A Pool Boiling Map: Water on a Horizontal Surface at Atmospheric Pressure

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A POOL BOILING MAP: WATER ON A HORIZONTAL SURFACE AT ATMOSPHERIC PRESSURE

BY

O. BURR OSBORN
B.S., Mississippi State University, 1983

THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in the Graduate Studies Program of the College of Engineering University of Central Florida Orlando, Florida

Fall Term
1986
ABSTRACT

The purpose of this study was to develop a boiling map for liquid water at atmospheric pressure. The map indicates the type of pool boiling phenomenon will initially occur in terms of the bulk temperature of the liquid and the interfacial contact temperature. The geometry studied was that of a flat horizontal plate with boiling incipient from the upward facing surface.

Previous analytical and experimental studies have been conducted concerning pool boiling. Several of these studies are summarized in this thesis and it is seen that there are gaps and limitations in the existing state of knowledge. Therefore, further research is required especially in the area of subcooled departure from nucleate boiling and the related heat transfer mechanisms.

A pool boiling map was constructed within the bounds of the available information. The results agree with available experimental data.
ACKNOWLEDGEMENTS

The author wishes to thank the faculty and the personnel of the Mechanical Engineering Department. He also wants to acknowledge the help of Dr. F.S. Gunnerson, Dr. E.R. Hosler, and Dr. L.A. Anderson who served on his committee. But special appreciation is extended to Dr. F.S. Gunnerson for his assistance, interest, suggestions, and literature sources without which this study would have been impossible.
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<th>Description</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>vapor film thickness</td>
<td>ft</td>
</tr>
<tr>
<td>b</td>
<td>in-growth coefficient</td>
<td>1/sec</td>
</tr>
<tr>
<td>c</td>
<td>specific heat capacity</td>
<td>BTU/lbm F</td>
</tr>
<tr>
<td>ΔG</td>
<td>free energy of formation</td>
<td>ft lbf</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity</td>
<td>ft/sec</td>
</tr>
<tr>
<td>g_c</td>
<td>proportional constant</td>
<td>32.2</td>
</tr>
<tr>
<td>h</td>
<td>specific enthalpy</td>
<td>BTU/lbm</td>
</tr>
<tr>
<td>h_film</td>
<td>convective film coefficient</td>
<td>BTU/hr f² sec</td>
</tr>
<tr>
<td>J</td>
<td>rate of embryo formation</td>
<td>sites/ft sec</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity</td>
<td>BTU/hr F</td>
</tr>
<tr>
<td>k</td>
<td>Boltzmann constant (eqn V-12)</td>
<td>1bm ft²/sec² R</td>
</tr>
<tr>
<td>m</td>
<td>wave number</td>
<td>ft</td>
</tr>
<tr>
<td>m</td>
<td>mass of one molecule (eqn V-17)</td>
<td>lbm</td>
</tr>
<tr>
<td>N</td>
<td>number of nuclei per unit volume</td>
<td>sites/ft³</td>
</tr>
<tr>
<td>p</td>
<td>momentum</td>
<td>ft lbm/sec</td>
</tr>
<tr>
<td>P</td>
<td>pressure</td>
<td>psia</td>
</tr>
<tr>
<td>P_crit</td>
<td>critical pressure</td>
<td>psia</td>
</tr>
<tr>
<td>P_max,r</td>
<td>pressure at T_{max,r}</td>
<td></td>
</tr>
<tr>
<td>P_r</td>
<td>reduced pressure P/P_crit</td>
<td>BTU/hr f²</td>
</tr>
<tr>
<td>q</td>
<td>heat flux</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>r</td>
<td>radius of vapor bubble</td>
<td>ft</td>
</tr>
<tr>
<td>r*</td>
<td>radius of a stable vapor bubble</td>
<td>ft</td>
</tr>
<tr>
<td>s</td>
<td>frequency function (eqn II-9)</td>
<td>1/sec</td>
</tr>
<tr>
<td>t</td>
<td>temperature (eqn II-1 to II-4)</td>
<td>°F</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td>°F</td>
</tr>
<tr>
<td>T_{\text{crit}}</td>
<td>critical temperature</td>
<td>°F</td>
</tr>
<tr>
<td>T_{\max,\text{r}}</td>
<td>reduced maximum liquid superheat</td>
<td>°F</td>
</tr>
<tr>
<td>T_{\max}</td>
<td>maximum liquid superheat</td>
<td>°F</td>
</tr>
<tr>
<td>T_{r}</td>
<td>reduced temperature T/T_{\text{crit}}</td>
<td>°F</td>
</tr>
<tr>
<td>T_{\text{min}}</td>
<td>minimum surface temperature by Henry's equation</td>
<td>°F</td>
</tr>
<tr>
<td>T_{\text{min,I}}</td>
<td>minimum surface temperature during film boiling as predicted by Berenson</td>
<td>°F</td>
</tr>
<tr>
<td>v</td>
<td>velocity</td>
<td>ft/sec</td>
</tr>
<tr>
<td>V</td>
<td>volume</td>
<td>ft^3</td>
</tr>
<tr>
<td>V_{r}</td>
<td>reduced volume V/V_{\text{crit}}</td>
<td></td>
</tr>
<tr>
<td>V_{\max,\text{r}}</td>
<td>reduced volume at T_{\max,\text{r}}</td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>collision frequency</td>
<td>1/sec</td>
</tr>
<tr>
<td>\alpha</td>
<td>thermal diffusivity</td>
<td>ft^2/sec</td>
</tr>
<tr>
<td>\beta</td>
<td>constant</td>
<td></td>
</tr>
<tr>
<td>\delta</td>
<td>average bubble height</td>
<td>ft</td>
</tr>
<tr>
<td>\rho</td>
<td>density</td>
<td>lbm/ft^3</td>
</tr>
<tr>
<td>\nu</td>
<td>viscosity</td>
<td>lbm/ftsec</td>
</tr>
<tr>
<td>\lambda</td>
<td>wavelength</td>
<td>ft</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------</td>
<td>------------</td>
</tr>
<tr>
<td><code>\nu</code></td>
<td>specific volume</td>
<td>ft/lbm³</td>
</tr>
<tr>
<td>σ</td>
<td>surface tension</td>
<td>lbf/ft</td>
</tr>
<tr>
<td>θ</td>
<td>time</td>
<td>sec</td>
</tr>
</tbody>
</table>

**Subscripts**
- crit: critical
- d: dangerous
- f: liquid
- g: gas
- i: interface
- L: liquid
- min: minimum
- s: surface
- sat: saturation
- sh: superheat
- sub: subcooled
- v: vapor
CHAPTER I
INTRODUCTION TO THE BOILING PHENOMENON

Boiling is the formation of vapor from the liquid phase of a substance. The phenomenon of two-phase boiling heat transfer is considerably more complicated than single-phase heat transfer. This is because not only the variables associated with single-phase heat transfer are involved, but also those variables associated with the phase change must likewise be considered. As a result of the large number of variables, neither general equations describing all the boiling processes or general correlations of the boiling heat transfer have been developed.

There have been, however, many studies done on the various boiling regimes. These studies usually specifically address a given set of specific conditions associated with a boiling regime and cannot be applied to the overall boiling phenomenon since the heat transfer mechanisms differ greatly from regime to regime. It is therefore necessary to analyze each region separately.

For the discussion of each regime the pool boiling concept will be used. Pool boiling is defined as boiling from a heated surface submerged in a volume of stagnant liquid. When the bulk of the liquid is below its boiling temperature, the process is called subcooled pool boiling. When the liquid is
at its boiling temperature, it is referred to as saturated pool boiling. For the purposes of explanation, the physical situation that will be addressed is that of a flat plate submerged horizontally in a stagnant pool of water at atmospheric pressure. The boiling to be studied will be that occurring from the top of the horizontal plate.

The first complete characteristics of pool boiling were reported by Nukiyama (1934). He summarized his results on what has now become known as the classical pool boiling curve (Figure I-1). In the first region from point A to B, the temperature of the liquid in contact with the surface is either below the boiling temperature or does not exceed it by more than a few degrees. In this region, heat is transferred to the liquid near the surface by free convection. As the temperature of the surface is increased, the required superheat needed for bubble formation is met and nucleate boiling begins. This region of boiling, illustrated in Figure I-1, is from point B to point C. In nucleate boiling, bubbles form at many favored sites on the surface called nucleation sites. These sites consist of scratches, pits, or crevices in the surface. The vapor bubbles are at first small at lower wall temperatures, and they will condense soon after leaving the surface. At the lower surface temperatures, the individual bubbles can be distinguished. There is little interaction between the bubbles generated at different nucleation sites. As the temperature of the surface is increased
Figure I-1. The classical pool boiling curve.
even more, the rate of bubble generation will become so high that continuous columns of vapor appear.

At the apex of the nucleate boiling regime, point D, departure from nucleate boiling has been reached. As point D is approached from the nucleate boiling region the rate of bubble growth increases, as does the frequency of formation. As the rate of bubble production increases the bubbles will tend to collide and coalesce, forming continuous vapor columns. At point D the number of vapor columns has increased to the point where they now restrict the flow of fluid to the surface.

Partial film boiling or transition boiling, region D to E, occurs after departure from nucleate boiling has been reached. The vapor columns and vapor film are unstable and collapse and reform. Large vapor patches originate at the outer edge of the film and at the fluctuating locations where the liquid actually touches the surface. As the surface temperature increases, the actual wetted area decreases, resulting in a reduced heat flux.

Point E represents the minimum film boiling point or the Leidenfrost point. At the minimum film boiling point a continuous vapor film just covers the surface. Stable film boiling occurs beyond point E. Vapor is released from the film periodically in the form of regularly spaced bubbles. In this region, heat transfer occurs mainly by conduction and convection
through the vapor film. At higher surface temperatures, radiation heat transfer becomes significant.

Changes in the system's parameters can effect the classical boiling curve of Nukiyama. The result of increasing the amount of subcooling on the boiling curve is illustrated by Figure I-2. From points A to B, single phase convection is occurring and the effect of subcooling is weak. The temperature needed to initiate nucleate boiling at point B increases with the amount of subcooling, but once again the effect is weak for water (Engelberg-Forster and Grief 1959). The temperature necessary to reach departure from nucleate boiling, along with the temperature needed to reach the minimum film boiling point, will increase with an increasing amount of subcooling to a differing degree.

The surface temperature needed to initiate each boiling regime for various amounts of subcooling will be one of the focal points of this work.

Another factor which will effect the pool boiling curve is the surface condition present. The physical properties have a bearing on how the material will be affected by the finishing process and what characteristic cavity size will result. The surface coating, the amount of oxidation, or fouling can greatly affect the wettability and the effective cavity size. The predominant effect of the surface conditions, as seen in Figure I-3, is in the nucleate boiling and transition boiling regions.
Figure I-2. Effect of subcooling on the classical pool boiling curve.
Surface Temperature

Figure I-3. Effect of surface roughness on the classical pool boiling curve.
Here the bubble formation is greatly enhanced due to the increase in the number of nucleation sites corresponding to an increase in the surface roughness. There is little effect in the film boiling region due to the limited liquid-solid contact.
CHAPTER II

METHOD OF TEMPERATURE ANALYSIS

Introduction

One objective of this study was to determine the type of boiling that may occur on the surface of a horizontal flat plate. This was done in terms of the bulk temperature of the liquid water and the instantaneous contact temperature between the liquid water and the flat plate. A boiling map was then constructed showing the boiling regimes in terms of these two temperatures.

The following results will be expected: when the instantaneous contact temperature of the interface is less than the boiling temperature of the liquid, no boiling will take place. If the instantaneous contact temperature is slightly greater than the boiling temperature and has the required superheat needed for bubble formation, then nucleate boiling will be expected. Nucleate boiling will continue until the interface temperature reaches the temperature at which departure from nucleate boiling (DNB) takes place. For the situation where the contact temperature is greater than the temperature required for DNB yet is less than the minimum temperature required to sustain a stable vapor film, the Leidenfrost temperature, then transition boiling will occur. At contact temperatures greater than the
Leidenfrost temperature, film boiling will result. Likewise, film boiling must occur if the interface contact temperature is greater than the maximum superheat limit of the liquid. The maximum superheat limit of the liquid is the maximum temperature limit beyond which the liquid phase can no longer exist.

**Method of Approach**

The previous discussion has focused on predicting the surface temperature required for a certain type of boiling. This generally requires using the thermal and physical properties of both the surface and the liquid water. A method is required to translate the surface temperature based on the thermal and physical properties of the situation to a general interface temperature. Once this interface temperature is found it may be applied to any surface regardless of the material. The boiling map could then be used for the more general situations without the need to account for the surface material.

**Analysis**

For analysis, semi-infinite geometries as illustrated in Figure II-1 are initially considered. The analysis can likewise be applied to finite geometries for very short periods of time where the heating or cooling effects at the surface have not yet propagated far into the material. For the analysis, it is assumed that the liquid is brought instantaneously into contact with the hot surface so that the surface experiences a step
Figure II-1. Temperature profiles for two semi-infinite geometries.
change in its temperature. The problem may be expressed from the parabolic heat conduction equation:

\[ \frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \frac{\partial t}{\partial \theta} \]  

(II-1)

where \( t \) represents temperature, \( \partial t \) represents time, \( x \) the position in the material, and \( \alpha \) the thermal diffusivity.

Using the following boundary and initial conditions for the liquid phase

\( t(0, \theta) = t_i \)  

(II-2)

\[ \lim_{x \to \infty} t(x, \theta) = t_L \]  

(II-3)

\( t(x, 0) = t_L \)  

(II-4)

and defining \( T = t - t_L \), then equation (II-1) becomes

\[ \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial \theta} \]  

(II-5)

and

\( T(0, \theta) = T_i - T_L \)  

(II-6)

\[ \lim_{x \to \infty} T(x, \theta) = 0 \]  

(II-7)
Transforming equation (II-5) with respect to $\Theta$

$$\int_{0}^{\infty} e^{-s \Theta} T_{xx} \, d\Theta = \frac{1}{\alpha} \int_{0}^{\infty} e^{-s \Theta} T_{\Theta} \, d\Theta$$  \hspace{1cm} (II-9)

The remaining portions of the derivation are left for the enjoyment of the reader and are detailed in Appendix A. The resulting equation for the interface contact is

$$T_{i} = \frac{T_{s} \sqrt{(kpc)}_{s} + T_{L} \sqrt{(kpc)}_{L}}{\sqrt{(kpc)}_{s} + \sqrt{(kpc)}_{L}}$$  \hspace{1cm} (II-10)

Equation (II-10) is valid when the liquid or solid are not undergoing a phase change. Carslaw and Jaeger (1959) showed that when the liquid undergoes a phase change, equation (II-10) may be written in the following form

$$T_{i} = \frac{T_{s} \sqrt{(kpc)}_{s} + T_{L} \sqrt{(kpc)}_{L} \text{erf} \phi}{\sqrt{(kpc)}_{s} + \sqrt{(kpc)}_{L} \text{erf} \phi}$$  \hspace{1cm} (II-11)

where $\text{erf} \phi$ is evaluated from the transcendental expression

$$\frac{C_{L} (T_{s} - T_{L})}{h_{fg} \sqrt{\pi}} = \phi \, e^{\phi^{2}} \left[ \frac{k_{s} \, (\rho c)_{L}}{k_{L} \, (\rho c)_{s}} + \epsilon \text{erf} \phi \right]$$  \hspace{1cm} (II-12)
For known initial temperatures equations (II-10), (II-11), and (II-12) may be used to determine the interface temperature.
CHAPTER III
ONSET OF NUCLEATE BOILING

Objective

The purpose of this chapter is to determine when the onset of nucleate boiling (ONB) will occur in terms of the interface contact temperature and the bulk temperature of the water.

Introduction

For vapor bubbles to form at nucleation sites, the local temperature must be at least slightly greater than the saturation temperature of the liquid. In actuality, the surface will be many degrees above the saturation temperature. This is due to the fact that in order to form and exist the vapor bubble must do work on the surrounding liquid, that is pushing the liquid from the region where the vapor bubble is to be; and once formed, to overcome the surface tension effects. The energy to do this must be supplied by the surface and is referred to as the surface superheat. The surface temperature may be expressed as

$$T_s = T_{sat} + T_{sh}$$  \hspace{1cm} (III-1)

where the surface temperature is $T_s$, the saturation temperature is $T_{sat}$ and the superheat required to form and sustain a vapor bubble is $T_{sh}$. 
Following bubble formation and growth, the temperature of the vapor bubble required to maintain its mechanical stability may be estimated. Figure III-1 illustrates a hemispherical vapor bubble formed at a nucleation site that is in mechanical and thermal equilibrium. Mechanical equilibrium requires that the net force on the bubble must be zero so the following equation applies

$$P_V - P_L = \frac{2\sigma}{r} \quad (III-2)$$

where $P_V$ is the pressure inside the vapor bubble, $P_L$ is the pressure of the surrounding liquid, and $r$ is the stable radius of the vapor bubble. The saturation temperature and pressure may be related using the Clapeyron relationship

$$\frac{h_{fg}}{v_{fg}T} = \frac{dP}{dT} \quad (III-3)$$

For low and moderate pressures, the quantity $\frac{h_{fg}}{v_{fg}T}$ is approximately a constant. When the temperature, $T$, is equal to the saturation temperature equation (III-3) may be integrated with respect to temperature and pressure of the liquid and vapor phases to yield

$$(T_V - T_{sat})\left(\frac{h_{fg}}{v_{fg}T_{sat}}\right) = P_V - P_L \quad (III-4)$$
Figure III-1. Stable vapor bubble.
Combining equations (III-4) and (III-2) gives

\[ T_v - T_{sat} = \frac{T_{sat} \nu_{fg} 2\sigma}{h_{fg} r} \]  

(III-5)

Equation (III-5) determines how much higher the vapor temperature must be than the saturation temperature for the vapor bubble to exist. Since the surface transfers heat to the vapor in the bubble, the minimum surface temperature may be thought of as being slightly greater than the vapor temperature predicted by equation (III-5.)

The Situation Addressed

For the situation where the nucleation site is filled with a gas, an expression for the superheat needed to form a vapor bubble was developed by Bergles (1981)

\[ T_v - T_{sat} = \frac{T_{sat} \nu_{fg} 2\sigma}{h_{fg} r} \left( \frac{p_f - p_g}{r} - p_g \right) \]  

(III-6)

where \( p_g \) is the pressure of the gas at the nucleation site. The pressure of the gas is difficult to determine. For the purposes of the onset of nucleate boiling, it will be assumed that there is no gas at the nucleation site and that the water completely wets the surface.

Prior to boiling in a subcooled liquid, hot jets of fluid are projected from the superheated boundary layer into the
cold bulk fluid. As the heat flux is increased, vapor bubbles will appear at the heated surface, but if the liquid is still greatly subcooled they will collapse before they are released. Finally, when the bulk temperature of the liquid is near the saturation temperature, the bubbles will be formed and released from the heated surface.

In calculating the surface temperature and corresponding interface temperature necessary for the onset of nucleate boiling, the assumption will be made that there will be a sufficient heat flux present to induce the onset of nucleate boiling regardless of the amount of subcooling that is present.

**Review of Literature**

There is an abundance of literature available on the onset of nucleate boiling for flowing systems, however for the pool boiling situation there is a remarkable lack of analytical work. As a result of this, the region on the boiling map representing the onset of nucleate boiling is determined from the experimental work described in the literature.

Engelberg-Forster and Grief (1959) stated that the rate of heat transfer from a surface to a liquid in the presence of nucleate boiling depends upon the degree of superheat \((T_s - T_{sat})\) but does not depend upon the degree of subcooling \((T_{sat} - T_L)\). This statement is based on experimental data summarized in their paper.
Fand and Keswani (1974) did experimental work on determining the surface temperature at the onset of nucleate boiling for varying bulk temperatures and heat fluxes. Some of their data are listed in Table III-1

### TABLE III-1

**SURFACE TEMPERATURES FOR THE ONB FOR DIFFERENT SUBCOOLINGS**

<table>
<thead>
<tr>
<th>$T_s$ (F)</th>
<th>$T_L$ (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>227</td>
<td>178</td>
</tr>
<tr>
<td>233</td>
<td>169</td>
</tr>
<tr>
<td>236</td>
<td>162</td>
</tr>
<tr>
<td>238</td>
<td>150</td>
</tr>
</tbody>
</table>

The data in Table III-1 were taken at atmospheric pressure in degassed and distilled water. The curve determined by these data points was extrapolated to further subcoolings.
CHAPTER IV
DEPARTURE FROM NUCLEATE BOILING

Objective

Following the nucleate boiling region boiling curve is the region of transition boiling. The onset of transition boiling is called departure from nucleate boiling. The purpose of this chapter is to determine the temperature of the surface when departure from nucleate boiling begins.

Introduction

In the nucleate boiling region, as the temperature of the horizontal surface is increased, there is a corresponding increase in the heat flux leaving the surface. This effect will continue until the surface has reached a sufficiently high temperature to produce a very large rate of vapor generation.

The rate of bubble formation is so great and the vapor bubbles are so tightly packed together that the surrounding liquid is restricted from easily reaching the surface. The heat transfer properties for this occurrence are very poor and become even worse with increasing surface temperature. The unstable situation will occur where with increasing surface temperature there will be a corresponding decrease in the heat transfer coefficient.
The condition at which this unstable situation begins is known as departure from nucleate boiling, and it marks the beginning of transition boiling.

Transition boiling is characterized by the partial presence of an unstable vapor film over the heated surface. This vapor film releases large patches of vapor from the film at irregular intervals resulting in the intermittent wetting of the surface. Because of the difficulty in defining and understanding transition boiling, there exists no general theory or formulation explaining adequately the characteristics of this region. As a result of the difficulty in understanding the transition boiling regime, there is a deficiency in the available information.

**Review of Literature**

The heat flux at departure from nucleate boiling is known as the critical heat flux. The critical heat flux at saturation conditions was determined by Zuber (1961) to be

\[ q^{11} = 0.131 h_{fg} \rho_v \frac{\sigma (\rho_L - \rho_v) g \rho_c \frac{1}{2}}{2 \rho_v} \]  

(IV-1)

Rohsenow (1973) recommended that the constant .131 be changed to 0.18 to better fit the available data. For a flat plate, Bergles (1981) determined that the constant suggested by Rohsenow should be multiplied by 1.14.
Experiments have shown that the critical heat flux is strongly dependent on the degree of subcooling present. Ivey (1962) developed the following equation for finding the critical heat flux for water

\[ q_{\text{crit}} = q_{\text{crit}} \left[ 1 + \frac{1}{\rho_v \rho_L} \left( \frac{c_p}{c_L} \rho_L \frac{(T_{\text{sat}} - T_L)}{h_{fg} \rho_v} \right)^{1/4} \right] \]  

Equation (IV-2) is an expression for the critical heat flux as a function of the bulk temperature of the water. There is, however, no indication as to what the surface temperature must be to initiate this critical heat flux or departure from nucleate boiling. For a given heat flux the surface temperature could be determined by using Newton's law of cooling if the heat transfer coefficient were known. For a flowing fluid there have been a number of studies done on predicting this heat transfer coefficient, but there are no known studies for the pool boiling situation. The approach taken is to determine the surface temperature by means of using empirical equations used for predicting the surface temperature for fully developed nucleate boiling. These equations then will be evaluated at the critical heat flux as determined by equation (IV-2). These results will be anticipated to yield surface temperatures which are slightly below those which would actually exist at the departure from nucleate boiling. This is due to the fact that the heat transfer
characteristics during nucleate boiling are significantly better than during transition boiling.

The only equation which appears to apply to the situation addressed is an equation developed by Thom (1965). This equation was developed for a flowing fluid in fully developed nucleate boiling, however Collier (1981) stated that the surface temperature arrived at through Thom's equation was not markedly different from those obtained from other pool boiling correlations. Thom's correlation is valid only for water

$$T_s = T_{sat} + 22.65 (q^{11})^{.5} e^{-P/87}$$ (IV-3)

The temperatures for this equation are in degrees Celsius, the pressure is in bars, and the heat flux has units of megawatts per square meter.

**Experimental Data**

The only pool boiling data found for departure from nucleate boiling and the corresponding critical heat flux were those data taken by Sakurai and Shiotsu (1974). Their relevant data are condensed in Table IV-1. The last two surface temperatures were evaluated using equation (IV-3) to give additional data points.
TABLE IV-1

CHF AND CORRESPONDING SURFACE AND BULK TEMPERATURES

<table>
<thead>
<tr>
<th>CRITICAL HEAT FLUX, $q'$ (BTU/hr-ft$^2$)</th>
<th>$T_L$ (F)</th>
<th>$T_s$ (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.83 \times 10^5$</td>
<td>212</td>
<td>257</td>
</tr>
<tr>
<td>$4.50 \times 10^5$</td>
<td>194</td>
<td>259</td>
</tr>
<tr>
<td>$6.56 \times 10^5$</td>
<td>176</td>
<td>(271)</td>
</tr>
<tr>
<td>$1.04 \times 10^5$</td>
<td>140</td>
<td>(286)</td>
</tr>
</tbody>
</table>
CHAPTER V

FILM BOILING

Objective

The purpose of this chapter is to determine the surface temperature at which film boiling will be taking place following the transition boiling regime.

Introduction to the Film Boiling Phenomenon

In 1756, Leidenfrost studied the evaporation of a small water droplet on a hot ladle. From his observations, there is a surface temperature at which the liquid droplet's evaporation time is the greatest. This has become known as the Leidenfrost point.

For pool boiling this point may be viewed as the surface temperature at which the heat flux is a minimum for film boiling. If the heat flux and associated temperature are increased beyond this point a stable vapor film will exist on the heater surface, illustrated in Figure V-1. For boiling near the Leidenfrost condition, there will be three mechanisms of heat transfer of importance for water; those being conduction through the vapor film, convection at the liquid-vapor interface, and most significantly conduction into the liquid at the small points of transient contacting.
Figure V-1. Stable vapor film on a heated surface.
During film boiling wave-like oscillations of the liquid vapor interface occur. This behavior has been applied to the film boiling phenomenon by Zuber and Tribus (1958) and Berenson (1961). Their work assumed that heat removal from the surface was characterized by the behavior of the bubbles being formed, which in turn is related to the occurrence of hydrodynamic Taylor instabilities of the liquid-vapor interface.

Theory

Vapor Removal

Berenson, Zuber, and Tribus used the idea of the Taylor instability to develop correlations for film boiling at saturation conditions on a horizontal flat plate. If a dense fluid lies above a less dense one a Taylor instability will develop as shown by Figure V-2. Wave-like action will occur at the interface between the media causing the film to rupture at the antinodes allowing the vapor to rise as bubbles. The actual derivation of the wavelength at which the bubbles will break away from the antinodes is given in Appendix B. The wavelength has been determined to be equal to

$$\lambda_d = 2\pi \left[ \frac{3\sigma g_c}{g(\rho_L - \rho_0)} \right] = \lambda_{\text{crit}} \sqrt{3} \quad (V-1)$$

where $\lambda_{\text{crit}}$ is the critical wavelength and $\lambda_d$ is the most dangerous wavelength. When the interfacial wavelength is less
Figure V-2. Taylor instabilities at a liquid-vapor interface.
than the critical wavelength, the interface is stable. Where the wavelength is greater than the critical wavelength, the interface becomes increasingly unstable until it ruptures at the most dangerous wavelength. The equation for the dominant wavelength indicates the bubble spacing during film boiling. At atmospheric pressure, the dominant wavelength for saturated water is about 2.7 centimeters, which is in good agreement with an experimentally observed value of about 3.0 centimeters (Hosler and Westwater 1962).

Liquid-solid Contacts

When the two-phase boundary begins to grow, the growth will continue until a bubble leaves the antinode. After the bubble leaves, the interface snaps back toward the solid surface. This leads to transient wetting of the surface near the minimum film boiling point as shown by Figure V-3. The liquid contacts and locally wets the surface, and the local surface temperature may momentarily fall below that value needed to sustain a vapor film. This cooling effect of the surface is referred to as microlayer evaporation. To model this occurrence the concepts of transient contacting and microlayer evaporation need to be considered.

Transient contacting during film boiling is of a very short duration and conduction is the major mode of heat transfer as posed by Henry (1973). Henry showed that the initial
Figure V-3. Liquid-solid contact.
temperature of the surface needed to sustain film boiling could be expressed in terms of the combination of equation (II-10) and the minimum temperature predicted by Berenson for an isothermal surface as shown in Appendix B. This leads to the following equation for the minimum temperature to sustain the onset of film boiling, accounting for the slight amount of transient contacting that occurs

\[ T_{\text{min}} = T_{\text{min}, I} + (T_{\text{min}, I} - T_L) \sqrt{\frac{(kpc)_L}{(kpc)_S}} \quad (V-2) \]

When the liquid touches the surface and boils away a thin liquid film will remain. This microlayer will continue to cool the surface as it is evaporated. The amount of cooling depends on several factors such as the amount of liquid present in the microlayer, the latent heat of vaporization, and the temperature of the surface. To quantify these parameters involved in transient contacting and microlayer evaporation, Henry performed a least squares curve fit on the experimental data to arrive at

\[ \frac{T_{\text{min}} - T_{\text{min}, I}}{T_{\text{min}, I} - T_L} = 0.42 \left[ \sqrt{\frac{(kpc)_L}{(kpc)_S}} \cdot \frac{h_{fg}}{C_s(T_{\text{min}, I} - T_{\text{sat}})^{0.6}} \right] \]

\[ (V-3) \]

Equation (V-3) may be applied to determine the temperature of the surface at the minimum film boiling point at not only saturation conditions, but also subcooled conditions. This is based on the fact that Henry used data points in his curve fit that included
many subcooled conditions. The temperatures arrived at using equation (V-3) will be used to construct the minimum film boiling line on the pool boiling map. This line will also serve as the maximum limit for the transition boiling regime. To find the maximum upper limit for the onset of film boiling the concept of the maximum superheat limit needs to be considered.

Maximum Superheat Limit

Due to the fact that the minimum temperature difference necessary to sustain highly subcooled film boiling has not been studied in detail for the flat plate geometry, the thermodynamic approach will be considered to determine the maximum temperature beyond which the liquid cannot exist in its liquid phase. This is called the limiting superheat temperature. If the liquid–surface interface is at or above this temperature, then film boiling will occur regardless of the amount of subcooling present. Two approaches for determining this maximum superheat are from thermodynamic considerations and from the kinetic theory of homogeneous nucleation.

**Thermodynamic Limit of Superheat**

To determine when boiling begins the pressure-volume, diagram in Figure V-4 is used. The area to the left of the two-phase envelope is the region of a stable subcooled liquid, and to the right is the region of a stable superheat vapor. The boundary of the envelope represents a saturated liquid left of
Figure V-4. Thermodynamic limit of superheat on a P-V diagram.
the critical point, and a saturated vapor to the right of the critical point. The horizontal line through the vapor dome represents an equilibrium mixture of saturated liquid and saturated vapor.

A liquid, however, can be superheated and a vapor subcooled, without the occurrence of a phase transformation even though its state lies within the two-phase region. Under these conditions the isotherm in Figure V-4 extends into the stable two-phase region as shown by curve BC and EF. In this region the superheated liquid and subcooled vapor are in a metastable state. The superheated liquid will tend to flash to vapor in the presence of nucleation sites or with the occurrence of turbulence in the liquid. Likewise, the subcooled vapor will tend to condense to a saturated liquid. The dashed line within the two-phase region is called the spinodal and represents the locus of the maxima and minima of all the isotherms. The boundary of the spinodal to the left of the critical point represents the upper limit temperature of a superheated liquid. At this maximum superheat the following relationship applies

\[
\left. \frac{dP}{dV} \right|_T = 0 \quad (V-4)
\]

The maximum superheat may be determined from the equation of state. Assuming that the liquid obeys the following Van der Waals equation of state
Pr = \frac{8T_r}{3(V_r - 1/3)} - \frac{3}{(V_r)^2} \quad (V-5)

Applying equation (V-4) to equation (V-5)

\frac{dPr}{dV_r} = 0 = \frac{-8T_{\text{max},r}}{3(V_{\text{max},r} - 1/3)} + \frac{6}{V_{\text{max},r}^3} \quad (V-6)

8T_{\text{max},r} = \frac{18(V_{\text{max},r} - 1/3)}{V_{\text{max},r}^3} \quad (V-7)

Substituting equation (V-7) into equation (V-5), a final expression for the reduced pressure at the point of maximum superheat is obtained

P_{\text{max},r} = \frac{1}{V_{\text{max},r}^2} \left[ \frac{2(3V_{\text{max},r} - 1)}{V_{\text{max},r}} - 3 \right] \quad (V-8)

For pressures much lower than the critical pressure, the reduced pressure may be approximated as being zero and equations (V-8) and (V-5) simplify to lead to

T_{\text{max},r} \sim \frac{27}{32} \quad (V-9)

From equation (V-9), the maximum superheat may now be expressed in terms of the critical temperature
\[ T_{\text{max}} \approx \frac{27}{32} T_{\text{crit}} \]  

(V-10)

For water, the maximum superheat limit from equation (V-10) has been determined to approximately 600°F.

**Kinetic Theory of Homogeneous Nucleation**

Another way of estimating the maximum superheat of a liquid is by the kinetic theory of homogeneous nucleation. Homogeneous nucleation considers the situation where the vapor phase is created and grows at the expense of the surrounding liquid. This process assumes that only a very small number of molecules have an energy level considerably greater than the average energy level of the liquid molecules. These higher energy molecules have a finite probability of forming a cluster of molecules resulting in a vapor embryo being formed. The probability of the formation of vapor nuclei of the necessary size can be calculated using the Boltzmann equation for the distribution of molecular clusters as a function of size (Döring 1937).

\[ \frac{-\Delta G(r)}{kT_L} = N(r) = N_0 e^{-\frac{\Delta G(r)}{kT_L}} \]  

(V-11)

where \( N \) is a constant that is equal to the molecular density, \( k \) is Boltzmann's constant, and \( G(r) \) is the free energy of
formation of a nucleus of radius $r$. The Gibb's free energy of formation may be expressed as

$$\Delta G(r) = 4\pi r^2 \sigma - \frac{4}{3} \pi r^3 \left( P_g - P_L \right)$$ \hspace{1cm} (V-12)

In equation (V-12), the first term may be thought of as the energy needed to create the surface of the vapor embryo and the second term is the energy needed to displace the liquid and create a hole in the fluid. If equation (V-12) has equation (III-2) substituted into it, the following expression for the Gibb's free energy is found

$$\Delta G(r) = 4\pi r^2 \sigma \left( 1 - \frac{2r}{3r^*} \right)$$ \hspace{1cm} (V-13)

where $r^*$ represents the radius of a vapor embryo that is in mechanical equilibrium. The Gibb's free energy is maximum when the radius $r$ is equal to $r^*$ which leads to

$$\Delta G(r^*) = \frac{4}{3} \pi r^*^2 \sigma$$ \hspace{1cm} (V-14)

If the bubbles formed have a radius that is less than $r^*$, the vapor bubble will collapse upon itself; and if the radius of the vapor bubble is greater than $r^*$, the bubble will grow spontaneously. This is due to the fact that only a vapor bubble with a radius of $r^*$ is stable. For a lower radius, the vapor
pressure in the bubble is not great enough to support the surface of the bubble, and with a greater radius the vapor pressure is too great, thus overwhelming the opposing surface tension effect. Homogeneous nucleation will occur if a stable vapor embryo or very small vapor bubble accepts or absorbs an additional molecule.

The rate of embryo formation or nucleation in a liquid of temperature $T$ is given by the product of nuclei per unit volume and the collision frequency, $w$,

$$ J = wNe $$

where $J$ is the rate of embryo formation. The collision frequency term suggested by Bernath (1952) is

$$ w = \frac{1}{(\frac{2\sigma r c}{\pi m})^2} $$

where $m$ is the mass of one molecule.

Table V-1 shows the rate of embryo formation and the corresponding bulk temperature of the liquid found from equation (V-16).

Simpson and Walls (1965) suggested that significant nucleation occurs for values of $J$ between $2.83 \times 10^7$ sites/ft$^3$-sec and $2.83 \times 10^{11}$ sites/ft$^3$-sec, this corresponds to a temperature of approximately $588^\circ F$. This
temperature represents the maximum superheat of water arrived at using the kinetic theory of homogeneous nucleation. The maximum superheat arrived at by both the thermodynamic and homogeneous approaches are close in numerical value due to the fact that they both represent the same physical occurrence.

TABLE V-1

RATE OF EMBRYO FORMATION
FOR A GIVEN LIQUID TEMPERATURE

<table>
<thead>
<tr>
<th>$T_L$ (F)</th>
<th>$J$ (sites/ft$^3$-sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>555</td>
<td>$1.83 \times 10^{-53}$</td>
</tr>
<tr>
<td>573</td>
<td>$2.75 \times 10^{-10}$</td>
</tr>
<tr>
<td>577</td>
<td>$4.59 \times 10^{-2}$</td>
</tr>
<tr>
<td>583</td>
<td>$3.88 \times 10^4$</td>
</tr>
<tr>
<td>588</td>
<td>$2.08 \times 10^9$</td>
</tr>
<tr>
<td>590</td>
<td>$4.23 \times 10^{11}$</td>
</tr>
</tbody>
</table>
CHAPTER VI

SUMMARY

The concepts introduced in chapters one through five were used to develop the Boiling Map, Figure VI-1. Each region on the boiling map will now be summarized.

The minimum temperature below which vapor and therefore vapor bubbles could not exist at atmospheric pressure is $212^\circ$ F. This is represented by the lowest horizontal line on the boiling map and is the minimum temperature for the onset of nucleate boiling. In actuality the minimum temperature will be greater than this due to the effects of surface tension and the work needed to form the bubble. Since there were no relationships to account for these effects at differing subcoolings, the limit was found by performing a linear regression on the data taken by Fand (1974). The resulting equation generated by this procedure is as follows

$$T_s = 297.0 - 0.3856 T_L$$  \hspace{1cm} (VI-1)

where all the temperature are in degrees Fahrenheit. This appears to be reasonable based on the fact that Engelberg-Forster and Grief (1959) found that the effect of subcooling on the onset of nucleate boiling was weak. The error, as determined by the
Figure VI-1. A pool boiling map for liquid water on a horizontal flat surface.
linear regression procedure, between the data points and the line representing the onset of nucleate boiling was five percent. An error bar was placed on the line indicating this difference.

The line representing the departure from nucleate boiling was very difficult to determine. The heat flux at which this phenomenon is initiated appears to be understood to some degree by a number of researchers in the field, however the corresponding surface temperature seems to have defied most studies. This is due mainly to the fact that the heat transfer process at the point of departure from nucleate is not completely understood. The approach will be taken to evaluate the surface temperature at the heat flux for departure from nucleate boiling is using Thom's correlation for fully developed nucleate boiling. The veracity of using Thom's correlation for the physical situation present is uncertain since it was developed from experimental data for upward flow. Collier (1981), however, states that the correlation does give results that were in good agreement with pool boiling data. For small subcoolings the correlation appears to give results which are consistent with those obtained experimentally by Sakuria (1974).

The error associated with determining the departure from nucleate boiling line was determined in the following manner. The data points were on the average ten degrees Fahrenheit below those predicted by Thom's equation. This difference was extended through the remainder of the subcoolings.
The film boiling region on the boiling map is more easily defined than that of transition boiling. The superheat limit of liquid water represents an upper bound on the film boiling region. This is a result of the fact that liquid water cannot exist above this temperature. For situations where liquid water is placed on a surface where the interface temperature is equal to or greater to this temperature, film boiling will occur. The minimum boundary on the film boiling region was determined using Henry's correlation. Henry's correlation is valid over a wide range of geometries and subcoolings and was developed for pool boiling situations. In determining the onset of film boiling, the minimum temperature (as predicted by Berenson) was used. This temperature has an error of ten percent, so it was assumed that the temperature arrived at by Henry's correlation would have at least this amount of error.
APPENDIX A

DERIVATION OF THE INTERFACE CONTACT TEMPERATURE

When two materials of differing temperatures are brought into contact, their interface will reach an intermediate temperature whose magnitude is related to the thermal physical properties of the two materials. The following is a derivation of this interface temperature. For this derivation it is assumed that the two contacting mediums will act as semi-infinite geometries. This is a good assumption for the situation where the contact period is short and the thermal penetration is small. Also for the analysis it will be assumed that the liquid is brought instantaneously into contact with the higher temperature surface so that the surface will experience a step change in its temperature. The problem may be expressed from the parabolic heat conduction equation.

\[
\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \frac{2t}{2\theta} \tag{A-1}
\]

with the following boundary conditions

\[
t(0, \theta) = t_i \tag{A-2}
\]

\[
\lim_{x \to \infty} t(\infty, \theta) = t_L \tag{A-3}
\]


\[ t(x,0) = t_L \]  \hspace{1cm} (A-4)

Defining \( T = t - t_L \), then the above equations become

\[ \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{2T}{2\theta} \]  \hspace{1cm} (A-5)

and

\[ T(0,\theta) = T_i - t_L \]  \hspace{1cm} (A-6)

\[ \lim_{\theta \to 0} T(\theta, \theta) = 0 \]  \hspace{1cm} (A-7)

\[ T(x,\theta) = 0 \]  \hspace{1cm} (A-8)

Transforming equation (A-1) with respect to \( \theta \)

\[ \int_{\theta=0}^{\infty} e^{-s\theta} Txx \, d\theta = \frac{1}{\alpha} \int_{\theta=0}^{\infty} e^{-s\theta} T_0 \, d\theta \]  \hspace{1cm} (A-9)

According to the Leibnitz's rule, the \( x \) differentiation may be removed from the left-hand integral. When this is done and the right-hand side of the expression is integrated by parts the following equation is obtained

\[ \frac{d^2}{dx^2} \int_{\theta=0}^{\infty} e^{-s\theta} Td\theta = \frac{1}{\alpha} \left[ Te^{-s\theta} \right]_{\theta=0}^{\infty} + \frac{s}{\alpha} \int_{\theta=0}^{\infty} e^{-s\theta} Td\theta \]  \hspace{1cm} (A-10)
The remaining integrals are just $\hat{T}(x,s)$, which will be written as $\hat{T}$

$$\frac{d^2\hat{T}}{dx^2} = -\frac{1}{\alpha} \hat{T}(x,0) + \frac{s}{\alpha} \hat{T} \quad (A-11)$$

Substituting equation (A-8) into equation (A-11), the following equation results

$$\frac{d^2\hat{T}}{dx^2} - \frac{s}{\alpha} \hat{T} = 0 \quad (A-12)$$

Transforming equations (A-6) and (A-7)

$$T(0,s) = (t_i - t_L) \frac{1}{s} \quad (A-13)$$

Taking the limit as $x$ goes to infinity

$$\lim_{x \to \infty} \hat{T}(x,s) = 0 \quad (A-14)$$

The solution to equation (A-12) will have the general form

$$\hat{T}(x,s) = A \exp(-\sqrt{s/\alpha}) + B \exp(\sqrt{s/\alpha}) \quad (A-15)$$

Boundary condition equation (A-14) requires that $B = 0$. When this is substituted into equations (A-13) and (A-15), the following equation appears
which leads to

\[ \hat{T}(x, \theta) = (t_1 - t_L) \frac{1}{s} \exp(-x \sqrt{\frac{\theta}{\alpha}}) \]  

(A-17)

Equation (A-17) may be written in terms of the complimentary error function

\[ T(x, \theta) = (t_1 - t_L) \text{erfc} \frac{x}{2 \sqrt{\alpha \theta}} \]  

(A-18)

or

\[ t(x, \theta) = t_L + (t_1 - t_L) \text{erfc} \frac{x}{2 \sqrt{\alpha \theta}} \]  

(A-19)

Equation (A-19) is the solution to the semi-infinite solid problem given by equations (A-5) through (A-8). However, the interface temperature needs to be clearly represented in terms of the surface temperature and the liquid temperature. To accomplish this, the complimentary error function will be written in the form

\[ \text{erfc} \frac{x}{2 \sqrt{\alpha \theta}} = \frac{2}{\pi} \int_{\sigma = \frac{x}{2 \sqrt{\alpha \theta}}}^{\infty} e^{-\sigma^2} \, d\sigma \]  

(A-20)
The heat flux into the liquid may be written as

\[ q^{ll} = - \left[ k \frac{2t}{2x} \right]_{x=0} \quad (A-21) \]

Substituting equation (A-20) into equation (A-21) and then using Leibnitz's rule for differentiating an integral, the heat flux becomes

\[ q^{ll} = - k(t_i - t_L) \frac{2}{\sqrt{\pi}} \left[ - \frac{1}{2\sqrt{\alpha \theta}} - \frac{x^2}{4\alpha \theta} \right]_{x=0} \quad (A-22) \]

Equation (A-22) may be simplified to the following form

\[ q^{ll} = \frac{\sqrt{k \rho c}}{\sqrt{\pi}} \frac{(t_i - t_L)}{\sqrt{\theta}} \quad (A-23) \]

The heat flux into the liquid may be written now as

\[ q^{ll}_L = \frac{1}{\sqrt{\pi}} \frac{t_i - t_L}{\sqrt{\theta}} \sqrt{(k \rho c)}_L \quad (A-24) \]

and the heat flux out of the surface may be expressed by the equation

\[ q^{ll}_s = - \frac{1}{\sqrt{\pi}} \frac{t_i - t_s}{\sqrt{\theta}} \sqrt{(k \rho c)}_s \quad (A-25) \]
Since the heat flux leaving the surface must equal the heat flux entering the liquid, equation (A-24) may be equated to equation (A-25) to yield

\[ \frac{t_i - t_L}{t_s - t_i} = \frac{\sqrt{(k \rho c)_g}}{\sqrt{(k \rho c)_L}} \]  

(A-26)

This is the equation to determine the interface temperature in terms of the liquid and surface temperature and the thermal physical properties of each medium.
SUMMARY OF BERENSON'S WORK ON HORIZONTAL FILM BOILING

Berenson (1961) was one of the first to study the phenomenon of film boiling in detail. The following is a brief condensation of his work concerning film boiling from a horizontal flat surface.

Between the transition boiling region and the film boiling region there is a point where the heater surface is almost entirely covered by a vapor film. At this point the heat flux has reached its minimum value and is called the minimum film boiling point. The surface temperature at which this occurs may be expressed as the saturation plus the heat flux divided by the total heat transfer coefficient

\[ T_s = T_L + \frac{\dot{q}}{h_{film}} \]  

(B-1)

Berenson's work was to determine the heat flux at this minimum film boiling point along with the corresponding heat transfer film coefficient. The following is a list of assumptions which he made in developing his relationships for these terms.

1. Near the minimum heat flux, the bubble spacing is unaffected by the vapor velocity and the vapor film thickness.
2. The vapor flows radially into the bubble.

3. The momentum forces are small compared to the viscous forces.

4. The flow is laminar.

5. The change in height of the vapor-liquid boundary is negligible compared to the average height of the bubble above the interface.

6. The kinetic energy of the vapor is negligible in comparison with the enthalpy change.

7. The average value of the properties are equal to those at the average temperature of the hot surface and the saturated liquid.

8. Heat is transferred through the vapor film by conduction alone, the effects of radiation being small.

Using these assumptions, Berenson developed his expression for the minimum surface temperature at the onset of film boiling. The following is a brief overview of his derivation. For one-dimensional viscous flow the momentum equation may be reduced to the following.

\[
\frac{dP}{dr} = \frac{\beta \nu_f V_v}{g_c a^2}
\]  

(B-2)

where beta is a constant equal to twelve if the vapor liquid boundary has the same effect as a stationary wall. Berenson showed that the vapor velocity varies with the radius according
to the following equation.

\[ V_v = \frac{k_v \Delta T}{e_{V_f} H_{fg} a^2} \left( \frac{\lambda^2}{2 \pi} - \frac{\pi r^2}{2 \pi r} \right) \]  \hspace{1cm} (B-3)

Combining equations (B-2) and (B-3) will give the following equation.

\[ dP = \frac{\beta \mu V_f k_v \Delta T}{g_c a^4 e_{V_f} H_{fg}} \left( \frac{\lambda^2}{2 \pi} - \frac{\pi r^2}{2 \pi r} \right) dr \]  \hspace{1cm} (B-4)

Equation (B-4) may be evaluated using the following expressions for the limits of integration on \( r \), the radius. The initial radius has been obtained by experimental measurements to be equal to

\[ r_1 = 2.35 \sqrt{\frac{g_c \sigma}{g(\rho_L - \rho_v)}} \]  \hspace{1cm} (B-5)

The final radius may be expressed as

\[ r_2 = \sqrt{\frac{a}{2\pi}} \]  \hspace{1cm} (B-6)

The wavelength that grows the fastest and, therefore, dominates, is that which maximizes \( b \), where for this case \( b \) is given by

\[ b = \frac{g(\rho_L - \rho_v)m}{\rho_L + \rho_v} - \frac{g_c \sigma m^2}{\rho_L + \rho_v} \]  \hspace{1cm} (B-7)
If equation (B-7) is differentiated with respect to \( m \) and solved for the value that maximizes \( b \), the following expression for \( m \) is arrived at.

\[
m = \left[ \frac{g(\rho_L - \rho_v)}{3g_c \sigma} \right]
\]  

(E-8)

Equation (B-4) can now be integrated and combined with equations (B-5), (B-6), and (B-8) to give the following pressure difference.

\[
P_2 - P_1 = \frac{8\beta}{\pi} \frac{\mu_f k_{vf}}{4g_c a^4 \rho_v h_{fg}} \Delta T \frac{g_c \sigma}{g(\rho_L - \rho_v)}
\]  

(E-9)

Equation (E-9) may be written in terms of the average height of the bubble above the film interface

\[
P_2 - P_1 = (\rho_L - \rho_v) \frac{g_c \delta}{g_c} - \frac{2\sigma}{r_1}
\]  

(E-10)

where the average height is expressed as

\[
\delta = 3.2 \sqrt{\frac{g_c \sigma}{g(\rho_L - \rho_v)}},
\]  

(E-11)

Combining equations (E-11), (E-10), and (B-5) will give the following expression for the pressure difference

\[
P_2 - P_1 = 2.34 \frac{g_c}{g_c} (\rho_L - \rho_v) \sqrt{\frac{g_c \sigma}{g(\rho_L - \rho_v)}}
\]  

(E-12)
Equating equation (B-12) and (B-9) and solving for the vapor film thickness yields

\[
a = [1.09 \beta \frac{\mu_f k_{V_f} \Delta T}{h_{fg} \rho_f g (\rho_L - \rho_v)} \sqrt{\frac{g c \sigma}{g (\rho_L - \rho_v)}}]^{\frac{1}{4}} \quad (B-13)
\]

Evaluating the constant with experimental results will give the following expression for the film thickness.

\[
a = 2.35 \left[\frac{\mu_f k_{V_f} \Delta T}{h_{fg} \rho_f g (\rho_L - \rho_v)} \sqrt{\frac{g c \sigma}{g (\rho_L - \rho_v)}}\right]^{\frac{1}{4}} \quad (B-14)
\]

The heat transfer coefficient may be defined by using the following equation.

\[
\dot{q} = \frac{k_{V_f} A \Delta T}{a} = h_{film} A \Delta T \quad (B-15)
\]

If equation (B-15) is combined with equation (B-14) and then solved for the film coefficient, the film coefficient will be equal to

\[
h_{film} = 0.425 \left[\frac{k^3_{V_f} h_{fg} \rho_f g (\rho_L - \rho_v)}{\mu_f \Delta T \sqrt{\frac{g c \sigma}{g (\rho_L - \rho_v)}}}\right]^{\frac{3}{4}} \quad (B-16)
\]

This is an expression for the film coefficient at the minimum film boiling point.
Berenson also developed the following equation for the minimum heat flux at the minimum film boiling point

\[ q_{11} = 0.09 \rho V_f h_{fg} [g(\rho_L - \rho_v) \frac{1}{2} \frac{g_c \sigma}{g(\rho_L - \rho_v)}]^{1/4} \] (B-17)

Now combining equations (B-17), (B-16) and (B-1), the expression for the surface temperature at the minimum film boiling point is obtained.

\[ T_{min,I} = T_{sat} + 0.127 \frac{\rho V_f h_{fg}}{k_v} \frac{g(\rho_L - \rho_v)}{\rho_L + \rho_v} \frac{2}{3} \frac{g_c \sigma}{g(\rho_L - \rho_v)} \frac{1}{k_v} \frac{\nu_f}{\rho_L} \] (B-18)

It is important to note that Berenson did not account for any liquid solid contact at the minimum film boiling point and he assumed that the surface was, therefore, isothermal.

This actually is not valid at temperatures near the minimum film boiling point where there is a substantial amount of liquid solid contact occurring. Henry, as shown in Chapter V, accounted for this occurrence in terms of Berenson's surface temperature.
REFERENCES


