Agent-Based and System Dynamics Hybrid Modeling and Simulation Approach Using Systems Modeling Language

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AGENT-BASED AND SYSTEM DYNAMICS HYBRID MODELING AND SIMULATION APPROACH USING SYSTEMS MODELING LANGUAGE

by

ASLI SOYLER AKBAS

B.S. Systems Engineering, Yeditepe University, 2006
M.S. Industrial Engineering and Management Systems, University of Central Florida, 2011
M.S. Modeling and Simulation, University of Central Florida, 2013

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Modeling and Simulation in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida

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Major Professor:
Waldemar Karwowski
ABSTRACT

Agent-based (AB) and system dynamics (SD) modeling and simulation techniques have been studied and used by various research fields. After the new hybrid modeling field emerged, the combination of these techniques started getting attention in the late 1990’s. Applications of using agent-based (AB) and system dynamics (SD) hybrid models for simulating systems have been demonstrated in the literature. However, majority of the work on the domain includes system specific approaches where the models from two techniques are integrated after being independently developed. Existing work on creating an implicit and universal approach is limited to conceptual modeling and structure design.

This dissertation proposes an approach for generating AB-SD hybrid models of systems by using Systems Modeling Language (SysML) which can be simulated without exporting to another software platform. Although the approach is demonstrated using IBM’s Rational Rhapsody® it is applicable to all other SysML platforms. Furthermore, it does not require prior knowledge on agent-based or system dynamics modeling and simulation techniques and limits the use of any programming languages through the use of SysML diagram tools. The iterative modeling approach allows two-step validations, allows establishing a two-way dynamic communication between AB and SD variables and develops independent behavior models that can be reused in representing different systems. The proposed approach is demonstrated using a hypothetical population, movie theater and a real–world training management scenarios. In this setting, the work provides methods for independent behavior and system structure modeling. Finally, provides behavior models for probabilistic behavior modeling and time synchronization.
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CHAPTER ONE: INTRODUCTION

Even though the benefits to integrating the agent-based (AB) and system dynamics (SD) modeling techniques are recognized in literature, the current body of knowledge lacks research on studies focusing on common approaches in methodologies. Furthermore, the issues that arise from their integration are evaluated using existing simulation platforms from each individual research domain. However, utilizing a new external platform, such as Systems Modeling Language (SysML) – that has been found beneficial for both discrete and continuous modeling techniques separately – has recently been evaluated under this research effort. This dissertation describes contributions to the field of AB-SD hybrid modeling and simulation technique. It describes an approach and demonstrates its potential applications in population dynamics modeling and project management using hypothetical and real-life scenarios, respectively. It uses Systems Modeling Language (SysML) for modeling and simulating multi-method simulation model development on a software platform Rational Rhapsody® by IBM which can also be implemented on any other SysML platforms.

Research Background

Agent-based (AB) and system dynamics (SD) modeling techniques have separately been considered among effective modeling methods in literature. However, their combination can be
considered the least studied among published literature on hybrid models. The majority of the reviewed work from this domain includes examples and methods of two techniques being modeled separately, as sub-models of each other. The two models would later be combined to simulate the conditions of the dominant technique – as a dependent component – driven from its sub-model’s behavior – as an independent component. In studies using this structure, the dynamic information exchange is often one-way – from the technique with independent behavior to the dependent one.

Unified Modeling Language (UML) representations are considered a common practice, particularly in computer science, for AB, SD and AB-SD simulations. However, limited work has been published on using its extension, i.e., Systems Modeling Language (SysML). According to the existing literature two distinct groups of practices emerged. While some researchers and practitioners still prefer UML diagrams for conceptual modeling, some studies from system sciences has captured systems using SysML. However, use of SysML is limited to conceptual modeling. In the second group, SysML is evaluated as a platform for modeling systems which could later be exported to external statistical simulation tools such as Matlab or Modelica. Limited studies are published on the topic and only one modeling technique was used for exploratory purposes.
Problem Statement

The literature review revealed an ongoing argument on AB-SD hybrid modeling technique. Some studies in literature advocate potential benefits that can be achieved through the integration of the two techniques, whereas some describe the issues arising from their differences of most basic modeling notions. Time and event synchronizations, continuous versus discrete behaviors, top-bottom versus bottom-up approaches are among examples of these issues. The majority of the existing literature on the topic consists of one school evaluating the other’s performance as an alternative modeling approach using the same or a similar case. Furthermore, existing knowledge on AB-SD modeling methodology has provided case specific approaches rather than a generalized methodology.

The need for identifying a common platform and a universal approach for AB-SD hybrid modeling and simulation has often been mentioned. However, existing literature is limited to studies using approaches where the two techniques are integrated after independently being modeled. Furthermore, AB-SD hybrid modeling and simulation within an external platform to both domain applications, such as SysML, has not been evaluated. Finally, potential benefits of an approach adapting model-based systems engineering (MBSE) methodologies for managing complexity and changes has not been researched.
Objective

As a result, the main objective of this dissertation is to develop an approach for agent-based and system dynamics hybrid modeling and simulation using Systems Modeling Language (SysML) to be used for understanding and studying system’s emerging behavior over time.

Contributions

This dissertation demonstrates an approach, which implicitly develops and simulates an AB-SD hybrid model of a system without requiring any prior knowledge on either modeling techniques. It uses SysML diagrams and objects to minimize the use of programming languages and adapts model-based systems engineering (MBSE) methodologies to create a holistic approach that can be applied to different domains or fields.

The approach starts from the problem identification phase of modeling and simulation methodology. Conducts input analysis through requirement analysis and distributes findings in multi dimensions. Specifically, in the proposed approach first, problem scope and boundaries, system limitations and expected behavior are analyzed. Second, gathered knowledge is used to identify physical components of the system. Finally derived behavior is merged and distributed over the physical components of the system. This methodology allows establishing a two-way dynamic continuous link between AB and SD mathematical models. Adapted MBSE approach
provides a top-bottom modeling approach that is the basic notion for SD modeling. The bottom-up approach required for ABM is captured through the proposed process flow in behavior analysis phase. Furthermore two step validation approaches recommended by both AB and SD modeling techniques are supported by individual behavior validations in behavior analysis and overall model validations after structure analysis. SysML provides the external platform where the two techniques are combined, which is found beneficial in literature in supporting AB and SD modeling efforts separately.

In addition to its contribution to AB-SD hybrid modeling, the proposed approach also provides methods that can be adapted by general modeling concepts. Specifically, through modularized behavior analysis, it allows changing, verifying and validating behavior independently. Furthermore, this allows modeling generic behavior rather than developing case-specific applications. As a result, it provides modeled behavior that can be re-used and customized for different applications. Overall the proposed methodology will:

- Provide a generalized AB-SD modeling and simulation framework
- Extend the MBSE approach for systems modeling using hybrid simulation platforms
- Propose an approach for modeling reusable behavior
- Provide alternative hybrid system architectures
- Develop case studies to demonstrate potential applications
Proposed approach provides an output from distributed behavior composed of previously analyzed and integrated inputs. Output from the simulated systems will aid stakeholders in understanding behavioral and structural dependencies and impact of decisions or external events. Thus, in overall the results collected through this approach will;

- Support stakeholders by providing the capability to run strategic what-if scenarios
- Support system analysis efforts through long term dynamic behavior analysis
- Identify factors that has the highest impact on the behavior caused by direct and/or indirect relations

Document Outline

This dissertation starts with a brief introduction on the topic and outlines the findings from literature review on each related field.

Later describes the Methodology in four main phases, requirements analysis, behavior analysis, structure analysis and validation and verification, which are further grouped according to the common phases used in MBSE approach.

In Methodology Verification, this dissertation provides an approach for modeling probabilistic behavior in SysML and compares the outputs with results collected from another simulation
platform AnyLogic. In addition, proposes an approach for managing time synchronization issues arising from AB and SD integration and verifies the overall approach by testing the significance of correlation and autocorrelation between independently-modeled agents using a hypothetical movie theater system.

The proposed approach first is demonstrated using a hypothetical giraffe population observation system for modeling and simulating population dynamics which is a common application area in both modeling techniques. Second, applies the approach on a real-world case study for training management. Through this case study this section demonstrates how the behavior is derived and distributed over the two system components, employee and organization. It shows the verified and validated overall model of the training management system and uses the model to study the change in count of people waiting for training over a four year period.

Finally, in Conclusion, contributions of the proposed methodology and possible extensions for future work are discussed.
CHAPTER TWO: RELATED WORK

The literature review starts with a brief description on agent-based (AB), system dynamics (SD) and AB-SD combined simulation techniques. Later, model based system engineering approaches that can be applied to modeling for simulation and existing literature on applications using Systems Modeling Language (SysML) are reviewed.

System Dynamics Modeling and Simulation

System dynamics (SD) is a technique to present, understand and explain complex problems (Radzicki et al., 2008). A critical factor in a system dynamics model is the identification of its objective (Forrester, 1987). It is efficient in modeling complex systems since it is based on nonlinear dynamics and feedback control. SD has diverse application areas such as transportation (Haghani, Lee, & Byun, 2003), healthcare (Homer & Hirsch, 2006), project management (Sterman, 1992) and so on.

SD utilizes human behavior by incorporating social psychology, organization theory and economics (Sterman, 2001). Models created by system dynamics are generalizable and enable the processing and analysis of graphically depicted data. These properties make system dynamics attractive for organizational models (Popova and Sharpanskykh, 2010). For example, SD was
shown to support identifying the gap between organizations and individuals learning and later used this understanding in reducing fragmented learning (Romme and Dillen, 1997, Dangelico et al. (2010)). The model analyses the district evolution according to a multiple dimensions such as institutional, economical, and social issues. Van Olmen et al. (2012) introduce a framework for health systems research, which can be used in two different applications of health systems. Schwaninger and Rios (2008) use system dynamics with viable system model for modeling organizational cybernetics. The main goal of the model is increasing the capabilities of the users in dealing with challenging issues in organization and society. Robbins (2005) proposes a system dynamics model with interdependent parameters as a support tool for decision-makers in nation building to investigate different sets of decision approaches at a regional level.

Different approaches in SD modeling have been suggested in literature. For example, Coyle (2001a) suggests using five stage approach where Towill (1993) further separates them in to nine stages. However, a common approach in all is the iterative nature of the overall process. Compared to methodology approaches, validation techniques in SD modeling is not a common topic in the domain (Barlas, 1996). Although this is in some ways contradicted by Sterman (1992), there is a gap in provided validation techniques that are specifically customized for SD. Barlas (1996) suggests a two-phase validation approach, where structure-oriented behavior and resulting behavior patterns are validated separately.

Overall, the principles of system dynamics modeling, such as the ability to study the effects of individual variables and their interactions, provide a pragmatic and holistic nature (Romme and
Dillen, 1997) that is found useful in modeling humans as social systems that are characterized by “dynamic complexity” (Senge, 1990).

**Agent Based Modeling and Simulation**

Agent-based modeling and simulation (ABMS) is an approach for modeling complex systems composed of autonomous actors, interactions of actors, the environment in which these actors interact and the rules defining the interactions (Macal, 2010). Actors in ABMS are named as ‘agents’. Agents are autonomous and they interact with each other according to the protocols defining their behaviors (Bandini, 2012). These protocols generally consist of simple rules. However, the combination of agents and their interactions creates a complex structure, which is used to understand the behavior of systems under various conditions. Therefore, ABMS is applicable to complex models, where traditional modeling tools are generally not sufficient (Macal, 2010). ABMS also incorporates features using advances in computational power and data storage capabilities. These technological improvements enable enhancements in modeling the complexity designed through ABMS by bridging macro and micro levels of a system (Macy and Willer, 2002).

ABMS is an active research area with numerous applications, such as organizations (Bonabeau, 2002, Van Dam et al., 2007), economics (Charania et al., 2006), epidemics (Carley et al., 2006), social systems modeling (Kohler and Gummerman, 2001), influence (Marsell et al., 2003) and so
on. One of the emerging concepts in ABMS research is organizational management and human behavior modeling. Rojas-Villafane (2010) use ABMS to create a model named Team Coordination Model (TCM), which estimates the performance of a team according to its composition, coordination mechanisms and characteristics of the job. The rules defining the behaviors of agents in TCM are individual team design factors and the overall performance of the model is validated by comparisons against real team statistics. As hierarchical structures are increasingly adopted by organizations and most of the activities are automatized, ABMS can be used to model organizations efficiently. Montealegre Vazquez and López (2007) develop a model for open hierarchical organizations, in which each member of the organization is modeled as an agent and the norms are used to define the behavior of agents. The organizational culture model by Harrison and Carrol (2006) also models the members of the organization as agents. In this model, interactions of the agents are modeled as social influences and the observed organizational property of the model is the cultural heterogeneity in the organization. Rivkin and Siggelkow (2003) use ABMS to model the decision behavior of top management agents in an organization. They observe properties of vertical hierarchy in organizations and identify circumstances in which vertical hierarchies may lead to inferior long-term performance.

ABMS uses agent-oriented approach rather than process oriented, which is not common to most simulation approaches (Macal & North, 2010). Although majority of the literature agrees on the high-level modeling phases, a common modeling technique that could represent different types of applications has not yet been identified (Gilbert & Bankes, 2002). The limited work on design concept standardization and protocols has been identified as an issue in very recent studies.
(Collins, Petty, Vernon-Bido, & Sherfey, 2015). A commonality in all reviewed literature is the ground-up approach (e.g., Masad & Kazil, 2015, Macal & North, 2007), which starts with simplest agent and extends it according to problem description.

**AB-SD Hybrid Models**

Availability of data and improvements in computational power has increased the use of simulation in various fields in academia and government industry. This trend is also observed in hybrid simulation platforms, especially in the area of manufacturing (e.g., Jahangirian, Eldabi, Naseer, Stergioulas, & Young, 2010). System dynamics (SD) and discrete event simulation (DES) combinations consists the majority of the published research. However, agent based modeling and simulation (ABMS) and SD combinations are found less researched and understood (Swinerd & McNaught, 2012) even though each separately are considered to be among the most important methods (Lättilä, Hilletofth, & Lin, 2010). Scholl (2001) points out this gap in literature, and discusses potential benefits of their combinations to the common applied research fields.

In addition to techniques used in hybrid modeling, one can also find commonalities in individual AB and SD modeling methods. For example, Coyle (2001b) describes a method for SD modeling which starts by identifying system actors and their possible states. Later, he continues by identifying rules and conditions for state transitions. However, the basic notion in their approach can be categorized as to be completely opposite of one another. Where ABM uses ground-up
approach SD is modeled using top-down notion (Macal & North, 2007). Among the first to be published in the domain, Phelan (1999) identifies three core differences between the two modeling techniques as their agenda, technique basis and epistemology. However, more differences have been argued by researchers in later years (Pourdehnad, Maani, & Sedehi, 2002 and Figueredo & Aickelin, 2011). Conceptual models are commonly used for identifying scope, interactions and behavioral dependencies of systems in literature (e.g., Gilli, Mustapha, Frayret, Lahrichi, & Karimi, 2014 and Größler, Stotz, & Schieritz, 2003). Furthermore, Unified Modeling Language (UML) is often used to represent agent states in studies from computer science fields (e.g., Borshchev & Filippov, 2004). Existing literature include studies that are in its early design phases (Gilli et al., 2014) or providing result from exploratory applications (e.g., Akkermans, 2001).

**Model Based Systems Engineering (MBSE) Approach**

Model-based design has been identified as an approach that can aid in issues arising from human-system interaction (Sage and Rouse, 2009). There is not a standardized methodology for MBSE approach (Ramos, Ferreira, & Barcelo, 2012); however, the majority of the well-known MBSE approaches utilize the Vee-Model (Figure 1) (Sellgren, Törngren, Malvius, & Biehl, 2009) and extend it according to their domain. Harmony SE is one these approaches (Hoffmann, 2014) where, prior to modeling, behavior is decomposed and modeled individually according to requirements and later after architectural design phase, are allocated to the responsible parts of
the system. Potential benefits from adapting MBSE is pointed out by a questionnaire conducted by Pastrana (2014) where later, a roadmap is suggested for designing conceptual models of distributed and hybrid simulation systems.

![Vee-Model INCOSE, 2011](image)

Figure 1 Vee- Model (INCOSE, 2011)

**Systems Modeling Language (SysML)**

The holistic approach required in modeling complex systems are supported by four key modeling facets, called pillars including nine diagrams, that consist of requirements, behavior, structure and parametric relationships (Ramos et al. 2012). Figure 2 captures the representation of diagrams published by Object Management Group (OMG) included in each pillar (Hause, 2006). Although Package and Use-Case diagrams are not included in this representation they are also considered a part of structure and behavior pillars, respectively.
Modeling and Simulation with SysML

Recent capabilities introduced by IBM’s Rational Rhapsody provides a platform for modeling continues dynamics using SysML. According to Euler’s method (Huntsville, 2014) one can solve a differential equation by approximating its solution at a discrete sub-division, referred to as steps, of a continuous time interval. This can be expressed as:
\[ f(P) = \frac{dP}{dt} \approx \frac{\Delta P}{\Delta t} = f(P_n) \]

Furthermore, this approximation is used to approximate the change, and hence, predict the future value of continuous function \( P \) from its initial or current value. The discrete equation is expressed as:

\[ P_{n+1} = P_n + \Delta t f(P_n), \]

where \( n \) is the computation count and \( t \) is the time step. Johnson et al. (2007) propose a methodology using Modelica internal behavior equations to create relationships among components where they represented algebraic equations with conditional logic, which add capability to add stakeholder requirements to system behavior (Johnson et al., 2011). McGinnis and Ustun (2009) demonstrate method for linking SysML with a simple discrete simulation model using Arena where they create a simulation from its conceptual model.

Among reviewed literature, the three most common diagrams used to capture behavior are, Parametric Diagrams (ParD) (T. Johnson, Paredis, & Burkhart, 2011 and T. A. Johnson, Jobe, Paredis, & Burkhart, 2007), Sequence Diagrams (SeqD) (David, Idasiak, & Kratz, 2010) and Statechart Diagrams (STM) (Silhavy, Silhavy, & Prokopova, 2011). In studies using ParD, equations are added as parametric constraint blocks with a composition relation to the owner block. This is consistent to composition relation between the agents and their behavior suggested by Bersini (2012). Furthermore, when used, SeqD and STM are added to the owning block. The main
commonality among these studies is that the behaviors are created after the structure analysis phase.

Majority of the proposed designs in literature-focusing on architectural design for different types of simulation- revealed two distinct perspectives: proposing a design of the actual system and of the conceptual model for the actual system’s simulation model. Studies from the first group, such as the block definition diagram (BDD) suggested by Johnson et al. (2011), decompose the system according to the actual components of the system. This is also common to studies suggesting a multi–level approach for modeling hybrid models (Basole & Bodner, 2015). The decomposition approach in studies belonging to the second group is based on the components of the model, which is similar to approach used in software development. For example, Swinerd & McNaught (2012) propose three design structures for SD-ABM models, which are decomposed according to SD and ABM parts of the system. There are few studies that captured both perspectives such as the mapping of domain and analysis meta-models proposed by Huang, Ramamurthy, & Mcginnis (2007). Additional to SysML, studies using Unified Modeling Language (UML) (such as Bersini, 2012), are also reviewed to capture alternative proposals for developing a universal ontology.

Existing research on single type models showed SysML being used either to support conceptual model development, similar to UML (Silhavy et al., 2011), or as foundation for models that could be exported to other simulation software such as Modelica (Johnson, Jobe, Paredis, & Burkhart, 2007) or Arena (Mcginnis & Ustun, 2009).
Even though there is an increasing interest in literature, where SysML is used to support modeling efforts, a gap exists in the domain, which adapts MBSE methodologies for modeling and simulating systems within SysML. Furthermore, an approach which implicitly drives an agent-based and system dynamics hybrid model of a system has not been provided. The few studies published on agent-based and system dynamics hybrid modeling and simulation domain use SysML to design the architectural components of a system’s model.
CHAPTER THREE: METHODOLOGY

Commonly used agent-based (AB) and system dynamics (SD) modeling techniques and alternative workflow suggestions are summarized in Chapter 2. Even though each separately is considered to be effective methods (Lätilä et al., 2010), there is very little research on agent-based and system dynamics (AB-SD) combinations. Furthermore, majority of the work focuses on model conceptualization and formulation and does not provide an approach that can consistently be used all throughout the modeling and simulation workflow.

Computing power advancements paired with large amount of data collected over the years significantly increase AB-SD modeling and simulation capabilities. However, these advancements also increase the intricacy and the scale of modeled environments and introduce three core challenges. First, high complexity is difficult to be included using the ground-up approach. Second, the involvement of stakeholders-from various fields and backgrounds-introduces additional needs and expectations, each facing unavoidable changes due to shifts in environmental conditions. Finally, the need to maintain the coherency and efficiency of validated models through structural or behavioral change requests that arise from emerging variables, constraints or states. This research proposes an approach for modeling and maintaining AB-SD hybrid models of systems using Systems Modeling Language (SysML).
This section describes the methodology in four main phases. As shown in Figure 3, it starts with requirements analysis and is followed by behavioral and structural design. Finally, it explains the methods for validation and verification.

A generic package diagram is created to capture this relation between the behavior and the responsible part of the environment in Figure 4. The two packages, Pkg Structure and Behavior Analysis, represent the high-level folders in the SysML project tree. An Agent block captured under Pkg Structure Analysis is used to represent the physical entity which is a part of the environment with a specific behavior that is defined under Pkg Behavior Analysis.
For simplicity, only high-level, potential components were used where both behavior blocks were allocated to a single part of the model. However, since the level of behavioral complexity and the associated structure is unique to each environment under study, a component may be responsible for more than one behavior. With the same token, more than one component may be involved in executing one behavior. This is further discussed in Structure Analysis section of this chapter.

Through the remainder of this dissertation the word agent is used to describe all environment members which or who are simulated using agent-based simulation technique. Furthermore, the word actor is used as a specific role to describe persons or systems who are external to system under development (Ramos et al., 2012). Finally, the word location is used to describe the area where agents and/or actors exist.
Requirements Analysis

Grouping similar requirements is a common approach both in academia and private industry (Friedenthal, Moore, & Steiner, 2009). Method uses five main groups for capturing the identified capabilities and conditions expected from the model. The first two of five can be classified as system-driven. These two groups include behavioral and structural requirements of the system.

The third and fourth groups can be classified as program-driven. Third group consists of translation rules that are used for building the designed model in the selected simulation environment or language. If the modeler is using the same two software consistently and neither has gone through any significant updates, no change in the specifications is expected and therefore can be imported for all new model designs. The fourth group captures model validation and verification test specifications and includes a list of the variables and their expected values that will be used within statistical tests. The final group can be classified as customer-driven. It is used to list the variables, values of which must be collected for output analysis.

Different methodologies used in requirements analysis and management are not covered within the scope of this dissertation. Further reading on the topic can be found in most SysML and MBSE books (e.g. Weilkiens, 2006 and Friedenthal et al., 2009).
Define Behavior

The developed process flow for behavior analysis can be grouped in seven phases as captured in Figure 5. Phase 1 starts with Use Case Diagram (UCD) design, and is followed by the next phase where each behavioral requirement is linked to associated use case(s). In the third phase, activities, involved per each use case, are mapped using the Activity Diagram (ActD). Then, the interaction between environment components and between actors and environment components are generated using Sequence Diagram (SeqD). Ports and interfaces are created in the fifth phase in order to establish the connection for message exchange between all members. In the sixth phase, initial Statechart Diagrams (STMs) are created and finally the model is compiled for behavior verification. The following sub-sections of behavior analysis follow the order of phases captured in Figure 5.

Figure 5 Behavior Analysis
Create Use Case Diagrams (UCD)

UCD is used to identify environment boundaries, scope, and model behavior and any internal and external interactions defined within the project scope. A flow chart is developed for creating the UCD. First the modeler identifies actors, their relation with the system and the types of their behavior, referred to as functions. Later, similar actions are repeated to identify the emphasis, and impact of location conditions and events if any are included within the environment boundaries.

The process starts by adding all members of environment, which are involved in, have impact on or simply observe outcomes. These can include stakeholders, external systems, agents and even locations other than the one considered within the focus. Later, by iterating a series of decisions, the modeler identifies the actors’ relations to the modeled system and their time or SD driven behaviors. Agents who are identified as a part of the environment are not added to UCD as actors. However, their behaviors are added as functions within the system boundary box. Later in section Structure Analysis, these are added as a part of the environment and designed behaviors are allocated to each responsible party.

Location of agents may play an important role in the design depending on the type of environment scenario. For example, studies focusing on influenza outbreak (eg. Lukens et al., 2014) often derive contact rate from the distance between agents. In such cases, location of each agent is considered as a factor impacting experiment results and therefore may be included in
UCD. After completing the process for an actor, modeler goes back to beginning and repeats the decisions with the new actor selected. UCD is completed once all the actors, functions and their associations are linked. The activity flow capturing this description is represented in Figure 6.

There is not a specific order suggested for actor selection. However, leaving the actors who are the focus of interest, to the last is recommended. This may help modelers to clarify environment boundaries and some of the assumptions prior to decisions requiring more details.

**Link Requirements to UCD’s**

The specific relation type between identified requirements and the model elements can be added manually or using a matrix view. In this phase, a generic relation “trace” can be used to map the use cases to the corresponding requirements. UCD can be used for visual verification to confirm that all required behaviors have been captured. Furthermore, it can be used as a map to add “satisfy” relation to the corresponding behavior block created from identified use cases. Multiple matrix views focusing on specific behavior or part can be created to simplify table contents when modeling complex systems.
Figure 6 Use Case Diagram Development Process
Create Activity Diagrams (ActD)

The activity diagram is used to capture the sequence of actions that needs to be executed in order to satisfy the goal defined by a use case (Weilkiens, 2006). The path of sequence execution is represented using control or object flows depending on the type of information necessary for executing an activity. If a system consists of activities common to more than one use case, they can be designed either explicitly as an operation or in groups as behaviors. Furthermore, an activity can be an action state or a message. Although multiple actions can be represented as embedded code within a single activity, it is not recommended. This method would not simplify the modeling of system behavior complexity, therefore would eliminate the benefits that can be achieved using MBSE approach.

Developed flow (Figure 7) starts by adding the actions of the selected use case and placing them in the diagram in a sequential order. A decision, fork and join nodes are later added if necessary to represent conditional reactions of the system. In the third step, the variables, which will be used either at the decision nodes or within actions, are added to the associated behavior block. Common variables must be added only once and to the responsible behavior block. For example, simulation time variable would only be added to the update time behavior block. Later during structural design these common variables will be allocated to all parts of the system. A star is added to this step to indicate that it is optional. The modeler can also use the sequence diagrams to identify variables and add them to the associated behavior block. Remaining steps focus on capturing internal and external message exchange.
First actions belonging to actors, who or which are external to the system scope and trigger a behavior sequence, are added as messages. From system’s perspective, these are incoming messages from an external source, therefore are represented using an inwards direction at the actor pin. These steps are not performed if there are any actions that are waiting for an action to
be completed by a different behavior block within the system. The waited actions are captured only at the ActD of the behavior block responsible of performing the action. Therefore, when modeling systems with complex behavior, activity diagrams must be created simultaneously. Instead of waiting to complete one ActD, when identified, the required action can be added as a message to the ActD of the responsible behavior block. Last group of steps focuses on identifying and adding such actions as messages. Process flow of the described method is captured in Figure 7.

Required ActDs such as “update_time” or “update_dynamics” can be used to start the modeling in this phase. If this is the first time this methodology is being used, modeler would create them manually and save the project. If not, a previously saved project with only the two use cases and their behavior blocks, can be imported using the “Add to model” menu option in Rhapsody (IBM, 2014).

The ActD for “update_time” behavior consists of one action, “increment_clock”. Furthermore, it is responsible of starting the overall system execution and updating the internal clock. As a result it consists of two message actions and one action with embedded code that will increment the clock (Figure 8). A variable named “Tnow” is added to the block representing the time of the simulation in days.
The second ActD created or imported satisfies the “update_dynamics” use case behavior. This is the behavior that is used to model the system dynamics parts of the model. Hence, it consists of an action named “update_dynamics” that will be executed after receiving the new time message “send_update”. This has the code embedded for updating variables identified as stock and dynamic. The second action has the code for updating rates per time increment measure $t$ (e.g. weeks, days) after receiving the corresponding messages from those behavior blocks.

**Generate Sequence Diagrams (SeqD)**

Harmony Profile allows automated generation of sequence diagrams (SeqD) from created ActDs, including operations such as:

- Generate operations from action names
- Create events
• Create interface
• Add corresponding operation and event realizations (Hoffmann, 2014).

One or more SeqDs can be created for a behavior block. However, to maintain modularity at least one SeqD per behavior block should be created. If Harmony profile is not used for SeqD generations, each listed operation has to be completed manually. Later in the SeqD operations and events should be assigned to message and event tools, simultaneously, as realizations. Only the messages exchanged between the system and actors are shown in initial SeqDs since these are created from the black-box activity diagrams. Internal messages are added to the SeqD after the actions are allocated to the responsible system parts during architectural design phase. Depending on the level of detail required, the behavior and conditional rules can be planned using SeqD. Although this is not required, it would lay the grounds for mapping the rules for state-based behavior and support designing efforts. Rhapsody diagram tools can be used to add conditions and logic for operation sequence. All types of operator based interactions added to SeqD are only added as a visual guidance and are not included in the compiled simulation execution file (IBM, 2014).

Create Ports and Interfaces

Similar to SeqD generation, ports and interfaces can be created automatically using the Harmony toolkit. This option will move all external events to corresponding interfaces and add receptions
to the receiving party. Finally, it will add parts of the behavior block and interacting actors to capture their communication using an Internal Block Diagram (IBD). Each behavior block created up until this phase will have its own IBD. The main purpose is to identify the specific behavior block, where the overall system is required to interact with an actor in the environment surrounding itself.

**Define States**

In behavioral design phase decomposed blocks are treated individually. Therefore one state diagram is created for each behavior block. The states and transition conditions are added according to the logic identified in SeqDs. The modeler can embed the code for operations during any state after SeqD design. However, all remaining code should be embedded during state definition. In order to maintain modularity, elements from the Rhapsody toolbar should be used rather than embedding complex conditions or loops within one operation.

“UC_update_time” is designed to be used for representing the internal clock of the system. As a result, it is set to be incremented once per day continuously. However, for simulations that are time bounded, an end state can be added using a conditional trigger for the final transition. As captured in Figure 9, only one of the operations defined in Figure 8 Update Time is used at this step. Any internal messages such as “sim_start” or “send_update” are added after the system is decomposed to its parts during structure analysis phase.
After establishing system clock, the simulation time units for continuous variables are modeled according to user preference. The graphical representation for the population net flow can be shown as in Figure 10.

Then the equation for a population at time $t$ using 1 week increments can be expressed as:

$$ \text{population}_{(t+1)} = \text{population}_{(t)} + birth\_rate_t - death\_rate_t $$  

(3.1)

In system dynamics, birth and death rates of a population are assumed to be proportional to the population (Cellier, 1991). This relation is captured using a feedback from the population to corresponding rates as shown in Figure 11.
Therefore, the population equation, where $BR$ and $DR$ represent the birth and death rate proportions at time $t$, is used as:

$$\text{population}_{(t+1)} = \text{population}_{(t)} + (BR_{(t)} \times \text{population}_{(t)}) - (DR_{(t)} \times \text{population}_{(t)})$$  \hspace{1cm} (3.2)

And therefore:

$$\text{population}_{(t+1)} = \text{population}_{(t)} + ((BR_{(t)} - DR_{(t)}) \times \text{population}_{(t)})$$  \hspace{1cm} (3.3)

Even though in Eq. (3.3) the two rate proportions are represented as dynamic variables, they can also be assumed as constant over time for the focused population type when there is a lack of contradicting evidence.

On the other hand, AB-SD hybrid modeling technique can be used to derive these rates from the simulated agent behavior, allowing the modeler to eliminate the proportion estimations and any associated errors. As a result, Eq. (3.1) must be used in operations when modeling stock variables.
that depend on agent behavior, such as “update_population()”. In order to maintain validity after this elimination, the modeler is required to provide more detailed information about the population at time 0 compared to SD modeling.

**Behavior Verification**

Similar to previous phases, the verification of decomposed behavior is done individually. First, developed model is compiled using simulated time in MSVC environment with C++ language and any possible issues are fixed. Later the program is executed and the individual behavior of each block is observed using simulated statecharts and sequence diagrams (IBM, 2014). As the final step, properties of all variables are checked for any errors.

Overall, the purpose of behavior analysis can be summarized as following:

- Identify system requirements
- Identify system scope and boundaries
- Identify the modularized actions and reactions of the system to the external triggers
- Identify its interaction with the surrounding environment and conditions
- Derive resulting behavior from findings gathered above for verification
Define Structure

The process flow for structure analysis is grouped in three phases that are system decomposition, behavior allocation and verification and validation as captured in Figure 12. Behavior allocation is further completed in four sub-phases where names have been kept the same on purpose to point out the shared diagrams between the two analyses.

Figure 12 Structure Analysis

Create Block Definition Diagrams

In literature review, the two approaches used in system decomposition for system modeling were discussed. During initial research efforts the selected system was decomposed according to its conceptual model parts. (Soyler Akbas, Mykoniotis, Angelopoulou, & Karwowski, 2014). Hence, the training system was decomposed as Agent-Based Model and System Dynamics Model (Figure 13). However as SD and ABM parts were further decomposed; behavior allocation and
maintenance became more complex. Furthermore, overall model design became too customized for providing quick changes to significant behavioral adaptations and for the capability to export specific behavior to be used in other systems.

![Initial Block Definition Diagram of a Training System](image)

**Figure 13 Initial Block Definition Diagram of a Training System**

Main focus of the modeling effort must be used to identify the best approach for system decomposition. If the goal is to study the behavior of a system itself, using a SD-ABM simulation technique rather than conceptualizing its model, the system must be decomposed according to its physical components. The main goal of this research is proposing a methodology for modeling system behavior over a time period. Therefore, this work discusses and showcases systems that are decomposed into its physical components. Three high level simple system structures are created to guide component identification. They are grouped according to the differences in main focus and information exchange between its components (Table 1).
### Table 1 System Decomposition Types

<table>
<thead>
<tr>
<th>Decomposition Type</th>
<th>Information Type</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>One Way</strong></td>
<td></td>
</tr>
</tbody>
</table>
|                    | Agents to Location | - Main system focus is the location  
|                    |                  | - Common location shared by all agents  
|                    |                  | - Changes in environment do not impact agent behavior  
|                    |                  | - Changes in agent behavior impact the location  |
|                    | **One Way**      |             |
|                    | Location to Agents | - Main system focus is agents  
|                    |                  | - Unique location per agent  
|                    |                  | - Changes in location impact agent behavior  
|                    |                  | - Changes in agent behavior do not impact their location  |
|                    | **Two Way**      |             |
|                    | Agents to Location & Location to Agents | - System focus is both  
|                    |                  | - Common location shared by all agents  
|                    |                  | - Changes in environment impact agent behavior  
|                    |                  | - Changes in agent behavior impact their environment  |
The “Agent” is used to represent unique objects, people, locations, which can be grouped under one goal. Similarly, “Location” represents a physical or conceptual location common or unique to agents. Both can include SD models. With the same token, both or sub-parts of both can be modeled as agents in AB models. This is further explained in the following section under each category.

Decomposition Type I

This type consists of models focusing on locational factors changing due to agent behavior independent of the location. Both location and agent can represent more than one unique part of the system. However, this layout assumes no interaction between individuals existing in different locations. The method provides the use of this structure only if there is a possible scope change in the future to include agents within the focus or they share conditions that impact both of their behavior in the environment over time. If not, Agents must be represented as actors under UCD, as externals only impacting the system. The farmers’ impact on ecological carbon and nitrogen stock model introduced by Gaube et al., 2009 is an example of this type. In this study one can see the impact of farmers’ work on the flows however the impact of nitrogen and carbon on an individual farmer is negligibly small and therefore not included.
**Decomposition Type II**

Type II can be used when the system focus is completely opposite to described in Type I. Hence, must be used when simulating systems where the change in an agent is driven by the changes in its location or locations. This design assumes each location is unique to an agent therefore, the system focus does not include location based interactions between agents. Simple supply-chain models can be given as examples of this type. Manufacturers’ decision making process at a micro level driven from the status of the raw material in their area of service can be modeled using this structure.

**Decomposition Type III**

In systems that require two way dependencies between its agent(s) and location(s) the model must be structured with parallel hierarchy using type III. This structure can allow actors to share the existing location conditions or resources and locations to drive their change based on individual and combined behavior simultaneously. Most of the population studies can be given as examples in this group such as the model proposed by Chaim, 2008. Using this structure, location dependent agents with unique SD or AB behaviors can be modeled at a micro level where their impact on the population can be modeled at a macro level under location.
**Complex Decompositions**

A combination out of the three proposed decomposition types can be used when modeling complex systems. Systems should be studied according to interdependencies among its components and the project scope to find the most suitable combination. For example a supply chain system including buyers, product manufacturers and raw material manufactures shared by all high level manufacturers can be decomposed using two Type II and one Type I decomposition structures as shown in Figure 14. However, if the original scope does not include the impact of factory locations they can be eliminated from the design.

![Complex Supply Chain System Decomposition](image)

This methodology can be useful for long term projects as they can be more open to project scope changes. Such a system model which originally consists of a single type, can be later extended to include micro details.
Allocate Behavior

Modeler can merge behavior designed in the previous section with the main system block after the system is decomposed to its components. This action will copy all operations and attributes into the main system block with a trace relation added, linking it back to the original behavior block. Later, the behavior is allocated to each responsible component using the graphs previously created or duplicated. This is further explained in the following sections. Similar to behavior analysis, the modeler can choose to complete the remaining phases either manually or by using Harmony profile tools.

Create White-Box Activity Diagrams (ActD)

In this phase, first, previously created ActDs are duplicated and renamed as White-box ActDs. Later, a swim lane is added for each system part and operations are placed-by moving- under the responsible block.

Generate Sequence Diagrams (SeqD) and Create Ports and Interfaces

After each behavior is allocated to the responsible part of the system Harmony profile can be used for generating the SeqDs and for creating the ports and event interfaces. This is executed
by following the steps discussed under each corresponding topic of behavior analysis. Differently, in this phase, if any of the operations are modified this action’s impact on the verified behavior cannot be analyzed. Therefore, necessary changes must be applied on the responsible behavior block and all behavior must be re-merged and allocated.

Define States

In this step, a state chart is created for each part of the system. Later, previously modeled states of the behavior blocks are duplicated and placed within each, creating integrated state charts. Organization of states in these integrated statecharts is modeler’s choice. However, “and-states” for parallel behaviors should be used rather than complete integrations. This way, if conditions in one behavior change, the states for that behavior could easily be identified and modified without requiring any changes in the other sections.

Verify and Validate System

Verification of the overall model is done visually, in three steps using simulated SeqD and statecharts. First, events and message sequences are checked to verify the communication between the different parts of the system. Second step focuses on state transitions. In this step, time and rate based and probabilistic triggers are observed that belong to either a single part of
the system or to randomly selected objects of parts whose multiplicities are more than one. In the final step of verification, function executions are checked by observing the change in variable values over time.

Output variables identified for validation during requirements analysis phase are used to conduct statistical output analysis. A hypothesis test, such as difference of means, is used to calculate the significance of difference between the model output and collected data from the real system.
CHAPTER FOUR: METHODOLOGY VERIFICATION

Two challenges were faced when the methodology was applied using IBM’s Rational Rhapsody. This section describes these issues and proposes solutions for overcoming these limitations. It starts with probabilistic behavior under Variability and later continues with time synchronization.

Variability

Rhapsody is not designed as a simulation tool and default C++ package does not come with a predefined math library functions. Therefore a random number following a specific distribution cannot be generated, except for uniform distribution. To eliminate this limitation a “generate_variate” behavior is created for systems which consists of behaviors with defined distributions. This behavior block includes calculations adapted from random variate generation techniques as functions.

For example, the associated variables and operations allocated to a “rate” block and the pseudocode of the algorithm for creating duration based state change that is exponentially distributed with a rate 0.1 per day are presented in Figure 15 and Algorithm 1, respectively.
Algorithm 1 Generate Timeout

1: Date: Day count at timeout
2: Ln[100]: Array storing -ln(i/100) where int i ~ U(0,100)
3: U: Generated integer ~ U(0,100)
4: Uni: Value at Ln[U]
5: While on transition between states
6: if next state has distributed transition then
7: Generate uniform variable U
8: Set Uni Ln[U]
9: Calculate exponential variate Date with rate 0.1
10: end if
Exponential distribution has a CDF that can be invertible. As a result, variates in this example are generated using inverse transformation technique that can be recalculated for different rate values. Additional to inverse transformation technique, functions included within this behavior block also includes convolution and composition methods to support different distribution types.

Two population models, one in AnyLogic and the other in Rhapsody, are created for validation analysis with 50 agents and used to check for evidence of a statistical difference between the two software outputs. Table 8 in the Appendix captures the cumulative arrival transition frequencies recorded per software. These values are also plotted against days (Figure 16).

Figure 16 Cumulative Frequency of Arrival Transitions
In cases of correlated outputs, such as this, mean of differences recorded for $t = 0, 1..., 50$ can be used for testing statistical difference. Where $X_t$ and $Y_t$ represent the outputs recorded on day, $t$, from AnyLogic and Rhapsody, respectively, the test criteria are as follows:

$$H_0: D_t = 0, \text{where } D_t = X_t - Y_t$$

$$H_a: D_t \neq 0$$

$$\bar{D} = \frac{1}{50} \sum_{t=1}^{50} D_t \approx -0.27451$$

$$S_D = \sqrt{\frac{1}{(50 \times 49)} \sum_{t=1}^{50} (D_t - \bar{D})^2} \approx 1.3723$$

$$T_{\text{calculation}} \approx -0.200$$

$$T_{0.025,49} \approx 1.96$$

Since $T_{\text{calculation}}$ falls within $\pm 1.96$ there is not significant evidence supporting a statistical difference between the two outputs from AnyLogic and Rhapsody.
Rhapsody provides two configurations for simulating time (IBM, 2014). The modeler can either use real-time to trigger time-based events or simulated time option, which updates the time, based on event completion using a virtual clock. In models where only one behavior block includes time-triggered events, operations or states, these two preset configurations are very useful in simplifying the implementation process. However, in models where more than one behavior block have time-dependent simultaneous actions, either of the two default configurations result in verification issues during model execution due to asynchronous behavior.

Figure 17 captures the sequence of events in a simple movie theater scenario which was created to demonstrate this issue. The system in the scenario is composed of one part, Movie Theater that interacts with Guests actors. Furthermore, it is responsible of providing an environment where guests could use to watch a movie. As a result, one behavior block representing the watch_movie use case was created to capture this behavior. The scenario has three message exchanges between the guests and the movie theater. First, the movie theater gets a notification of arrival. When all the guests arrive, it sends “movie_start” message and starts the 120-min timer. A timer is also started at Guests, when they receive the message, which counts up until their exit time that is randomly distributed between 118 to 122 min. At the end of their exit time, they leave feedback by sending a message back to movie theater block.
Individual states of the movie theater, on the left, and guest agent, on the right, executing this behavior are captured in Figure 18. Number, one through four are used indicate the conditions, event triggers, and operations and their description are given as follows:

- **Movie Theater**
  1. In “WaitFor_arrival” state the theater counts the “arrive” messages guests send. After each message, the theater checks if the room capacity “count” has been reached and exits the state.
  2. When the room is full, sends the “movie_start” message to each guest.
3. “ShowMovie” state stays active for 120 minutes. This behavior is assigned using a
time trigger function “tm(duration)”. During this state it starts collecting the
feedback from guests who are leaving before the movie ends.

4. Waits until feedback is collected from all guests

- Guest Actor
  1. Each guest notifies the theater when they arrive.
  2. They wait for all guests to arrive.
  3. They leave the movie theater randomly between 118 to 122 minutes after the
     movie starts.
  4. They give a feedback with a score between 1 and 10 before exiting.
Figure 18 Movie Theater and Guest Agent Behavior

Figure 19 shows the expected sequence of events and states of guest actor and the movie theater. Verification of the model includes checking the correctness of the event sequence indicated by the rectangle box. The correct behavior is guests with “leaveTime” less than 120 minutes sending their feedback before and the remaining sending it after the movie is over. The scenario was executed 31 times representing a day with 11 shows using a 30 guest capacity. However, due to generated random numbers being the same, no difference was observed between 31 iterations.
Figure 19 Expected Output
Three groups of tests were conducted to eliminate design technique as the potential cause for errors. First, to confirm independency between show times for other tests, the significance of correlation between observed error counts (y) and show times (x) were tested. A hypothesis test was designed as follows:

\[ H_0 = \rho = 0 \text{ and } H_a = \rho \neq 0 \]

where \( \alpha = 0.05 \), \( t_{9,0.975} = \pm 2.262 \)

Correlation coefficient of the sample, \( r \), was calculated using the following formula (UA, 2015):

\[
r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{4.1}
\]

\[ r \approx -0.31105 \]

Confidence limits for \( r_{9,0.975} \) was calculated using the following formula (UA, 2015):

\[
r_{n-2,0.975} = \frac{-1 \pm t_{n-2,0.975}(\sqrt{n-2})}{n-1} \tag{4.2}
\]

\[ r_{9,0.975} \approx (-0.7786, 0.5786) \]
Where \( n \) is the sample size, calculated \( r \) is found within the confidence limits suggesting not enough evidence supporting a correlation between observed errors and show times. Later, the correlation between successive error counts is tested using first order autocorrelation coefficient. The correlation Eq. (4.1) is modified to test for observations with lag 1 for \( x_i = \text{Error Count}_i \) and \( x_{i+1} = \text{Error Count}_{i+1} \) (UA, 2015) as follows:

\[
r_1 = \frac{\sum_{i=1}^{n-1}(x_i - \bar{x}_i)(x_{i+1} - \bar{x}_{i+1})}{\sqrt{\sum_{i=1}^{n-1}(x_i - \bar{x}_i)^2} \sqrt{\sum_{i=2}^{n}(x_i - \bar{x}_{i+1})^2}} \approx -0.08139
\]

Similarly, confidence limits are calculated by adjusting Eq. (4.2) to test \( r \) at different levels of \( k \), time lag, as follows (UA, 2015):

\[
r_{n-k-1,0.975} = \frac{-1 \pm t_{n-k-1,0.975}(\sqrt{n-k-1})}{n-k} = (-0.7786, 0.5786)
\]

The two calculations are repeated for different values of \( k \) and the results are plotted as shown in Figure 20. Additional to confidence limits, the \( r \) values are also compared with simple approximation limits, \( \pm 2/\sqrt{n} \). All \( r \) values for lags 1 to 7 are found within the confidence limits suggesting not enough evidence to recognize an autocorrelation between occurrences.
Grouping observed errors according to the guest “leaveTime” shows that all errors occurred when the leaveTime is equal the movie duration, 120 seconds. Eight out of eleven shows has guests with wrong behavior and the highest guest count is observed during Show 8. A final test is conducted testing significance of correlation between counts of guests with leaveTime at 120 versus observed errors, which are captured in Table 2.

Magnitude of $r$ and confidence limits are found -0.008, (-0.7786, 0.5786), using (1) and (2) respectively. Tests results indicate not enough evidence of correlation between the two outcomes. Since all three tests have failed to reject $\rho = 0$ there is not enough evidence suggesting a design methodology error. Furthermore, with 95% confidence this is rather due to Rhapsody’s back-end event execution ordering logic.
Table 2 Experiment Results

<table>
<thead>
<tr>
<th>Show Id</th>
<th>$y = \text{Guest Count with wrong behavior}$</th>
<th>$x = \text{Guest Count with leaveTime at 120 minutes}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show 1</td>
<td>6 Guests</td>
<td>8 Guests</td>
</tr>
<tr>
<td>Show 2</td>
<td>4 Guests</td>
<td>11 Guests</td>
</tr>
<tr>
<td>Show 3</td>
<td>4 Guests</td>
<td>8 Guests</td>
</tr>
<tr>
<td>Show 4</td>
<td>0 Guests</td>
<td>6 Guests</td>
</tr>
<tr>
<td>Show 5</td>
<td>0 Guests</td>
<td>11 Guests</td>
</tr>
<tr>
<td>Show 6</td>
<td>2 Guests</td>
<td>4 Guests</td>
</tr>
<tr>
<td>Show 7</td>
<td>1 Guest</td>
<td>12 Guests</td>
</tr>
<tr>
<td>Show 8</td>
<td>7 Guests</td>
<td>8 Guests</td>
</tr>
<tr>
<td>Show 9</td>
<td>0 Guests</td>
<td>8 Guests</td>
</tr>
<tr>
<td>Show 10</td>
<td>2 Guests</td>
<td>4 Guests</td>
</tr>
<tr>
<td>Show 11</td>
<td>2 Guests</td>
<td>9 Guests</td>
</tr>
</tbody>
</table>

Maintaining correct sequence and synchronization between various behavior blocks, where more than one time based conditions, require adding extra messages or triggered operations within the modeled behavior. As a result, modifying derived behavior-in order to maintain its validity only for an issue within Rhapsody applications-can jeopardize efforts to capture the true representation of the actual system. Furthermore, they are required for all systems sharing the same behavioral patterns and not just the one example provided in this section. Therefore a generic solution is proposed rather than a quick work around adjusted to only one particular case. This solution also aligns with the purpose of this study for creating re-usable behavioral models.
**Proposed Solution**

Rather than cluttering the correct representation of the system with extra operations or events, in order to maintain synchronization when modeling SD-ABM systems, method provides an explicit behavior named “update_time” as demonstrated for movie theater system UCD captured in Figure 21.

A behavior is created which consists of an operation, “increment_clock” and a message “send_update” as captured in Figure 22. Although this behavior can be added with the remaining behavior at the initial UCD design phase, it can also be added to completed models. In such cases, the same design flow is used, and the behavior allocated to the location part of the main system.

During final STM update the behavior is added as a parallel state to existing states of location block and for all remaining parts and members of the environment it is added as an owner state named “Active”.

Figure 21 Update Time Use Case
For example, movie theater system has two members with time triggered behavior, the guests and the room. The room is responsible of updating the local time and updating the guests by sending a message as previously captured in Figure 22. As a result, the time trigger function “tm()” is only used once by the room to increment local clock in the system. As captured in Figure 23, this behavior is added as parallel states to the existing states of the room. On the other hand, in the guest STM the Active owner class is added to track any messages send from room and update the guest clock according to local time (see Figure 24). Finally, all the remaining tm() operations are changed to condition based triggers using local time “tnow” variable.
Additional to fixing synchronization issues, separation of this behavior can support modeling efforts with two main areas. First, if designed models are to be exported to a different simulation software, system behavior can be separated and exported explicitly. Furthermore, update time can also be exported explicitly to be used in modeling other systems. Rhapsody specific behavior such as “update_time” or “generate_variate” can be allocated to a unique system component or grouped under Location.
Figure 24 Guest State Diagram

```c
send_update/setTnow(params->tnow);

WatchingMovie
leaveTime=gen_uniform(118,122);
leaving(gen_uniform(1,10)) to proom
[tnow==leaveTime]/
FILE * pFile;
pFile = fopen
("results.txt","a+")
fprintf(pFile,
"Iteration %d %d\n",
it,leaveTime);
fclose (pFile);
printf("%d
%d\n",tnow,leaveTi
me)
```

waitingMovie
movie_start
arrival to proom
[it>10]
CHAPTER FIVE: POPULATION DYNAMICS CASE STUDY

This sections uses a hypothetical case study to demonstrate the approach for developing and simulating an AB-SD hybrid model of a selected system using SysML. Wild life has been a commonly studied area in AB modeling (Akbas et al., 2015). Therefore, a hypothetical example is created to demonstrate the methodology focusing on the status of giraffe population in Africa over time. Five facts (GCF, 2014) and two assumptions about giraffes are selected to describe specific procedures under different conditions.

- Leopard, lion, and hyena are among their predators.
- 60, 8 and 3% of calves are killed during their first, second and third year, respectively.
- Females mature at age 4 and gestation and nursing lasts for 57 to 65 and 4 to 52 weeks, respectively.
- Males start propagating after 7 years old.
- Average life span is 25 years.
- Assumed ratios for bull to cow and adults to calves are 1 to 1 and 10 to 1, respectively.

Requirements Analysis

According to the 5 requirement groups identified in Methodology chapter, all except first and last conditions listed above are grouped under behavioral requirements and the remaining two
are grouped under structural. On the other hand, the output of interest -status of population- would be added to the output requirements. A screenshot from Rhapsody® model tree capturing these requirements and their groups are shown in Figure 25.

Figure 25 Giraffe Population Scenario Requirements

Define Behavior

The behavior of the giraffe population observation system is modeled in seven steps. It starts with Use Case Diagram (UCD) design and demonstrates the process flows within the developed approach.
Create Use Case Diagrams (UCD)

The first step in the process flow is adding all actors identified by the stakeholders to the UCD. As a result, all agents identified in requirements analysis, such as Leopard and Giraffe are added as actors to UCD, as can be seen in Figure 26.

![Figure 26 Use Case Diagram - Action 1](image)

Once all are added, the possibility for grouping any actors is investigated. Given the scope of the scenario, the stakeholder’s interest in leopards, lions and hyenas do not go further than their total hunting success. Therefore, even though they were originally listed separately, these three actors can be grouped under the role “Predators” and represented as one actor. The resulting logic flow and UCD are captured in Figure 27 and Figure 28.
The modeler now can start with function identification per each actor. In this system, the actor identified as the Stakeholders is a type of giraffe conservation society and has two main duties. First, they are responsible of providing scientific findings on giraffe population and second, act as the observers who are interested in the outcomes of the model. Provided information include initial conditions within the environment that have an impact on system behavior, such as initial population count and male to female ratio. This behavior is represented using “set_initial_conditions” use case and an association link is added from the stakeholder actor. At any time during UCD design, modeler can add a list of these variables as a requirement under the
output requirements group if not added during requirements analysis phase. Figure 29 highlights the path taken back to Pick Actor action after analyzing Stakeholders actor.

The second actor, Predators, interacts with the environment by killing the giraffes. However, this behavior is explained from giraffes’ perspective using probability of death. Therefore their impact is not a part of the main focus in the environment. For this scenario, both situations for the final decision can be true. If the stakeholders suggest a possibility for model extension in the future focusing on any predator behavior, the modeler would keep this actor and take the path shown in Figure 30. On the other hand, if an extension towards this
direction is not within stakeholder interests, this actor can be deleted. It is important to note that, this should only be considered if no interaction exists between the selected actor and any member of the environment.

The final actor Giraffe is a part of the main environment scope, therefore is not included as an external member in the UCD. Later in architectural design this will be added as a part of the system structure. Provided assumptions suggest two functions, “reproduce“ and “die” and no information is given about the effect of their location information nor is included within environment interests. Reproduce is defined as a duration triggered function and die is defined as a success rate changing over time. Since the time is used by more than one a use case named “update_time” is added to keep the behavior synchronized. This is further explained in Time

Figure 30 Use Case Diagram Predator Definition
Synchronization section of chapter Methodology Verification. Finally, to capture the change in population count over time, “update_dynamics” use case is added.

After identifying all actors and use cases, each actor is connected with the corresponding use cases via the “Association” link to represent the relations. The resulting UCD and corresponding decision path is captured in Figure 31 and Figure 32, respectively.

![Figure 31 Finalized Use Case Diagram](image-url)
Figure 32 Giraffe Action Flow
Link Requirements to UCD's

A trace relation between identified requirements and the model elements such as “die” use case are added to demonstrate the matrix view as shown in Figure 33. After adding the relations, individually entered requirements can be brought to the UCD to verify that all has been captured and linked with the appropriate relation type to the associated use case as shown in Figure 34.

Figure 33 Requirements Matrix View
Figure 34 Finalized Use Case Diagram
Create Activity Diagrams (ActD)

The ActD for system specific use cases includes the actions belonging to “reproduce” and “die” behaviors as captured in Figure 35 and Figure 36 simultaneously. According to system description, pregnancy lasts between 57 to 65 weeks after propagation. To capture this duration “pregnancy_duration” and “pregnancy_status” variables are added to the reproduce behavior block.

![Reproduce Activity Diagram](image)

Figure 35 Reproduce Activity Diagram

There are four conditions that result in the death of a giraffe. Three of them are their chance of survival after a predator attack. If they survive all, they will die at the end of their natural life span. To capture this behavior, four variables, “age”, “survival_chance”, “survival_duration” and “life_span” are added. If the modeler is using Rhapsody with the Harmony Profile, embedded code for each action can be added during state definition phase.
The ActDs for remaining behavior blocks, such as update_time and update_dynamics, are not created nor modified for this example. After UCD design, previously modeled behavior of those blocks are imported to be reused for modeling the giraffe observation system.

**Generate Sequence Diagrams (SeqD)**

Depending on the level of detail required, the behavior and conditional rules can be planned using SeqD. Although this is not required, it would lay the grounds for mapping the rules for state-based behavior and support designing efforts. For example, the default SeqD generated for “die” ActD using Harmony profile would include RNDsurvival() operation as captured in Figure 37. Note that the “die” message- originally included in the black-box ActD- is an internal message and is not included in the initial SeqD.

According to system definition, the chance of survival increases as calves grow older. The ones who survive first year get a new value for the survival_chance and this loop continues until they die because of old age. Since the embedded code used in RNDsurvival() operation does not
change, to avoid clutter in the model, attack success can be created as the operation parameter as shown in Figure 37.

![Figure 37 Sequence Diagram of Die Behavior](image)

Figure 37 Sequence Diagram of Die Behavior

Figure 38 captures the modified SeqD for the die ActD. First, die behavior block is responsible of identifying the survival_chance for a newborn calve with a 60% attack_success value. At age 52 weeks, RNDsurvival(8) is executed to calculate their chance of survival during second year. The same logic is applied throughout their lives with decreasing attack_success rates. During any age, if the outcome of their survival chance is 0, they die at the end of their survival_duration value.

Following the same procedure the SeqD for reproduce behavior block is created. There are four conditions the system must satisfy before executing the propagate() operation. After female calves reach the end of fourth year, if they are not pregnant and there are adult males in the system, they initiate propagation. Following the pregnancy duration, they give birth. This sequence repeats until they reach the end of their life spans.
Figure 38 Modified Sequence Diagram of Die Behavior
Pregnancy_duration and pregnancy_status variables were already identified and added during ActD design phase. Additional to these, four more variables gender, age, life_span and adult_male_count were identified using the interaction operators. Age and life_span variables and their condition iteration were already added to the overall system when they were added to the die behavior block. Hence, this loop operation was not included in the final SeqD. Remaining two variables were added to the reproduce behavior block (Figure 39).

The SeqDs for update_dynamics and update_time behavior blocks are explained in the Time Synchronization section of Methodology Verification chapter.
Create Ports and Interfaces

System’s reaction to any predator was not included in the system focus identified during the UCD design. As a result, the associated behavior blocks do not require a message exchange (Figure 40). Therefore, generated IBDs only include the parts of behavior blocks without any connection to an actor.

![IBD Diagram](image)

Figure 40 Die Behavior Internal Block Diagram of Actual Scenario

This would be different if further information was available on predators, such as their attack frequencies or impact of attack success on time between attacks. Such interaction would initially be captured in the die behavior block ActD as a message action and later be added to the SeqD. Additional to the block part (sender), the resulting IBD would have included the predator actor (receiver) as can be seen in Figure 41.
Define States

Statecharts belonging to the giraffe population observation system, update_dynamics, set_initial_conditions, die and reproduce are shown in Figure 42, Figure 43, Figure 44 and Figure 45, respectively.

In the sample giraffe population system the rates and stock variables are set to update once per week (tm(7)) as captured in Figure 42.
According to the requirements identified during the first part of the proposed process flow, the giraffe population requires two unique operations for setting the initial conditions for actors, which are define_gender() and define_initial_age() as can be seen in Figure 43. For the purpose of this study ages of the alive giraffes at time 0 were assumed uniformly distributed between new born and 22 years. Similarly the gender was assigned randomly following U(0,1), keeping the 1 to 1 bull to cow ratio.
Die behavior is designed to have two states as alive or dead. According to requirements, while alive, the actors can die due to an attack or old age. The change in survival probability per age group was explained in detail in the previous section. Three conditions were added triggering a transition to dead state in order to capture the corresponding sequence and logic (Figure 44). For example, if they survived the attacks for a year, their survival chances are recalculated. With the same token, if they survived the attacks encountered during their lifetime, they finally die after reaching their lifespan.

![Die State Diagram](image)

Figure 44 Die State Diagram

Finally, reproduce behavior is designed to have three states. First the female waits until reaching maturity. After pregnancy they stay in the nursing state for the duration they were assigned. However, a control condition was added for the triggers leaving waiting_age and nursing states, checking the availability of mature enough adult males in the system (Figure 45).
It is important to note the impact of the requirements analysis step in the process flow. For example, the current design assumes none of the cows are pregnant or nursing at time 0. STM design would have been different in cases where further information on pregnant to nursing ratios among cows are provided and is an interest to the stakeholders.

**Behavior Verification**

Simulated SeqDs and statecharts are used to verify independently modeled behavior. A screenshot from the simulated reproduce behavior output can be seen in Figure 46. Following the age requirement fulfillment, the cow executes propagate() operation and transitions to pregnancy state. After waiting till the end of pregnancy, behavior transitions into nursing state, confirming the designed behavior. An end state is only included in the set_initial_conditions
behavior, therefore all remaining behaviors loop between different states according to existing conditions. As the final step, properties of all variables are checked for any errors.

Figure 46 Simulated Sequence Diagram for Reproduce Behavior

Overall, the purpose of behavior analysis can be summarized as following:

- Identify system requirements
- Identify system scope and boundaries
- Identify the modularized actions and reactions of the system to the external triggers
- Identify its interaction with the surrounding environment and conditions
- Derive resulting behavior from findings gathered above for verification
Define Structure

Structure modeling starts with decomposing the system according to its physical components using Block Definition Diagrams. Later previously modeled independent behavior is merged and distributed to the responsible part of the system.

Create Block Definition Diagrams

Giraffe population observation system is decomposed using Type III decomposition structure due to the two way dependency between the giraffes and their location. Initially eighty giraffes are created sharing one location, Africa, as can be seen in Figure 47.

Figure 47 Block Definition Diagram
Allocate Behavior

After system decomposition independent behavior blocks are merged under the main system block “Giraffe_Population_Observation”. Later using white-box ActDs each activity is allocated to the responsible system component.

Create White-Box Activity Diagrams (ActD)

First, previously created ActDs are duplicated and renamed as White-box ActDs. Later, a swim lane is added for each system part, such as Giraffe, and operations are placed-by moving- under the responsible block. For example, one operation RNDSurvival() is identified and an event message “die” under the die behavior (Figure 48 (a)). Since chance of survival after an attack is unique to each giraffe, the owner of the operation is itself. In the previous section, provided methodology for modeling rate attributes was described, such as death_rate being derived from the population behavior. As a result the message event “die” is placed to the receiver component, location, leaving an activity under Giraffe (Figure 48 (b)).
These steps are repeated for all remaining ActDs, `update_time`, `set_initial_conditions`, `update_dynamics` and `reproduce`, as can be seen Error! Reference source not found., Figure 49, Figure 50, Figure 51 and Figure 52, respectively.
Figure 50 White-Box Activity Diagram of Set Initial Conditions

Figure 51 White-Box Activity Diagram of Update Dynamics Behavior

Figure 52 White-Box Activity Diagram of Reproduce Behavior
Generate Sequence Diagrams (SeqD) and Create Ports and Interfaces

After each behavior is allocated to the responsible part of the system Harmony profile is used for generating the SeqDs and for creating the ports and event interfaces.

Define States

Location part of the system is responsible of executing two behaviors. After starting the simulation it transitions to an “Active” state where it performs Update_time and update_dynamics behaviors, which are integrated as and-states. Update_dynamics is responsible of updating the population variable and the weekly rates on the seventh day every week (Figure 53).

Figure 53 Location State Diagram
The behaviors of giraffes are activated when the simulation clock starts running at the location. The initially deployed giraffes, who are assumed to be already in the environment, are assigned a random gender, age and survival_chance (based on the initial age group) to represent a uniformly distributed population. Furthermore, gender of a calf born after time 0 is randomly selected based on the cow to bull proportions within the population. This logic is designed to satisfy the ratio requirement identified in problem description.

Contradictory to using “and-states”, such as in Figure 53, the reproduce behavior was added as a subset to being alive (see Figure 54). Later, alive state was further divided into two “and-states” and the top portion was used to show the gender of a giraffe after maturity to demonstrate simulated view when in parallel states.

**Verify and Validate System**

Three tests are conducted to validate the behavior designed for Giraffe Population Observation System. First the model is verified using the simulated SeqD and statecharts. Later, results from 30 simulation iterations are collected using an initial population size of 140 giraffes for the second and third tests. Three values at any given time $t$ in days for population count, births and deaths per week, are collected and their averages are plotted in Figure 55. Two expected behaviors are tested as follows:
Figure 54 Giraffe State Diagram
• Correct representation of agent (giraffe) behavior was captured- When creating initial population, none of the female giraffes are pregnant. Therefore, the output is checked for any births occurring before the minimum pregnancy duration, 57 weeks. As can be seen from the plotted output, no births can be observed up until the minimum required pregnancy duration indicated with an orange arrow.

• Correct representation of population dynamics was captured- The synchronization between birth/death rates and population count are checked to confirm correctness of the SD calculations. For example, during the time indicated within the grey box, three giraffes die on different weeks of the sixth year. Overall population count also decreases by three by the end of year six as a result of the change in death rates on matching weeks.
Figure 55 Giraffe Population Behavior
CHAPTER SIX: TRAINING MANAGEMENT CASE STUDY

In this section the application of the developed approach is demonstrated using a real-life case study focusing on a training management project which was planned and executed when a large-scale company had decided to adapt a new software technology in 2009. The project required 1255 employees’ attendance from different divisions and backgrounds. This new technology was the same as to what engineers had already been using; however, the processes were changed. When they were used to working on locally saved files, and sharing these documents mostly using emails, employees were asked to do all using this new technology. In addition to the extra work created by the efforts spent on a new technology, each employee was asked to attend an eight-hour (full-day) training. At the end of four fiscal years (FY), in 2012, only 1007 employees out of 1255 were trained where the total training capacity was over 2300 seats (Figure 56). Furthermore, by 2011, more than 29% of trained were returning for a second training. Obtained data included versions of a Microsoft Excel sheet saved at different times over the project duration, created for attendance tracking. The purpose of the modeling effort focusses on supporting project management team by simulating the training bubble to be used for training scheduling.
Two characteristics that are most commonly observed in complex systems are emergence of a pattern and continual appearance of new entity kinds (Levin, 2002) or large number of interacting entities (Morel and Ramajujam, 1999). Emergence was explained as being dynamic behavior of balanced negative and positive feedback rather than being the absence of tension (Newell, 2008). Because of the variety in forms of complexity, one cannot conclude that all complex systems are adaptive (Levin, 2002). Furthermore, complexity in systems cannot be explained by chaos (Bak, 1996, p. 31), meaning systems with simple dynamics can be very complex thus they do not have to be chaotic to be accepted complex (Morel and Ramanujam, 1999).
The existence of complexity in learning systems, a phrase introduced by Davis and Simmt (2003) describing collective classroom components, is advocated by also other researchers (Burns and Knox, 2011, Davis and Sumara, 2006). Newell (2008), following Davis, Simmt and Sumara’s published arguments on how individual learner and teacher dynamics interacts and emerges as learning, evaluates the potential benefits and challenges of accepting this theory.

Unlike immediate training climate, studies on organizations as systems has a longer history, and today, they are accepted as “dynamic systems of adaptation and evolution that contain multiple parts which interact with one another and the environment” (Morel and Ramanujam, 1999). Furthermore, their nested structure continuously interacts with other macro and micro, systems and sub-systems, respectively (Folke & Folke, 1992). New systems may arise from emerging dynamics as part of the system, due to change processes occurring with an organization (Dooley and Van de Ven, 1999). Bot (2012) has listed the most common properties of complex systems in a study where he looked into the complexity of learning a third language. Training management was evaluated with respect to each property listed by Bot, and the findings were captured in Table 3. Explanations and case examples were supported with findings from literature. The findings support the theory of training management emerged as complex adaptive-derived from its evolution through a life-cycle iterations-system that interacts with other complex adaptive systems such as technology and economy.
### Table 3 Complex System Properties, Adapted from Bot, 2012

<table>
<thead>
<tr>
<th>Complex System Properties</th>
<th>Training Management System Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex systems are sets of interacting variables.</td>
<td>Training management interacts with organization system (macro) and knowledge transfer variables (micro).</td>
</tr>
<tr>
<td>In many complex systems, the outcome of development over time cannot be predicted … because the variables that interact keep changing over time.</td>
<td>Although there are techniques to support training planning often times changes in duration, cost, training performance occur.</td>
</tr>
<tr>
<td>Dynamic systems are always part of another system, going from sub-molecular particles to the universe.</td>
<td>Training management system is part of knowledge transfer system.</td>
</tr>
<tr>
<td>As they develop over time, dynamic subsystems appear to settle in specific states, which are preferred but unpredictable, so-called ‘attractor states.’</td>
<td>Employees within an organization create a unique knowledge share structure creating a culture which emerges individual and organization’s learning state. Weick (1979)</td>
</tr>
<tr>
<td>Systems develop through iterations of simple procedures that are applied over and over again, with the output of the preceding iteration as the input of the next.</td>
<td>Training is applied in organizations in iterations, the lessons learned from each experience (outputs) feeds the following management strategy as inputs. (Armstrong, 2003)</td>
</tr>
<tr>
<td>The development of a dynamic system appears to be highly dependent on its beginning state. Minor differences at the beginning can have dramatic consequences in the long run. …</td>
<td>If started without well planning the effects of each variable and their interactions, training efforts will fail costing the investments and time of the stakeholders.</td>
</tr>
<tr>
<td>In dynamic systems, changes in one variable have an impact on all other variables that are part of the system: systems are fully interconnected.</td>
<td>In training management, change in one variable for instance organization’s climate or available resources will trigger a change in the whole system will affect outcomes.</td>
</tr>
<tr>
<td>In natural systems, development is dependent on resources: … all natural systems will tend to entropy when no additional energy is added to the system.</td>
<td>Training management rely on the resource availability, depletion of any resource will trigger system’s state to change to ‘steady-state’.</td>
</tr>
<tr>
<td>Systems develop through interaction with their environment and through internal self-reorganization.</td>
<td>Training has emerged from interaction of systems such as learning, organization and technology. Through time its internal interactions derived management variables (Dooley and Van de Ven, 1999)</td>
</tr>
<tr>
<td>Because systems are constantly in flow, they will show variation, which makes them sensitive to specific input at a given point in time and some other input at another point in time.</td>
<td>Due to continuous change in it is variables such as humans and technology same management approaches will result in varying outputs.</td>
</tr>
</tbody>
</table>
Model Development

Structural and behavioral characteristics of the systems, such as input analysis, are identified first, through requirements analysis. Later identified behavior is individually modeled under behavior analysis. After verification, the behavior is integrated and allocated to parts of the system in structure analysis. Finally, the model is verified and validated and the findings are discussed.

Requirements Analysis

The majority of the work completed in this phase consists of input analysis. Each of the original training status report snapshots included training dates of ranging from 3,000 to 10,000 employees, which also consisted of contradicting or duplicate information. First, using the latest report, a skeleton list including the generated id’s of employees located in US, who had either attended training or was required to attend in the future is generated. Later all ID’s in each snapshot that were not included in this list are deleted. Each duplicate id is deleted after confirming all attendance data are successfully copied in to the remaining. Due to the length of the project, some sessions had been renamed over the years, and to avoid double-counting, a total of 13,173 data points are checked individually for potential duplicate attendance information. Using the data population increase, training attendance, attendance probability and training schedule are studied as further explained respectively in the following sections.
**Population Increase**

A change in the population count was observed according to the snapshots taken randomly during the four year project timeline. During project kick-off in October 2009, the initial number of employees, who would be invited for trainings, was 476. This number almost tripled by the end of fiscal year (FY) 2012. Although there were new hires, the majority of this increase was driven by the changes in project scope. As time progressed employees from additional departments were also included. The dates of the snapshots and corresponding count of employees included in each report is shown in Table 4.

Table 4 Population Increase per Snapshot

<table>
<thead>
<tr>
<th>Snapshot Date</th>
<th>Invited Trainee Count</th>
<th>Cumulative Time in Between (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/1/2009</td>
<td>476</td>
<td>0</td>
</tr>
<tr>
<td>7/26/2010</td>
<td>784</td>
<td>571</td>
</tr>
<tr>
<td>8/27/2010</td>
<td>785</td>
<td>603</td>
</tr>
<tr>
<td>10/14/2010</td>
<td>865</td>
<td>651</td>
</tr>
<tr>
<td>3/2/2011</td>
<td>948</td>
<td>790</td>
</tr>
<tr>
<td>7/1/2011</td>
<td>1,000</td>
<td>911</td>
</tr>
<tr>
<td>10/18/2011</td>
<td>1,058</td>
<td>1,020</td>
</tr>
<tr>
<td>11/2/2011</td>
<td>1,089</td>
<td>1,035</td>
</tr>
<tr>
<td>4/9/2012</td>
<td>1,136</td>
<td>1,194</td>
</tr>
<tr>
<td>8/1/2012</td>
<td>1,192</td>
<td>1,308</td>
</tr>
<tr>
<td>9/25/2012</td>
<td>1,255</td>
<td>1,363</td>
</tr>
</tbody>
</table>

A scatter plot of total employee count per date can be seen in Figure 57. According to the R-square value, more than 99% of the change in invitation count can be expressed in terms of days
passed. In other words, there is not enough evidence to reject that any future total invitation count could be predicted by the days passed.

As a result, the constant rate, 0.5661 per day, from the fitted equation can be expressed in SD as can be seen in Figure 58.

\[ y = 0.5661x + 476.39 \]
\[ R^2 = 0.992 \]

**Figure 57 Total Employee Count per Snapshot**

**Figure 58 SD Representation of Population Increase**
Training Attendance

The first focus in the attendance analysis is the behavior of employees who had attended a training more than once over the years. However, out of 1255, only eight employees had attended training three times and none had attended more than that. Due to the limit of this data and its ratio to the total sample size, the dates from their third attendance is not included in the study. The date of the first training and the time, in days, until the second training are mapped and checked for any outliers.

A scatter plot is created to test dependency between the first training date and the time until the second, Delay, as captured in Figure 59. The total sample size is 124 with a mean and standard deviation at 436 and 230 days, respectively. The R-Square value, 0.133, of the fitted equation is not significant enough to reject dependency.

![Figure 59 Delay vs Initial Training Date](image)
However, considering the high mean and standard deviation in delay, only a part of the data from FYs 2011 and 2012 could have been recorded. In other words, since the study has ended at the end of FY 2012, if any had attended their first training during that year any delay larger than 360 days would not be recorded. Furthermore, this limit for FY 2011 would be 720 days. As a result, as the initial training date increased, this limit had to decrease. To confirm this theory, the dependency between the two using the data from FY 2009 and 2010 is tested. The highest R-square value is observed using linear regression, at 0.0049, as captured in Figure 60. According to updated test statistics, dependency between the observed delay and the initial training date is rejected. As a result, the data collected from the first two FYs only is used for the remaining analysis under training attendance section.

![Time Till 2nd Training](image)

**Figure 60 FY 2009 and 2010 Delay vs Initial Training Date**

Next, the autocorrelations, $r$, for lag values, $j$, from 1 to 80 are calculated using equation (4.1). $r$ values for all lags including 1, which is 0.008 lower than the limit, are within the confidence
intervals calculated at 0.05 significance, using equation (4.1). A snapshot of the plot is shown in Figure 61.

![Autocorrelation Plot](image)

**Figure 61 FY '09 and '10 Autocorrelation Plot**

After completion of the independence tests, the data is ranked and imported into Arena’s Input Analyzer software to find the best fit. Figure 62 captures a histogram of data with the fitted distribution line and the best fit p-values are captured in Table 5.

![Delay Data Fit](image)

**Figure 62 Delay Data Fit**
Table 5 Attendance Distribution Fit Test Results

<table>
<thead>
<tr>
<th>Data Group</th>
<th>Sample Size</th>
<th>Distribution</th>
<th>Chi-Square P-Value</th>
<th>KS P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FY ’09 – ‘10</td>
<td>22</td>
<td>TRIA(113, 284, 1130)</td>
<td>0.227</td>
<td>&gt;0.15</td>
</tr>
</tbody>
</table>

Out of 1152 employees who attended training over the four-year project duration, 145 of them had attended a second training. As a result, identified delay distribution is distributed randomly only to 12.6% of the total trained.

Attendance Probability

The attendance probabilities is the second focus of the attendance studies. Different from the first focus explained in the previous section, where the behavior was distributed over the employees, the attendance counts are studied per training base. There are a total of 144 trainings offered over the four-year project, where, the attendance count mean and standard deviations are at 7.87 and 4.19, respectively. Following the same steps, a scatter plot of relation is created as can be seen in Figure 63. The highest R-square value, 0.0038, is achieved using a linear fit. There is significant evidence to reject dependency between attendance counts recorded and training dates.

Later a scatter plot is created to check for any autocorrelation for different lags, j, changing from 1 to 142 (Figure 64). Only one r value is observed slightly out of the confidence limits at lag 36.
Probability of getting an $r$ value within the confidence limits is therefore 0.993. At 0.05 significance, this suggests enough evidence to reject any autocorrelation.

Figure 63 Attendance Count per Training

Figure 64 Attendance Count Autocorrelation Plot
After completion of the independence tests, the data is ranked and imported into Arena’s Input Analyzer software to find the best fit. Figure 65 captures a histogram of data with the fitted distribution line and the best fit p-values are captured in Table 6.

![Figure 65 Attendance Count Data Fit](image)

### Table 6 Attendance Count Distribution Fit Test Results

<table>
<thead>
<tr>
<th>Data Group</th>
<th>Sample Size</th>
<th>Distribution</th>
<th>Chi-Square P-Value</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>FY '09 – ‘12</td>
<td>144</td>
<td>UNIF(0.999,16)</td>
<td>&gt;75</td>
<td>0.0081</td>
</tr>
</tbody>
</table>

During model validation and verification the average of total attendance collected at the end of FY 12, from 31 iterations, does not reveal significant evidence to reject the validity of the model. The t test result is -1.1529, between (+/-) 2.3556 t value at 0.05/2 significance. However, when the results are plotted comparing iteration average with the actual data, an unexpected behavior is observed. Figure 66 captures the resulting plot. While the simulated data followed a relatively linear increase in cumulative attendance the actual data showed an increasing increase.
Due to the scattered behavior observed at the tail of the its autocorrelation plot, initially, a serial correlation following a specific pattern, similar to a seasonal correlation, was not expected. However, when each FY year is separately analyzed multiple correlation coefficients over the confidence interval are observed for FY 09 and 10. The values are even higher when the two were combined. On the other hand, FY 11 and 12, when studied separately and combined, shows no significant correlation. The two correlograms are shown in Figure 67, on the left and right respectively.
FY 09 and 10 are tested for its significance in fitting a time dependent equation, due to failing independency requirement of data fitting. Highest R-square value, 0.3029 is achieved using an exponential relation between the time of training and attendance counts per training. Although R-square value does not find enough evidence to reject the data is a good fit, it is also not enough to confirm a good fit (The Pennsylvania State University, 2015). The first issue is that R-squared value displayed is calculated using continuous prediction values. The fitted equation can be used if the model is SD based only. Since the case model is AB-SD with continuous interaction, a decimal value would be rounded to the nearest integer, which might result in a lower R-squared. Hence, the sum of squares (SSR) and total sum of squares are calculated for the rounded values of predicted attendance (Table 11 in Appendix E).

The new R-squared value calculated from the rounded predictions is found by dividing the sum of SSR by SSTO. The new fitted equation is $y = 1.8206e^{0.0029x}$ with R-squared at 0.681. Attendance, rounded predicted attendance and the initial trend line is shown in Figure 68.
The remaining part of the data collected during FY 11 and 12 are later, combined and re-fitted to find the best distribution. With 0.0155 and 0.454 square error and Chi-square p-values, respectively, Uniform (0.999, 16) fit is again found the best fit.

Training Schedule

The project team had never picked a random date for a training, rather, most of the trainings were scheduled during the months without vacations or per request or according to a strategic decision. As a result, an input analysis is not conducted for the training schedule. Dates in between each training are calculated and used as is within the model. A full list of training dates and attendance counts can be found in APPENDIX C: TRAINING SCHEDULE.
Define Behavior

The first step in behavior definition is UCD creation. Following the proposed decision flow, employee agent is removed as an actor, since they are a part of the system scope. Later, their behavior, “attend_training” is added as a UC. Similarly, the company is also removed as an actor and its specific behavior “train” is added as a UC. Once all system specific behavior was captured, pre-modelled common behaviors such as “update_time”, “update_dynamics” and “set_initial_conditions are imported. The resulting UCD is shown in Figure 69.

![UCD Diagram](image)

Figure 69 Training Management System UseCase Diagram

After identifying the UCs, their relations to requirements are established. Once all functional requirements are traced back, their black box ActD are designed. First behavior is “attend_training” whose behavior is based on three conditions. First, class availability is checked and does not continue with registration unless a seat is available. Second, a decision whether to show-up to a registered class is made. Finally, re-taking training decision is made. If no-show or re-take decisions are made, the behavior goes back to the beginning and waits for a seat availability. This flow is captured in the black-box diagram as shown in Figure 70. The second
behavior “train” is responsible of sending invitations and calculating how many employees show-up at a training (Figure 71).

The final black-box, set_initial_conditions, is responsible of assigning an employee id to agents and importing training schedule that is previously identified in the requirements analysis (Figure 72). The remaining two behaviors update_time and update_dynamics use imported ActD that are previously captured in the Methodology section.
Any interaction or communication with an external agent, actor, is not identified during UCD design phase. As a result, generated SeqD only lists identified activities within each behavior block. With the same token, IBDs only shows the behavior itself without any connection between them and an actor. In this case study, all variables and operations are added during STM design phase. Algorithms and corresponding attributes used in each operation are as follows:

Algorithm 2 Check Class Availability

1: seat_id: seat number per training
2: class_capacity: 16
3: if seat_id < class_capacity then
4: Return 1
5: else
6: Return 0
7: end if

Algorithm 3 Register Session

1: registration_list[16]: array storing employee ids who have registered per training
2: $i$: registration_list array number
3: Set $\text{registration\_list}[i] = \text{employee\_id}$
4: $i++$

Algorithm 4 Check Re-take

1: $U_1$: Generated random number from $U(1,100)$
2: a: Lower end of triangular distribution, 113
3: b: Higher end of triangular distribution, 1130
4: c: Mean of triangular distribution, 284
5: retake_probability: identified value for retake percentage, 20.6
6: delay: waiting time till second attendance
7: total_retakers: retake_probability % of trained
8: if total_retakers $\leq$ retake_probability $\times$ trained then
9: Set retake_decision=1
10: Generate $U_1$
11: If $U_1 < ((c-a)/(b-a))$ then
12: Set delay $= b - \sqrt{(b-a) \times (c-a) \times (U_2/100))} + 0.5$
13: else
14: Set delay $= b - \sqrt{(b-a) \times (b-c) \times (1-(U_2/100)))} + 0.5$
15: end if
16: end if
Algorithm 5 Calculate No-Show

1: attendance_count: Generated random number from U(1,16)
2: total_attendance: Cumulative value of attendance_count
3: date: simulation time
4: b: Higher end of triangular distribution, 1130
5: c: Mean of triangular distribution, 284
6: retake_probability: identified value for retake percentage, 20.6
7: delay: waiting time till second attendance
8: Generate U1
9: if time <= 730 then
10:     Set attendance_count = 1.8206 * exp (0.0029 * date)
11: else
12:     Generate attendance_count
13: end if
14: Set total_attendance = total_attendance + attendance_count

Algorithms 2, 3 and 4 are placed in corresponding operations of attend_training behavior. Later each operation are added as a transition response or rule to STM diagram design. For example, behavior would not change to “Registered_for_training” unless the class had seats available and the behavior would proceed after registering for the upcoming session. The STM captured in Figure 73 shows these operations and three states of attend_training behavior. The SD part of the simulation includes two stock variables training_bubble and trained. When an employee moves to the “Trained” state they increase the “training_rate” by 1. Similarly, if they decide to
re-take training after waiting for assigned delay duration, they move to “Waiting_training” state, increasing “return_rate” by 1. Two operations, “get_trained” and “decide_retake”, are responsible of these actions, respectively.

![Attend Training State Diagram](image)

The “train” behavior STM is designed to have two states (Figure 74). When the day, tnow, matches the date on the imported training schedule, it transitions to “In_training” state. After waiting 1 day in training, it goes back to waiting state until the next training day. Remaining algorithm, “Calculate No-Show”, is executed before the training starts on the same day.
Indirectly, a two way dynamic interaction between the AB and SD models is established by implementing these behaviors. First, AB to SD dependency is created by checking the ratio of retakers to Trained stock variable in the population under check_retake operation. Second, two operations decide_retake and get_trained are responsible of updating the two of the rates in SD, return_rate and training_rate, respectively, creating SD to AB dependency. Final SD model is achieved when the STM for update_dynamics is finalized resulting in a relation, which is shown in Figure 75. Each behavior block is verified prior to moving to next phase following the visual verification techniques discussed in Methodology section.
Define Structure

During UCD analysis, two agents, employee and company are identified as parts of the system. Their relation is identified as Decomposition Type III due to the two way dependency between them. As a result the BDD of training management system is designed with two parts, which are also connected to one another (Figure 76).

![Diagram](image)

Figure 76 Training Management System Block Definition Diagram

After structure identification, all behavior blocks are merged with the main system block, Training_Management_System. Following the steps proposed under methodology, behavior is allocated to responsible system part. The resulting IBD of the system parts are captured in Appendix E, Figure 86. Finally, behavior states are distributed and integrated using and-states in part STMs.
Employee STM

Each employee is assigned a unique ID at the simulation start, which is allocated from set_initial_conditions behavior block. Until simulation end notification, each employee stays in active state. However, when in Active state, they wait for an invitation from the company in order to be included in the population. After receiving an invitation they start executing the attend_training behavior. Finally, at the end of simulation each writes training attendance date(s) to a text file with their unique employee ids (Figure 77).

Figure 77 Employee State Diagram
Company STM

The company is responsible from importing the training schedule at the simulation start which was allocated from set_initial_conditions behavior block. It remains in Active state until reaching simulation end time, 1455 days and sends the simulation end event to each employee. When in Active state, it simultaneously executes three sub-states. First sub-state includes the behavior from update_time behavior block. At the end of each day, it updates the stock variables and rates and writes them to a text file with the current day’s number. The second sub-state is responsible of sending the invitations for the next training to each employee currently in population. The final sub-state is responsible of running the training operation.

It is important to note the direct relation between input analysis conducted under requirements analysis section and corresponding behavior allocation. For example, attendance count is studied using the attendance count data per training and the fit is distributed among the trainings but not to employees. As a result Calculate No-show operation is allocated to the company and is used in its STM. The resulting STM is shown in Figure 78.
Figure 78 Company State Diagram
**Verify & Validate**

Additional to verification completed under behavior design phase, visual and statistical tests are conducted on the integrated model. First, randomly 10 employees, out of 1255 created, are selected. Their and company’s simulated statecharts are watched simultaneously for any potential logic errors. A screenshot capturing this process is shown in Figure 79.

Second, simulated sequence diagram is used with 30 random employees, different from the first 10, with the company to verify the behavior sequence. For example, an agent selected with a small employee ID is expected to register for a training while some other would be waiting for training. With the same token, due to population size changes, an agent might not be invited at all. Figure 80 shows an example of the three cases simulated using a SeqD.
Figure 79 Simulated Statecharts
After verifying the model built, population, total attendance and total trained averages from 33 iterations are visually compared to the actual data. Due to its deterministic nature population count is not used for validating the model. It is important to note that, total attendance is studied as an input, however, as separately for different behaviors distributed among employees and
trainings. In other words, total attendance depends on agents’ decision to attend a training the second time as much as the count of employees who show up to the training. As a result, while the two are considered as inputs, the value of total attendance is an output to that combined behavior. With the same token, total employees trained depends on total attendance due to class size limitation and their chance to show up—both directly and indirectly. Figure 81 captures the outputs versus actual data plotted over the duration of the project.

![Simulation Outputs](image)

**Figure 81 Simulation Outputs**
The validity of the model is statistically tested after completing visual checks. Since the input suggested a non-stationary system, from FYs 09 and 10 to FYs 11 and 12, over a finite time, two tests are conducted for the two outputs, at the end of FYs 10 and 12 null and alternative hypotheses tests are:

\[ H_0: \bar{X}(t)_{replications} = X(t)_{data} \text{ and } H_a: \bar{X}(t)_{replications} \neq X(t)_{data} \]

A table including each replication value at \( t \) 730 and 1460 days can be found in APPENDIX D: SIMULATION OUTPUT. Table 7 captures the actual data collected, average and standard deviation of the replications, t-test results and t-value at 0.05 significance for 32 degrees of freedom. Each test value is within the confidence interval. As a result, there is not enough evidence contradicting the validity of the model and the alternative hypotheses are all rejected.

### Table 7 Output Analysis

<table>
<thead>
<tr>
<th></th>
<th>Total Attendance</th>
<th>Total Attendance</th>
<th>Total Trained</th>
<th>Total Trained</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FY 10</td>
<td>FY 12</td>
<td>FY 10</td>
<td>FY 12</td>
</tr>
<tr>
<td>Collected Data at ( t )</td>
<td>497</td>
<td>1152</td>
<td>470</td>
<td>1007</td>
</tr>
<tr>
<td>Replication Mean</td>
<td>496.879</td>
<td>1147.576</td>
<td>470.636</td>
<td>1005.303</td>
</tr>
<tr>
<td>Replication Std. Dev.</td>
<td>3.879</td>
<td>34.710</td>
<td>3.471</td>
<td>33.005</td>
</tr>
<tr>
<td>t-test</td>
<td>-0.191</td>
<td>-0.732</td>
<td>1.053</td>
<td>-0.295</td>
</tr>
<tr>
<td>t-value at 0.025</td>
<td>± 2.352</td>
<td>± 2.352</td>
<td>± 2.352</td>
<td>± 2.352</td>
</tr>
</tbody>
</table>
Results

One of the outputs studied in the simulation is count of employees currently waiting for training, often referred to as “training bubble” (Enos, 2011). The bubble consists of two groups of employees. First group, referred to as group 1 in this section, includes employees hired during the project timeline or added to due training scope changes. This group is studied under population increase section of input analysis. Total population count with respect to total trained per snapshot is shown in Figure 82. The bubble values for group 1 are calculated by subtracting total trained from the total population and is represented with a line.

![Figure 82 Training Bubble from Population Increase](image)

Project team would use the values from group 1 to schedule future trainings. As a result, majority of the group 1 bubble behavior is explained when plotted with the training frequencies (Figure 83, highest bar representing 10 trainings per four weeks). For example, as a result of regressive trainings offered till the end of FY 11 – until Day 1065- project team was able to drop group 1
bubble almost to half. In FY 12, due to group 1 dropping to 250, the training frequency was dropped to as low as zero. However, during the first quarter of FY 12 there was an unexplained increase observed in the group 1 bubble. Furthermore, towards the end of FY 12 - although the training frequencies were increased - group 1 training rate has continuously decreased. This resulted in a higher group 1 bubble value compared to end of FY 11.

![Figure 83 Group 1 Training Bubble](image)

One of the factors affecting unexpected bubble behavior is the re-takers and it was not accounted for in training planning. 25.67% of the employees trained in FYs 09 and 10 had re-attended a training by the end of FY 12. At the end of FY 12, the ratio of re-takers was 12.59% of the total training attendance. However due to the large delay between two attendances, averaging at 500.26 days, this ratio may have increased if data had been collected also for FYs 13 and 14.

Simulated bubble values consists both employees from group 1 and re-takers. Validated model is used to simulate its change till the end of FY 12. The outputs are plotted using the same graph.
as captured in Figure 84. Two unexpected behaviors that were identified in group 1 bubble counts are highlighted using two rectangles on the figure. First one is the spike observed in beginning of FY 12, captured in left rectangle. The same spike is not recorded in the simulated bubble. On the contrary, a small decrease is observed which corresponded to the training frequencies. Similarly, increasing training frequency towards the end of FY 12 has actually showed a decrease in the simulated bubble. This is highlighted using the rectangle on the right. Decrease in simulated bubble during a population increase suggests that re-takers had the majority in each training class that was organized for the new hires. Although the training frequency was increased towards the end of FY 12 it was not enough to compensate for the amount of seats re-takers were using.

Figure 84 Simulated Training Bubble
CHAPTER SEVEN: CONCLUSIONS AND FUTURE WORK

This dissertation describes the work on developing a methodology for modeling and simulating systems with agent-based (AB) and system dynamics (SD) modeling techniques using Systems Modeling Language (SysML). A methodology is provided, which extends Model-based systems engineering (MBSE) approach establishes a two way dynamic and continuous communications within the hybrid platform. Hypothetical and real-world examples are developed on Rational Rhapsody to demonstrate proposed methodology.

Main challenges in model development can be grouped in four areas. First is the increasing variation in backgrounds of stakeholders. Every individual or group of individuals adds the know-how from their perspectives in collaborations. Furthermore, they expect to see how their input-either previous analysis results or pure data-is integrated into the model and impact the system outcomes. After input analysis, provided methodology combines findings in two dimensions. First analysis results are integrated within a behavior. Secondly, they are used for identifying responsible system components. Resulting system model provides an output of a distributed behavior composed of integrated inputs. Second challenge is the increasing complexity of modeled systems. This dissertation provides an approach for managing this complexity and proposes a technique for identifying and modeling particular behavior and responsibilities of system parts.
The third challenge is more specific to long term projects. It is the need to maintain the coherency and efficiency of verified and validated models through behavioral and structural change requests. Proposed approach allows changing, verifying and validating modularized behavior independently. In other words, an independent behavior block where a change is requested can be modified, verified and validated and re-allocated to the structural component without impacting the validity in other behavior blocks.

The final challenge is reusability of modeled behavior. Today, modeling efforts often start from scratch even if same behavior exists in a previously developed model for a different case by the same person or group. Provided methodology models behavior and allocates it to a particular structure. This approach allows modeling the generic behavior rather than the particular case. Therefore, each behavior block can be separately exported and imported later to be used for a different case.

Two challenges were faced while working with Rhapsody. First was modeling a behavior, which follows a probabilistic distribution. An operation implementing random variate generation technique was added to overcome this limitation. The output from generated variates was compared to values collected from a simulation software, AnyLogic, and not enough evidence was found suggesting difference in means. The second challenge was maintaining the simulation time synchronization between different components of the system. A behavior block was provided representing a virtual clock that was responsible of updating time within each component.
Additional to its support during model development, provided approach can play an important role in identifying key factors deriving the system behavior and in providing insight to measures that can be collected for system evaluation, and analysis. Furthermore, holistic nature of provided approach allows the proposed methodology to be applicable to different areas of research. This work demonstrated its application for population dynamics and scheduling problems. Future applications can include modeling crowd behavior based on geographical locations. Influenza outbreak modeling can be an example to such application. Furthermore, the provided method can be extended for hybrid models with alternative configurations. First, current method’s performance can be evaluated when applied to develop a selected configuration, such as discrete event simulation and system dynamics combinations. Later, if necessary, provided steps can be adjusted specifically for each hybrid modeling technique combinations.
APPENDIX A: EXAMPLE MODELS – MOVIE THEATER SEQUENCE DIAGRAM
Figure 85 Movie Theater Sequence Diagram
APPENDIX B: EXPONENTIAL VARIATE DATE
Table 8 Generated Date Frequency

<table>
<thead>
<tr>
<th>Days</th>
<th>AnyLogic Transitioning Agent Count</th>
<th>Days</th>
<th>AnyLogic Transitioning Agent Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>26</td>
<td>46</td>
</tr>
<tr>
<td>1</td>
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<td>27</td>
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<td>3</td>
<td>11</td>
<td>29</td>
<td>48</td>
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APPENDIX C: TRAINING SCHEDULE
Table 9 Training Dates and Attendance Counts

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Table 10 Simulation Results per Replication

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APPENDIX E: TRAINING MANAGEMENT SYSTEM INTERNAL BLOCK DIAGRAM
Figure 86 Training Management Internal Block Diagram
APPENDIX F: ATTENDANCE COUNT REGRESSION ANALYSIS
Table 11 Attendance Regression Analysis

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International.


