting target speed is to use the Discrete Fourier Transform (DFT). The DFT operates on a sequence of pulses and generates data on the spectral components present in the signal. The Fast Fourier Transform (FFT) allows for the efficient computation of the DFT. The relative desirability of the FFT over conventional narrow-band filters depends primarily on the time-bandwidth product of the signal. The larger the time-bandwidth product, the more favorable the FFT becomes (Rabiner and Gold 1975).

The Moving Target Detector

To illustrate some of the problems and solutions with a MTI, consider the Moving Target Detector (MTD) that the MIT Lincoln Lab designed. The MTD is a possible, low cost, update for in-place air traffic control (ATC) radars. The waveform had a one microsecond pulse width and a one millisecond interpulse spacing. The signal wavelength was 10.7 cm corresponding to S band. The interpulse spacing was required to yield the desired unambiguous range of at least
60 miles. The maximum target speed expected was 750 miles per hour which yielded a maximum Doppler shift of 6231 Hz. Since the pulse width is narrow, compared to the Doppler frequencies, the output of the phase detector looked like a series of pulses.

The signal processor consisted of both in-phase and quadrature channels, a three pulse canceler and an eight point FFT device. To eliminate blind speeds and resolve speed ambiguities, two PRFs were used: one of about 1000 Hz and one of about 1400 Hz. A ten pulse burst was transmitted at one PRF, then ten pulses were transmitted at the second PRF, prior to switching back to the first PRF. The first two pulses of each burst, sampled at each range bin, were used to fill the three pulse canceler. The last eight pulses out of the canceler underwent a FFT to determine their spectral components. Weighting was applied in the frequency domain to decrease the filter sidelobes.

A clutter map of the surrounding terrain was used to detect targets on crossing paths. This clutter map was built up recursively, adding one-eighth of the value of the zero speed filter in each of the range-angle bins to seven-eighths of the value stored in the memory. Hence, the clutter map changed as the clutter characteristics changed. As rain swept in, or the wind changed, the clutter map changed accordingly. Furthermore, a constant false alarm
rate detector allowed for detection of aircraft in rain, if their speed was sufficiently different from that of the rain.

The measured gain of the MTD over typical ATC radars was about 45 dB. This represented about a 20 dB increase over the conventional ATC radars using a three pulse canceler and a limiting IF amplifier. Additionally, the MTD had a narrower notch at zero speed and each of the blind speeds.

The results of this study indicate that, if properly utilized, the MTD could enhance the operation of the ARTS III automated ATC system. Valid MTD Doppler Data should be incorporated into the tracker algorithm to improve prediction accuracies. Also, the substantial informational content of the MTD primitive reports provides an effective technique for reducing tracker loads, discriminating between clutter and target returns and detecting second-time-around targets (Andrews 1975).

It is interesting to note why the FFT was used instead of narrowband filters. If we assume that N narrowband filters are required to achieve the appropriate speed accuracy, then 8N complex multiplications for each range bin had to be computed. Since the pulse width was one microsecond the maximum size of the range bin must also be one microsecond. Otherwise, a signal could be lost between samples. The duration between pulses was about one millisecond, so 1000 range bins must be processed in every eight milliseconds for real time operations. For eight Doppler bins
this implies that one complex multiplication must be performed every 125 nanoseconds. On the other hand, if a FFT is implemented only two multiplications are required to perform the eight point FFT for each range bin (Rabiner and Gold 1975). Hence, the required machine speed is reduced to one complex multiplication every 500 nanoseconds.
SUMMARY

The principles behind much of radar system engineering have been presented. Applicable design requirements and the driving forces behind those requirements have been discussed. Additionally, the report has been written on a level which is easy to understand, but not technically superficial.

This paper can be used as an overview of radar engineering for an individual who wants to learn more about radar. The combination of technical rigor and physical insight, makes this reference a good starting point. From here, the interested reader may pursue more mathematical, more detailed, or more specific publications.
LIST OF REFERENCES


Hulsmeyer, C. 1904. Hertzian-wave Projecting and Receiving Apparatus Adapted to Indicate or Give Warning of the Presence of a Metallic Body, Such as a Ship or a Train, in the Line of Projection of Such Waves. British Patent #13,170.


